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Some New Type of Contraction and Common Fixed Point Theorem in Fuzzy Metric Space 2022

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Abstract: *The main objective of writing this research paper is to prove the Common Fixed Point Theorem in Fuzzy Metric Space with a new type of contraction.*

Keywords: Compatibility mappings, Common fixed point, Fuzzy metric space, Self-mapping etc

I. INTRODUCTION

The **[5]** S. Banach fixed point theorem is important as a source of existence and uniqueness of theorem in different branches of analysis .In this way the theorem provides an impressive illustration of the unifying power of functional analytic method and of the usefulness of fixed point theorems in analysis .

The **[5]** S. Banach fixed point theorem or contraction principle concerns certain mappings of a complete metric space into itself .It states conditions sufficient for the existence and uniqueness of a fixed point .The theorem also gives an iterative process by which we can obtain approximations to the fixed point and error bounds etc. Compatibility is a powerful tool for generalization of **[5]** S. Banach contraction principle recently many authors generalized **[5]** S. Banach principle, use of commutatively and compatibility concept .Present paper introduce new type of contraction and proved common fixed point theorem in fuzzy metrics space.

The concept of fuzzy sets was introduced initially by Zadeh. Since then, it was developed extensively by many authors and used in variousfields. Specially**[7, 8]** introduce the concept of fuzzy metric spaces in different ways.

In present paper we use in**[8],** George and Veeramani modified the concept of fuzzy metric space which introduced by Kromosil and Michalek.

Further Jungck**[1]** introduced compatibility in metric space .He proved common fixed point theorems in to metric spaces .

1.1 Preliminary Definition

Definition: 1.A binary operation $*: [\mathbf{0},\mathbf{1}] \times [\mathbf{0},\mathbf{1}] \to [\mathbf{0},\mathbf{1}]$ is called a continuous t-norm if $([\mathbf{0},\mathbf{1}],*)$ is abelian topological monoid with 1 such that $\boldsymbol{a} * \boldsymbol{b} \le \boldsymbol{c} * \mathbf{d}$, whenever $\boldsymbol{a} \le \boldsymbol{c}, \boldsymbol{b} \le \boldsymbol{d}$ for all $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}$ in $[\mathbf{0},\mathbf{1}]$.

Definition:2. the 3-tuple $(\boldsymbol{X},\boldsymbol{F},*)$is called a fuzzy metric space if X is an arbitrary set , $*$ is a continuous t-norm and$\boldsymbol{M}$ is a fuzzy set on $\boldsymbol{X} \times \boldsymbol{X} \times (\mathbf{0},\infty)$ satisfying the following conditions:

(I) $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) > 0$,

(II) $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) = \mathbf{1}$ *if and only if* $\boldsymbol{x} = \boldsymbol{y}$,

(III) $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) = \boldsymbol{F}(\boldsymbol{y},\boldsymbol{x},\boldsymbol{t})$,

(IV) $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) * \boldsymbol{F}(\boldsymbol{y},\boldsymbol{z},\boldsymbol{s}) \le \boldsymbol{F}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{t}+\boldsymbol{s})$,

(V) $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},.): (\mathbf{0},\infty) \to [\mathbf{0},\mathbf{1}]$ *is continuous* ,

For *all* $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}$ *in* $\boldsymbol{X}$ *and* $\boldsymbol{t},\boldsymbol{s} > 0$.

COMPATIBILITY (1) [1]: Let A and B be a pair of self-maps of a complete fuzzy metric space$(\mathbf{X},\mathbf{M},*)$**.** Then $\boldsymbol{A}$ and$\boldsymbol{B}$ are said to be compatible if $\boldsymbol{limt}_{n\to\infty} \boldsymbol{F}(\boldsymbol{AB}\boldsymbol{x}_n, \boldsymbol{BA}\boldsymbol{x}_n, \mathbf{t}) = \mathbf{1}$, for all$\boldsymbol{t} > 0$whenever $\{\boldsymbol{x}_n\}$ is a sequence in X such that $\boldsymbol{limit}_{n\to\infty}\boldsymbol{A}\boldsymbol{x}_n = \boldsymbol{limit}_{n\to\infty}\boldsymbol{B}\boldsymbol{x}_n = \boldsymbol{z}$ for some $\boldsymbol{z} \in \boldsymbol{X}$.

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II. MAIN RESULT

Theorem (1): Let $(X, F, *)$ be a complete fuzzy metric space and let $f, g, A, B, C, D, H, I, J, K, L, M, N, P, Q, R, S, T, U, V, \delta, \beta$ be self mappings of X such that following conditions are satisfying:

$(I) f \subset A, f \subset C, f \subset H, f \subset J, f \subset L, f \subset N, f \subset Q, f \subset S, f \subset U, f \subset \delta$ $, g \subset B, g \subset D, g \subset I, g \subset K, g \subset M, g \subset P, g \subset R, g \subset T, g \subset V, g \subset \beta$

$(II) f, g, A, B, C, D, H, I, J, K, L, M, N, P, Q, R, S, T, U, V, \delta, \beta$ are continuous ,

(III) **Pairs are** $[f, B], [f, D]$ $[f, I], [f, K], [f, M], [f, P], [f, R], [f, T], [f, V], [f, \beta]$ and $[g, A], [g, C]$ $[g, H], [g, J], [g, L],$

$[g, N], [g, Q], [g, S], [g, U], [g, \delta]$ *compatible,*

$(IV) F(fx, gy, \alpha t) \geq$

$$\max.\{F(Ax,By,t), F(Cx,Dy,t), F(Hx,Iy,t), F(Jx,Ky,t), F(Lx,My,t), F(Nx,Py,t), F(Qx,Ry,t), F(Sx,Ty,t), F(Ux,Vy,t), F(\delta x,\beta y,t)\}$$

Where $0 < \alpha < 1$ and every x, y in X, $t > 0$. Then $f, g, A, B, C, D, H, I, J, K, L, M, N, P, Q, R, S, T, U, V, \delta, \beta$ have a unique common fixed point in X.

