

Weak Shock Wave Motion in Metals: A Theoretical Study

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Abstract: Shock waves in metallic materials play an important role in high-pressure physics, impact engineering, and materials science. The propagation of weak shock waves in metals is particularly significant because it determines the early stages of stress wave evolution during dynamic loading. In this work, a theoretical study of weak shock wave motion in metals is presented. The governing conservation equations, Rankine–Hugoniot relations, and constitutive behavior of metallic materials are discussed. The propagation characteristics of weak shocks are analyzed under small perturbation assumptions where entropy changes are minimal. Theoretical approaches describing the relationship between shock velocity and particle velocity are discussed along with implications for material deformation and energy dissipation. The study highlights the importance of weak shock theory in understanding wave attenuation, elastic–plastic transition, and dynamic response of metals under high strain-rate loading.

Keywords: Shock wave, weak shock, motion in metals, stress wave propagation, theoretical analysis

I. INTRODUCTION

Shock waves are compressive disturbances that propagate through a medium at speeds greater than the local speed of sound. When a shock wave travels through a metallic medium, it produces sudden changes in pressure, density, temperature, and particle velocity. These phenomena are important in fields such as explosive engineering, planetary impact studies, and high-velocity material processing.

Shock wave propagation in metals has been studied extensively since the mid-twentieth century. Early experimental work provided important relations between shock velocity and particle velocity in metals such as aluminum, copper, and zinc [1]. Later studies expanded the theory to include viscoplastic deformation and microstructural effects in porous or heterogeneous metals [2].

Weak shock waves represent a limiting case where the discontinuity across the shock front is small. In such cases, the entropy change across the shock is negligible and the wave can often be approximated as an isentropic compression wave[3]. This approximation simplifies theoretical analysis and allows analytical solutions for wave propagation in solids.

The purpose of this paper is to present a theoretical discussion of weak shock wave motion in metallic materials. The governing equations, theoretical models, and physical interpretations are described.

II. BASIC THEORY OF SHOCK WAVES IN METALS

Shock waves in solids obey the conservation laws of mass, momentum, and energy across the shock front. These conservation laws lead to the Rankine–Hugoniot relations that describe the jump conditions across the shock.

For a one-dimensional shock wave propagating through a material, the conservation equations can be written as

Mass conservation

$$\rho_1 U \square = \rho_2 (U \square - u \square)$$

Momentum conservation

$$P_2 - P_1 = \rho_1 U \square u \square$$

Energy conservation

$$E_2 - E_1 = \frac{1}{2} (P_2 + P_1)(V_1 - V_2)$$

where

ρ = density

P = pressure

U_{\square} = shock velocity

u_{\square} = particle velocity

E = internal energy.

These relations are widely used to determine the dynamic equation of state of metals from shock experiments. Experimental studies have shown that the relationship between shock velocity and particle velocity in many metals is approximately linear[4]

$$U_{\square} = C_0 + s u_{\square}$$

where

C_0 = bulk sound speed

s = material constant.

This relation forms the basis for describing shock compression in metallic materials.

III. WEAK SHOCK APPROXIMATION

When the shock strength is small, the difference between upstream and downstream properties is very small. Under this condition, the shock can be treated using weak shock theory.

In weak shock propagation:

- Pressure change is small
- Entropy change is negligible
- Shock thickness becomes large compared to strong shocks

Because the entropy change is small, the shock can be approximated as an isentropic wave [5].

The pressure perturbation can be expressed as

$$\Delta P \ll P_0$$

and the propagation speed approaches the local sound speed of the material.

Weak shocks are often observed in the early stage of explosive loading or during stress wave attenuation in solids. As the shock propagates through the medium, its strength gradually decreases and it eventually transforms into an elastic wave.

The theoretical treatment of weak shock propagation was developed using perturbation methods and characteristic analysis, allowing analytical solutions for shock decay and motion in compressible media[6].

IV. SHOCK MOTION IN METALLIC MEDIA

The propagation of shock waves in metals differs from gases due to the presence of elasticity, plasticity, and microstructural effects.

When a shock wave enters a metal:

An elastic precursor wave may propagate first.

The plastic wave follows behind the elastic front.

Material deformation and dislocation motion occur.

The internal structure of the shock wave depends on material properties such as viscosity, plasticity, and porosity. Studies have shown that pore collapse and micro-inertia effects significantly influence shock structure in porous metals[4]

The attenuation of shock waves in multi-metallic systems has also been investigated. Impedance-graded materials can reduce shock intensity and mitigate damage during high-velocity impacts[5]

V. THEORETICAL MODELING

Theoretical modeling of weak shock propagation in metals generally involves continuum mechanics and nonlinear wave theory.

The governing equations include:

- Conservation equations
- Constitutive relations for stress–strain behavior
- Equation of state for the material

For small perturbations, the nonlinear wave equation describing the pressure disturbance can be written as

$$\partial P / \partial t + c \partial P / \partial x + \alpha P \partial P / \partial x = 0$$

where

c = sound speed in the material

α = nonlinearity parameter.

This equation describes the evolution and steepening of compression waves into shock fronts.

Analytical solutions derived using perturbation methods help predict shock decay, wave speed, and energy dissipation during propagation.

VI. CONCLUSION

Weak shock waves play an important role in the dynamic response of metals subjected to high-rate loading. Theoretical models based on conservation laws and perturbation analysis provide valuable insights into the propagation characteristics of these waves. In metallic materials, shock propagation is strongly influenced by elastic-plastic behavior, microstructure, and material porosity. Understanding weak shock motion is essential for predicting material response in impact events, explosive loading, and high-pressure physics experiments. Future work may focus on integrating microstructural effects and numerical simulations to provide a more comprehensive description of shock wave dynamics in advanced materials.

REFERENCES

- [1] J. M. Walsh and R. H. Christian, "Equation of state of metals from shock wave measurements," *Physical Review*, vol. 97, no. 6, pp. 1544–1556, 1955.
- [2] G. B. Whitham, "On the propagation of weak shock waves," *Journal of Fluid Mechanics*, vol. 21, pp. 337–361, 1965.
- [3] J. H. S. Lee, *The Gas Dynamics of Explosions and Reactive Systems*. Cambridge University Press, 2008.
- [4] V. A. Levin and M. A. Lomunov, "The structure of steady shock waves in porous metals," *Journal of the Mechanics and Physics of Solids*, vol. 107, pp. 204–228, 2017.
- [5] M. B. Rubin et al., "Steady shock waves in porous metals: viscosity and micro-inertia effects," *International Journal of Plasticity*, vol. 135, 2020.
- [6] Y. Chen et al., "Shock wave propagation through impedance-graded multi-metallic systems," *International Journal of Mechanical Sciences*, vol. 178, 2020.