Proof: Let $x_0 \in X$ arbitrary and $f(X) \subset A(X)$ exist $x_1 \in X$ *such that* $fx_0 = Ax_1$ and for this, $x_1 \in X$ and $g(X) \subset B(X)$ *there exist* $x_2 \in X$ such that $gx_1 = Bx_2$ and for this, $x_2 \in X$ and $f(X) \subset C(X)$ *there exist* $x_3 \in X$ such $fx_2 = Cx_3$ and for this, $x_3 \in X$ and $g(X) \subset D(X)$ *there exist* $x_4 \in X$ such that $gx_3 = Dx_4$ and for this, $x_4 \in X$ and $f(X) \subset H(X)$ *there exist* $x_5 \in X$ such **that** $fx_4 = Hx_5$ and for this, $x_5 \in X$ and $g(X) \subset I(X)$ *there exist* $x_6 \in X$ such that $gx_5 = Ix_6$ and for this, $x_6 \in X$ and $f(X) \subset J(X)$ *there exist* $x_7 \in X$ such that $fx_6 = Jx_7$ and for this, $x_7 \in X$ and $g(X) \subset K(X)$ *there exist* $x_8 \in X$ such that $gx_7 = Kx_8$ and for this, $x_8 \in X$ and $f(X) \subset L(X)$ *there exist* $x_9 \in X$ such that $fx_8 = Lx_9$ and for this, $x_9 \in X$ and $g(X) \subset M(X)$ *there exist* $x_{10} \in X$ such that $gx_9 = Mx_{10}$ and for this, $x_{10} \in X$ and $f(X) \subset N(X)$ *there exist* $x_{11} \in X$ such that $fx_{10} = Nx_{11}$ and for this, $x_{11} \in X$ and $g(X) \subset P(X)$ *there exist* $x_{12} \in X$ such $fx_{11} = Px_{12}$ and for this, $x_{12} \in X$ and $f(X) \subset Q(X)$ *there exist* $x_{13} \in X$ such that $fx_{12} = Qx_{13}$ and for this, $x_{13} \in X$ and $g(X) \subset R(X)$ *there exist* $x_{14} \in X$ such **that** $gx_{13} = Rx_{14}$ and for this, $x_{14} \in X$ and $f(X) \subset S(X)$ *there exist* $x_{15} \in X$ such that $fx_{14} = Sx_{15}$ and for this, $x_{15} \in X$ and $g(X) \subset T(X)$ *there exist* $x_{16} \in X$ such that $gx_{15} = Tx_{16}$ and for this, $x_{16} \in X$ and $f(X) \subset U(X)$ *there exist* $x_{17} \in X$ such $fx_{16} = Ux_{17}$ and for this, $x_{17} \in X$ and $g(X) \subset V(X)$ *there exist* $x_{18} \in X$ such that $gx_{17} = Vx_{18}$ and for this, $x_{18} \in X$ and $f(X) \subset \delta(X)$ *there exist* $x_{19} \in X$ such $fx_{18} = \delta x_{19}$ and for this, $x_{19} \in X$ and $g(X) \subset \beta(X)$ *there exist* $x_{20} \in X$ such that $gx_{19} = \beta x_{20}$ In simple induction we construct a sequence $\{y_n\}$ in X *as follows*:

$fx_{20n-20} = Ax_{20n-19} = y_{20n-19}$, $gx_{20n-19} = Bx_{20n-18} = y_{20n-18}$, $fx_{20n-18} = Cx_{20n-17} = y_{20n-17}$, $gx_{20n-17} = Dx_{20n-16} = y_{20n-16}$, $fx_{20n-16} = Hx_{20n-15} = y_{20n-15}$, $gx_{20n-15} = Ix_{20n-14} = y_{20n-14}$, $fx_{20n-14} = Jx_{20n-13} = y_{20n-13}$, $gx_{20n-13} = Kx_{20n-12} = y_{20n-12}$, $fx_{20n-12} = Lx_{20n-11} = y_{20n-11}$, $gx_{20n-11} = Mx_{20n-10} = y_{20n-10}$, $fx_{20n-10} = Nx_{20n-9} = y_{20n-9}$, $gx_{20n-9} = Px_{20n-8} = y_{20n-8}$, $fx_{20n-8} = Qx_{20n-7} = y_{20n-7}$, $gx_{20n-7} = Rx_{20n-6} = y_{20n-6}$, $fx_{20n-6} = Sx_{20n-5} = y_{20n-5}$, $gx_{20n-5} = Tx_{20n-4} = y_{20n-4}$, $fx_{20n-4} = Ux_{20n-3} = y_{20n-3}$, $gx_{20n-3} = Vx_{20n-2} = y_{20n-2}$, $fx_{20n-2} = \delta x_{20n-1} = y_{20n-1}$, $gx_{20n-1} = \beta x_{20n} = y_{20n}$, for **n= 1,2,3,……..** .

By Use of condition (iv) with $x = x_{20n}$ and $y = x_{20n+1}$

$F(y_{20n+1}, y_{20n+2}, \alpha t) = F(fx_{20n}, gx_{20n+1}, \alpha t) \geq max.\{ F(Ax_{20n}, Bx_{20n+1}, t), F(Cx_{20n}, Dx_{20n+1}, t),$
$F(Hx_{20n}, Ix_{20n+1}, t), F(Jx_{20n}, Kx_{20n+1}, t), F(Lx_{20n}, Mx_{20n+1}, t), F(Nx_{20n}, Px_{20n+1}, t), F(Qx_{20n}, Rx_{20n+1}, t),$
$F(Sx_{20n}, Tx_{20n+1}, t), F(Ux_{20n}, Vx_{20n+1}, t), F(\delta x_{20n}, \beta x_{20n+1}, t) \}$

$F(y_{20n+1}, y_{20n+2}, \alpha t) \geq$
$max. \{ F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t),$
$F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t), F(y_{20n}, y_{20n+1}, t) \}$

$F(y_{20n+1}, y_{20n+2}, \alpha t) \geq F(y_{20n}, y_{20n+1}, t)$ ....(1)
Similarly we get,
$F(y_{20n+2}, y_{20n+3}, \alpha t) \geq F(y_{20n+1}, y_{20n+2}, t)$ ....(2)
By equation (1) and (2) we have

$$F(y_{n+1}, y_{n+2}, \alpha t) \geq F(y_n, y_{n+1}, t) \ldots\ldots\ldots\ldots (3)$$

In simple induction

$F(y_n, y_{n+1}, \alpha t) \geq F\left(y_n, y_{n-1}, \frac{t}{\alpha^n}\right) \geq F\left(y_{n-2}, y_{n-1}, \frac{t}{\alpha^n}\right) \geq \cdots \ldots \ldots \ldots \ldots \ldots \geq F(y_1, y_2, \frac{t}{\alpha^n}) \to 1$ as $n \to \infty$
So $F(y_n, y_{n+1}, \alpha t) \to 1$ as $n \to \infty$, for any $t > 0$.
Let each $\varepsilon > 0$ and $t > 0$, we can choose $n_0 \in N$ such that $F(y_n, y_{n+1}, \alpha t) \to > 1 - \varepsilon$ all $n > n_0$.
Let $m, n \in N$ suppose $m \geq n$. Thus we have

$$F(y_n, y_m, \alpha t) \geq F\left(y_n, y_{n+1}, \frac{t}{m-n}\right) * F\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right) * \ldots \ldots \ldots \ldots \ldots * F(y_{m-1}, y_m, \frac{t}{m-n})$$

$> (1-\varepsilon) * (1-\varepsilon) * (1-\varepsilon) * \ldots \ldots \ldots . . * (1-\varepsilon)$ {m-n time} $\geq 1-\varepsilon$
Hence $\{y_n\}$ is a Cauchy sequence in X.
$(X, F, *)$ is complete fuzzy metric space , $\{y_n\}$ converges some $z \in X$, and so
$\{fx_{20n-20}\}, \{gx_{20n-19}\}, \{Ax_{20n-18}\}, \{Bx_{20n-17}\}, \{Cx_{20n-16}\}, \{Dx_{20n-15}\}, \{Hx_{20n-14}\}, \{Ix_{20n-13}\},$
$\{Jx_{20n-12}\}, \{Kx_{20n-11}\}, \{Lx_{20n-10}\}, \{Mx_{20n-9}\}, \{Nx_{20n-8}\}, \{Px_{20n-7}\},$
$\{Qx_{20n-6}\}, \{Rx_{20n-5}\}, \{Sx_{20n-4}\}, \{Tx_{20n-3}\}, \{Ux_{20n-3}\}, \{Vx_{20n-2}\}, \{\delta x_{20n-1}\}, \{\beta x_{20n}\}$ also converges $\to z$.

Case I:

Let $x = Bx_{20n-17}$ and $y = Ax_{20n-18}$, **(iii)**
$fBx_{20n-17}, ABx_{20n-17}, CBx_{20n-17}, HBx_{20n-17}, JBx_{20n-17}, LBx_{20n-17}$
$NBx_{20n-17}, QBx_{20n-17}, SBx_{20n-17}, UBx_{20n-17} \to Bz,$
$gAx_{20n-18}, BAx_{20n-18}, DAx_{20n-18}, IAx_{20n-18}, KAx_{20n-18}, MAx_{20n-18}$
$PAx_{20n-18}, RAx_{20n-18}, TAx_{20n-18}, VAx_{20n-18} \to Az$

$F(fBx_{20n-17}, gAx_{20n-18}, \alpha t) \geq max.\{ F(ABx_{20n-17}, BAx_{20n-18}, t), F(CBx_{20n-17}, DAx_{20n-18}, t),$
$F(HBx_{20n-17}, IAx_{20n-18}, t), F(JBx_{20n-17}, KAx_{20n-18}, t), F(LBx_{20n-17}, MAx_{20n-18}, t),$
$F(NBx_{20n-17}, PAx_{20n-18}, t), F(QBx_{20n-17}, RAx_{20n-18}, t),$
$F(SBx_{20n-17}, TAx_{20n-18}, t), F(UBx_{20n-17}, VAx_{20n-18}, t), F(\delta Bx_{20n-17}, \beta Ax_{20n-18}, t) \}$ as $n \to \infty$
$F(Bz, Az, \alpha t) \geq max.\{ F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t),$
$F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t), F(Bz, Az, t) \}$
$F(Bz, Az, \alpha t) \geq F(Bz, Az, t)$. Hence
$Bz = Az$.
$F(Bz, z, \alpha t) \geq F(Bz, z, t)$
$Bz = z$ **. Such that** $Bz = Az = z$ **. (4)**
Let $x = Dx_{20n-16}$ and $y = Cx_{20n-17}$, (iii)
$fDx_{20n-16}, ADx_{20n-16}, CDx_{20n-16}, HDx_{20n-16}, JDx_{20n-16}, LDx_{20n-16}$
$NDx_{20n-16}, QDx_{20n-16}, SDx_{20n-16}, UDx_{20n-16}, \delta Dx_{20n-16} \to Dz,$
$gCx_{20n-17}, BCx_{20n-17}, DCx_{20n-17}, ICx_{20n-17}, KCx_{20n-17}, MCx_{20n-17}$

PCx_{20n-17} , RCx_{20n-17} , TCx_{20n-17} , VCx_{20n-17} $\beta Cx_{20n-17} \to Cz$

$F(fDx_{20n-16}, gCx_{20n-17}, \alpha t) \geq max.\{ F(ADx_{20n-16}, BCx_{20n-17}, t), F(CDx_{20n-16}, DCx_{20n-17}, t),$

$F(HDx_{20n-16}, ICx_{20n-17}, t), F(JDx_{20n-16}, KCx_{20n-17}, t), F(LDx_{20n-16}, MCx_{20n-17}, t),$

$F(NDx_{20n-16}, PCx_{20n-17}, t), F(QDx_{20n-16}, RCx_{20n-17}, t),$

$(SDx_{20n-16}, TCx_{20n-17}, t), F(UDx_{20n-16}, VCx_{20n-17}, t), F(\delta Dx_{20n-16}, \beta Cx_{18n-15}, t)\}\}$ as $n \to \infty$

$F(Dz, Cz, \alpha t) \geq max.\{ F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t),$

$F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t), F(Dz, Cz, t)\}$

$F(Dz, Cz, \alpha t) \geq F(Dz, Cz, t)$. Hence

$Dz = Cz$.

$F(Dz, z, \alpha t) \geq F(Dz, z, t)$

$Dz = z$ .Such that $Dz = Cz = z$. **(5)**

Let $x = Ix_{20n-14}$ and $y = Hx_{20n-15}$, $(iii) and\ (iv)\ we\ have$

$fIx_{18n-12}, AIx_{20n-14}, CIx_{20n-14}, HIx_{20n-14}, JIx_{20n-14}, LIx_{20n-14}$

$NIx_{20n-14}, QIx_{20n-14}, SIx_{20n-14}, UIx_{20n-14}, \delta Ix_{20n-14} \to Iz,$

$gHx_{20n-15}, BHx_{20n-15}, DHx_{20n-15}, IHx_{20n-15}, KHx_{20n-15}, MHx_{20n-15}$

$PHx_{20n-15}, RHx_{20n-15}, THx_{20n-15}, VHx_{20n-15}, \beta Hx_{20n-15} \to Hz$

$F(fIx_{20n-14}, gHx_{20n-15}, \alpha t) \geq max.\{ F(AIx_{20n-14}, BHx_{20n-15}, t), F(CIx_{20n-14}, DHx_{20n-15}, t),$

$F(HIx_{20n-14}, IHx_{20n-15}, t), F(JIx_{20n-14}, KHx_{20n-15}, t), F(LIx_{20n-14}, MHx_{20n-15}, t), F(NIx_{20n-14}, PHx_{20n-15}, t),$

$F(QIx_{20n-14}, RHx_{20n-15}, t), (SIx_{20n-14}, THx_{20n-15}, t), F(UIx_{20n-14}, VHx_{20n-15}, t), F(\delta Ix_{20n-14}, \beta Hx_{20n-15}, t)\}$

as $n \to \infty$

$F(Iz, Hz, \alpha t) \geq max.\{F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t),$

$F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t), F(Iz, Hz, t)\}$

$F(Iz, Hz, \alpha t) \geq F(Iz, Hz, t)$. Hence

$Iz = Hz$.$F(Iz, z, \alpha t) \geq F(Iz, z, t)$

$Iz = z$ .Such that $Iz = Hz = z$. **(6)**

Let $x = Kx_{20n-12}$and $y = Jx_{20n-13}$, $(iii) and\ (iv)\ we\ have$

$f\ Kx_{20n-12}, AKx_{20n-12}, CKx_{20n-12}, HKx_{20n-12}, JKx_{20n-12}, LKx_{20n-12}$

$NKx_{20n-12}, QKx_{20n-12}, SKx_{20n-12}, UKx_{20n-12}\ \delta Kx_{20n-12} \to Kz,$

$gJx_{20n-13}, BJx_{20n-13}, DJx_{20n-13}, IJx_{20n-13}, KJx_{20n-13}, MJx_{20n-13}$

$PJx_{20n-13}, RJx_{20n-13}, TJx_{20n-13}, VJx_{20n-13}, \beta Jx_{20n-13} \to Jz$

$F(fKx_{20n-12}, gJx_{20n-13}, \alpha t) \geq max.\{ FM(AKx_{20n-12}, BJx_{20n-13}, t), F(CKx_{20n-12}, DJx_{20n-13}, t),$

$F(HKx_{20n-12}, IJx_{20n-13}, t), F(JKx_{20n-12}, KJx_{20n-13}, t), F(LKx_{20n-12}, MJx_{20n-13}, t),$

$F(NKx_{20n-12}, PJx_{20n-13}, t), F(QKx_{20n-12}, RJx_{20n-13}, t),$

$(SKx_{20n-12}, TJx_{20n-13}, t), F(UKx_{20n-12}, VJx_{20n-13}, t), F(\delta Kx_{20n-12}, \beta Jx_{20n-13}, t)\}$ as $n \to \infty$

$F(Kz, Jz, \alpha t) \geq max.\{ F(Kz, Jz, t), F(Kz, Jz, t), F(Kz, Jz, t), F(Kz, Jz, t),$

$F(Mz, Jz, t), F(Kz, Hz, t), F(Kz, Hz, t), F(Kz, Hz, t), F(Kz, Hz, t), F(Kz, Hz, t)\}$

$M(Kz, Jz, \alpha t) \geq M(Kz, Jz, t)$. Hence

$Kz = Jz$.$F(Kz, z, \alpha t) \geq F(Kz, z, t)$

$Kz = z$ .Such that $Kz = Jz = z$. (7)

Let $x = Mx_{20n-10}$, and $y = Lx_{20n-11}$, $(iii) and\ (iv)\ we\ have$

$fMx_{18n-8}, AMx_{20n-10}, CMx_{20n-10}, HMx_{20n-10}, JMx_{20n-10}, LMx_{20n-10}$

$NMx_{20n-10}, QMx_{20n-10}, SMx_{20n-10}, UMx_{20n-10}, \delta Mx_{20n-10} \to Mz,$

$gLx_{20n-11}, BLx_{20n-11}, DLx_{20n-11}, ILx_{20n-11}, KLx_{20n-11}, MLx_{20n-11}$

$PLx_{20n-11}, RLx_{20n-11}, TLx_{20n-11}, VLx_{20n-11}, \beta Lx_{20n-11} \to Lz$

$F(fMx_{20n-10}, gLx_{20n-11}, \alpha t) \geq max.\{ F(AMx_{20n-10}, BLx_{20n-11}, t), F(CMx_{20n-10}, DLx_{20n-11}, t),$

$F(HMx_{20n-10}, ILx_{20n-11}, t), F(JMx_{20n-10}, KLx_{20n-11}, t), F(LMx_{20n-10}, MLx_{20n-11}, t),$

$F(NMx_{20n-10}, PLx_{20n-11}, t), F(QMx_{20n-10}, RLx_{20n-11}, t),$

$(SMx_{20n-10}, TLx_{20n-11}, t), F(UMx_{20n-10}, VLx_{20n-11}, t), F(\delta Mx_{20n-10}, \beta Lx_{20n-11}, t)\}$ as $n \to \infty$

$F(Mz, Lz, \alpha t) \geq max.\{ F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t),$
$F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t), F(Mz, Lz, t)\}$

$$F(Mz, Lz, \alpha t) \geq F(Mz, Lz, t)$$

$Mz = Lz$.
$F(Mz, z, \alpha t) \geq F(Mz, z, t)$
$Mz = z$ .Such that $Mz = Lz = z$. (8)
Let $x = Px_{20n-8}$, and $y = Nx_{20n-9}$, $(iii) and\ (iv)\ we\ have$
$fPx_{18n-6}, APx_{20n-8}, CPx_{20n-8}, HPx_{20n-8}, JPx_{20n-8}, LPx_{20n-8}$
$NPx_{20n-8}, QPx_{20n-8}, SPx_{20n-8}, UPx_{20n-8}, \delta Px_{20n-8} \to Pz,$
$gNx_{20n-9}, BNx_{20n-9}, DNx_{20n-9}, INx_{20n-9}, KNx_{20n-9}, MNx_{20n-9}$
$PNx_{20n-9}, RNx_{20n-9}, TNx_{20n-9}, VNx_{20n-9}, \beta Nx_{20n-9} \to Nz$
$F(fPx_{20n-8}, gNx_{20n-9}, \alpha t) \geq max.\{ F(APx_{20n-8}, BNx_{20n-9}, t), F(CPx_{20n-8}, DNx_{20n-9}, t),$
$M(HPx_{20n-8}, INx_{20n-9}, t), M(JPx_{20n-8}, KNx_{20n-9}, t), M(LPx_{20n-8}, MNx_{20n-9}, t), F(NPx_{20n-8}, PNx_{20n-9}, t),$
$F(QPx_{20n-8}, RNx_{20n-9}, t), (SPx_{20n-8}, TNx_{20n-9}, t), F(UPx_{20n-8}, VNx_{20n-9}, t), F(\delta Px_{20n-8}, \beta Nx_{20n-9}, t)\}$ as $n \to \infty$
$F(Pz, Nz, \alpha t) \geq max.\{ F(Pz, Nz, t), F(Pz, Nz, t), F(Pz, Nz, t), F(Pz, Nz, t),$
$F(Pz, Nz, t), F(Pz, Nz, t), F(Pz, Nz, t), F(Pz, Nz, t), F(Pz, Nz, t), , F(Pz, Nz, t)\}$

$$F(Pz, Nz, \alpha t) \geq F(Pz, Nz, t)$$

$Pz = Nz$.
$F(Pz, z, \alpha t) \geq F(Pz, z, t)$
$Pz = z$ .Such that $Pz = Nz = z$. (9)
Let $x = Rx_{20n-6}$, and $y = Qx_{20n-7}$, $(iii) and\ (iv)\ we\ have$
$fRx_{18n-4}, ARx_{20n-6}, CRx_{20n-6}, HRx_{20n-6}, JRx_{20n-6}, LRx_{20n-6}$
$NRx_{20n-6}, QRx_{20n-6}, SRx_{20n-6}, URx_{20n-6}, \delta Rx_{20n-6} \to Rz,$
$gQx_{20n-7}, BQx_{20n-7}, DQx_{20n-7}, IQx_{20n-7}, KQx_{20n-7}, MQx_{20n-7}$
$PQx_{20n-7}, RQx_{20n-7}, TQx_{20n-7}, VQx_{20n-7}, \beta Qx_{20n-7} \to Qz$
$F(fRx_{20n-6}, gQx_{20n-7}, \alpha t) \geq max.\{ F(ARx_{20n-6}, BQx_{20n-7}, t), F(CRx_{20n-6}, DQx_{20n-7}, t),$
$F(HRx_{20n-6}, IQx_{20n-7}, t), F(JRx_{18n-4}, KQx_{20n-7}, t), F(LRx_{20n-6}, MQx_{20n-7}, t), F(NRx_{20n-6}, PQx_{20n-7}, t),$
$F(QRx_{20n-6}, RQx_{20n-7}, t), (SRx_{20n-6}, TQx_{20n-7}, t), F(URx_{20n-6}, VQx_{20n-7}, t), F(\delta Rx_{20n-6}, \beta Qx_{20n-7}, t)\}$ as $n \to \infty$
$F(Rz, Qz, \alpha t) \geq max.\{ F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t),$
$F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t), F(Rz, Qz, t)\}$

$$F(Rz, Qz, \alpha t) \geq F(Rz, Qz, t)$$

$Rz = Qz$.
$M(Rz, z, \alpha t) \geq M(Rz, z, t)$
$Rz = z$ .such that $Rz = Qz = z$. (10)

Let $x = Tx_{20n-4}$, and $y = Sx_{20n-5}$, $(iii) and\ (iv)\ we\ have$
$fTx_{20n-4}, ATx_{20n-4}, CTx_{20n-4}, HTx_{20n-4}, JTx_{20n-4}, LTx_{20n-4}$
$NTx_{20n-4}, QTx_{20n-4}, STx_{20n-4}, UTx_{20n-4}, \delta Tx_{20n-4} \to Tz,$
$gSx_{20n-5}, BSx_{20n-5}, DSx_{20n-5}, ISx_{20n-5}, KSx_{20n-5}, MSx_{20n-5}$
$PSx_{20n-5}, RSx_{20n-5}, TSx_{20n-5}, VSx_{20n-5}, \beta Sx_{20n-5} \to Sz$
$F(fVx_{18n}, gSx_{20n-5}, \alpha t) \geq max.\{ F(AVx_{18n}, BSx_{20n-5}, t), F(CVx_{18n}, DSx_{20n-5}, t),$
$F(HVx_{18n}, ISx_{20n-5}, t), F(JVx_{18n}, KSx_{20n-5}, t), F(LVx_{18n}, MSx_{20n-5}, t), F(NVx_{18n}, PSx_{20n-5}, t),$
$F(QVx_{18n}, RSx_{20n-5}, t), (SVx_{18n}, TSx_{20n-5}, t), F(UVx_{18n}, VSx_{20n-5}, t), F(\delta Vx_{18n}, \beta Sx_{20n-5}, t)\}$ as $n \to \infty$
$F(Tz, Sz, \alpha t) \geq max.\{ F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t),$
$F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t), F(Tz, Sz, t)\}$

$F(Tz, Sz, \alpha t) \geq F(Tz, Sz, t)$
$Tz = Sz$.
$F(Tz, z, \alpha t) \geq F(Tz, z, t)$
$Tz = z$. Such that $Tz = Sz = z$. (11)

Let $x = Vx_{20n-2}$, and $y = Ux_{20n-3}$, $(iii) and\ (iv)\ we\ have$
$fVx_{20n-2}, AVx_{20n-2}, CVx_{20n-2}, HVx_{20n-2}, JVx_{20n-2}, LVx_{20n-2}$
$NVx_{20n-2}, QVx_{20n-2}, SVx_{20n-2}, UVx_{20n-2}$, $\delta Vx_{20n-2} \rightarrow Vz$,
$gUx_{20n-3}, BUx_{20n-3}, DUx_{20n-3}, IUx_{20n-3}, KUx_{20n-3}, MUx_{20n-3}$
$PUx_{20n-3}, RUx_{20n-3}, TUx_{20n-3}, VUx_{20n-3}, \beta Ux_{20n-3} \rightarrow Uz$
$F(fVx_{20n-2}, gUx_{20n-3}, \alpha t) \geq max.\{ F(AVx_{20n-2}, BUx_{20n-3}, t), F(CVx_{20n-2}, DUx_{20n-3}, t),$
$F(HVx_{20n-2}, IUx_{20n-3}, t), F(JVx_{20n-2}, KUx_{20n-3}, t), F(LVx_{20n-2}, MUx_{20n-3}, t), F(NVx_{20n-2}, PUx_{20n-3}, t),$
$F(QVx_{20n-2}, RUx_{20n-3}, t), (SVx_{20n-2}, TUx_{20n-3}, t), F(UVx_{20n-2}, VUx_{20n-3}, t), F(\delta Vx_{20n-2}, \beta Ux_{20n-3}, t)\}$ as $n \rightarrow \infty$
$F(Vz, Uz, \alpha t) \geq max.\{ F(Vz, USz, t), F(Vz, Uz, t), F(Vz, Uz, t), F(Vz, Uz, t),$
$F(Vz, Sz, t), F(Vz, Uz, t) , F(Vz, Uz, t) , F(Vz, Uz, t), F(Vz, Uz, t) , F(Vz, Uz, t)\}$
$F(Vz, Uz, \alpha t) \geq F(Vz, Uz, t)$
$Vz = Uz$.
$F(Vz, z, \alpha t) \geq F(Vz, z, t)$
$Vz = z$. Such that $Vz = Uz = z$. (12)
Let $x = \beta x_{20n}$, and $y = \delta x_{20n-1}$, $(iii) and\ (iv)\ we\ have$
$f\beta x_{20n}, A\beta x_{20n}, C\beta x_{20n}, H\beta x_{20n}, J\beta x_{20n}, L\beta x_{20n}$
$N\beta x_{20n}, Q\beta x_{20n}, S\beta x_{20n}, U\beta x_{20n}$, $\delta\beta x_{20n} \rightarrow \beta z$,
$g\delta x_{20n-1}, B\delta x_{20n-1}, D\delta x_{20n-1}, I\delta x_{20n-1}, K\delta x_{20n-1}, M\delta x_{20n-1}$
$P\delta x_{20n-1}, R\delta x_{20n-1}, T\delta x_{20n-1}, V\delta x_{20n-1}$,$\beta\delta x_{20n-1} \rightarrow \delta z$
$F(f\beta x_{20n}, g\delta x_{20n-1}, \alpha t) \geq max.\{ F(A\beta x_{20n}, B\delta x_{20n-1}, t), F(C\beta x_{20n}, D\delta x_{20n-1}, t),$
$F(H\beta x_{20n}, I\delta x_{20n-1}, t), F(J\beta x_{20n}, K\delta x_{20n-1}, t), F(L\beta x_{20n}, M\delta x_{20n-1}, t), F(N\beta x_{20n}, P\delta x_{20n-1} t),$
$F(Q\beta x_{20n} , R\delta x_{20n-1}, t), (S\beta x_{20n}, T\delta x_{20n-1}, t), F(U\beta x_{20n}, V\delta x_{20n-1}, t) , F(\delta\beta x_{20n} , \beta\delta x_{20n-1}, t) \}$ as $n \rightarrow \infty$
$F(\beta z, \delta z, \alpha t) \geq max.\{ F(\beta z, \delta z, t), F(\beta z, \delta z, t), F(\beta z, \delta z, t), F(\beta z, \delta z, t),$
$F(\beta z, \delta z, t), F(\beta z, \delta z, t) , F(\beta z, \delta z, t) , F(\beta z, \delta z, t), F(\beta z, \delta z, t) , F(\beta z, \delta z, t)\}$
$F(\beta z, \delta z, \alpha t) \geq F(\beta z, \delta z, t)$
$\beta z = \delta z$.
$F(\beta z, z, \alpha t) \geq F(\beta z, z, t)$
$\beta z = z$. Such that $\beta z = \delta z = z$. (13)

Such that equations (4) to (13) we have
$Az = Bz = Cz = Dz = Hz = Iz = Jz = Kz = Lz = Mz = Nz = Pz = Qz = Rz = Sz = Tz = Uz = Vz = \delta z = \beta z = z$. (14)
Let $x = gx_{20n-19}$, and $y = fx_{20n-20}$, $(iii) and\ (iv)\ we\ have$
$fgx_{20n-19}, Agx_{20n-19}, Cgx_{20n-19}, Hgx_{20n-19}, Jgx_{20n-19}, Lgx_{20n-19},$
$Ngx_{20n-19}, Qgx_{20n-19}, Sgx_{20n-19}, Ugx_{20n-19}$, $\delta gx_{20n-19} \rightarrow gz$,
$gfx_{20n-20}, Bfx_{20n-20}, Dfx_{20n-20}, Ifx_{20n-20}, Kfx_{20n-20}, Mfx_{20n-20}$
$Pfx_{20n-20}, Rfx_{20n-20}, Tfx_{20n-20}, Vfx_{20n-20}$, $\beta fx_{20n-20} \rightarrow fz$
$F(fgx_{20n-19}, gfx_{18n-18}, \alpha t) \geq max.\{ F(Agx_{20n-19}, Bfx_{18n-18}, t), F(Cgx_{20n-19}, Dfx_{18n-18}, t),$
$F(Hgx_{20n-19}, Ifx_{18n-18}, t), F(Jgx_{20n-19}, Kfx_{18n-18}, t), F(Lgx_{20n-19}, Mfx_{18n-18}, t), F(Ngx_{20n-19}, Pfx_{18n-18}, t),$
$F(Qgx_{20n-19}, Rfx_{18n-18}, t), (Sgx_{20n-19}, Tfx_{18n-18}, t), F(Ugx_{20n-19}, Vfx_{18n-18}, t), F(Ugx_{20n-19}, Vfx_{18n-18}, t)\}$
as $n \rightarrow \infty$

$F(gz, fz, \alpha t) \geq max.\{F(gz, fz, t), F(gzfz, t), F(gzfz, t), F(gz, fz, t),$
$F(gz, fz, t), F(gz, fz, t), F(gz, fz, t), F(gz, fz, t), F(gz, fz, t), F(gz, fz, t)\}$
$M(gz, fz, \alpha t) \geq M(gz, fz, t)$
$gz = fz$.
$F(gz, z, \alpha t) \geq F(gz, z, t)$
$gz = z$ .Suchthat $gz = fz = z$. (15)
By (14) and (15).
$fz = gz = Az = Bz = Cz = Dz = Hz = Iz = Jz = Kz = Lz = Mz = Nz = Pz = Qz = Rz = Sz = Tz = Vz = Uz = \delta z = \beta z = z$. Hence z is common fixed point of $f, g, A, B, C, D, H, I, J, K, L, M, N, P, Q, R, S, T, U, V, \delta$ and β.
Case II: let w be another common fixed point, also $z \neq w$ we have $x = z, y = w$ by (iv),
$F(fz, gw, \alpha t) \geq max.\{F(Az, Bw, t), F(Cz, Dw, t), F(Hz, Iw, t), F(Jz, Kw, t),$
$F(Lz, Mw, t), F(Nz, Pw, t) F(Qz, Rw, t), F(Sz, Tw, t), F(Uz, Vw, t), F(\delta z, \beta w, t)\}$ as $n \to \infty$
$F(z, w, \alpha t) \geq max.\{F(z, w, t), F(z, w, t), F(z, w, t), F(z, w, t), F(z, w, t),$
$F(z, w, t), F(z, w, t), F(z, w, t), F(z, w, t),, F(z, w, t)\}$
$F(z, w, \alpha t) \geq F(z, w, t)$.Hence we have,
$z = w$ which is contradict and z is a unique common fixed point of self mappings
$f, g, A, B, C, D, H, I, J, K, L, M, N, P, Q, R, S, T, U, V, \delta$ and β.

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REFERENCES

[1]. G.Jungck , Int. J.MAthematical& Math. Sci. 9 (1986), 771-79.
[2]. G. JUNGCK, B.E. Rhoades Internat. J. Math. & Math. Scl. VOL.16 NO. 3 (1993) 417-428, 417
[3]. G. Jungck, Commuting mappings and fixed points, Amer. Math.Monthly 83 (1976),261-263.MR 53 #4031
[4]. K. M. Das & K. V. NAIK Pro.Am. Mathematical Society, Vol. 77, No. 3, Dec. 1979
[5]. S. Banach, Sur les opérationsdans les ensembles abstractsetleur application aux équations intégrales(Thèseprésentée en juin 1920 al'Université de Léopol [= Lwów] pourobtenir le grade dedocteur en philosophie), Fund.Math. 3 (1922), 133-181.
[6]. D.Zike , Fuzzy psedoMetric spaces. J.Maths.Anals.Appl.86 (1982),74-95.
[7]. M.A. Erceg, Metric spaces in fuzzy set theory..Maths.Anals. Appl., 69(1979), 205-230.
[8]. A.George&P.Veeramani,On some results of analysis for fuzzy Metric spaces, Fuzzy sets and Systems, 90(1997),365-368.
[9]. Dr U.D. Tapi , V.K. Songara S.G.S.I.T.S. Indore : Study of Fixed Point Theory by Fuzzy Metric Space, Dissertation Thesis Fulfill Degree of Master of Science in Applied Mathematics and Computational Science 2009 .
[10]. Pooja mishra, Brajbhushan Singh Chouhan : Pair of twelve compatibility mappings and common fixed point theorem in metric space ,National Seminar RIT Indore, April 2018
[11]. Pooja mishra, Brajbhushan Singh Chouhan : Common fixed point in Fuzzy Metric Space , National Multi Conference LNCT Indore , November 2021.
[12]. Pooja mishra : Some Special Type Of Theorem In Fuzzy Metric Space And There Common Fixed Point ,National Conference On Modeling And Computational Simulation Dr. A.P.J. Abdul Kalam University Indore,2022.