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Steady- State Impulse Heating and Related Thermal Stresses

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Abstract: *The present manuscript deals with the heat transfer and thermal stress analysis of thick circular plate under steady temperature conditions. A circular plate is subjected to impulse heat flux applied on the upper surface. Heat dissipates by convection from the lower boundary surface into the surrounding. The thick curved surface of circular plate is thermally insulated. The integral transform methods are used for heat transfer analysis to determine temperature changes. The theory of linearized thermoelasticity based on solution of Navier's equation in terms of Goodier's thermoelastic displacement potential, Michell's function, and the Boussinesq's function for cylindrical co-ordinate system have been used to discussion and analysis of thermal stress. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically for different isotropic solid material.*

Keywords: Heat transfer analysis, Steady state, Impulse heating, Thermal Stress analysis

I. INTRODUCTION

Thermoelasticity is based on temperature changes induced by expansion and compression of the test part. Although this coupling between mechanical deformation and thermal energy has been known for over a century. After world war second, there was very rapid development of thermoelasticity, stimulated by various engineering sciences. Thermoelasticity contains the generalized theory of heat conductions, the generalized theory of the thermal stresses. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role.

Sipailov et al. [7] studied the effect of heat transfer in the impulse method of measurement on the value of the maximum temperature and on the time for achieving its half-value. Taking this heat transfer effect into account increases the accuracy of measuring the thermophysical properties. Lohle et al [3], analyze inverse heat conduction problems by the analysis of the system impulse response by the application of the non-integer system identification (NISI) method. Stakhanova et al [8] carried out heat transfer on model fragments of fuel rod claddings during impulse supply of power. The time taken for reaching steady state for different parameters of impulse and the values of heat-transfer coefficient are determined. Lee C.W. [2] obtained three dimensional series solution for elastic thick plate subjected to general temperature distribution. T. Hata [9] concerned with a method for calculating the thermal-stress distribution in a nonhomogeneous thick elastic plate under steady distribution of the surface temperature whose shear modulus and coefficient of thermal expansion are assumed to be functions of z. Kulkarni et al [1] determined the temperature changes and thermal stresses due to conduction of heat in the thick circular plate under transient temperature conditions and analyzed his analytical results for heat treatment given in the annular region which is described by Dirac-delta function.

II. HEAT TRANSFAR ANALYSIS

2.1 Formulation of the Problem

Consider a thick circular plate of thickness $2h$ occupying space D defined by $0 \le r \le a,\ -h \le z \le h$. An impulse heating is applied on the upper surface of the plate $(z = h)$. Heat dissipates by convection from the lower boundary surface $(z=-h)$ into the surrounding at the zero temperature. The circular edge $(r=a)$ is thermally insulated. Assume that the boundary of the circular plate is free from traction. Under these more realistic prescribed conditions, the quasi-static steady state thermal stresses are required to be determined.

The steady state temperature of the plate satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=0 \qquad (2.1.1)$$

with the boundary conditions

$$\frac{\partial T}{\partial r}=0 \qquad \text{at } r=a,\ -h\le z\le h \qquad (2.1.2)$$

$$\frac{\partial T}{\partial z}=T_0\left[\delta(r-r_1)+\delta(r-r_2)+\delta(r-r_3)+\delta(r-r_4)\right] \qquad \text{at } z=h,\ 0\le r\le a \qquad (2.1.3)$$

and

$$\frac{\partial T}{\partial z}-h_s T=0 \qquad \text{at } z=-h,\ 0\le r\le a \qquad (2.1.4)$$

where

h_s be the relative heat transfer coefficients on the lower surface of the thick circular plate

δ is impulse function and

T_0 is strength of impulse heating.

2.2 The Solution for Temperature Change

To obtain the expression for temperature $T(r,z)$ introduce the finite Hankel transform over the variable r and its inverse transform defined in [5] as

$$\overline{T}(\alpha_n, z)=\int_0^a rJ_0(\alpha_n r)T(r,z)dr \qquad (2.2.1)$$

$$T(r,z)=\sum_{n=1}^{\infty}\left(\frac{2J_0(\alpha_n r)}{a^2 J_0{}^2(\alpha_n a)}\right)\overline{T}(\alpha_n, z) \qquad (2.2.2)$$

where α_1, α_2 … are roots of the transcendental equation

$$J_1(\alpha a)=0 \qquad (2.2.3)$$

$J_n(x)$ is Bessel function of the first kind of order n.

This transform satisfies the relations

$$H\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]=-\alpha_n^2\overline{T}(\alpha_n, z) \qquad (2.2.4)$$

and

$$H\left[\frac{\partial^2 T}{\partial z^2}\right]=\frac{d^2\overline{T}}{dz^2} \qquad (2.2.5)$$

On applying the finite Hankel transform defined in the equation (2.2.1) and its inverse defined in the equation (2.2.2) to the equation (2.1.1) one obtain,

$$\frac{d^2\overline{T}}{dz^2}-\alpha_n^2\overline{T}=0 \qquad (2.2.6)$$

where $\overline{T}$ is the Hankel transform of T .

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Also the Hankel transform of impulse function defined in equation (2.1.3) is given by

$$H\left\{T_0\left[\delta(r-r_1)+\delta(r-r_2)+\delta(r-r_3)+\delta(r-r_4)\right]\right\}$$

$$=(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right] \qquad (2.2.7)$$

On solving equation (2.2.6) under the conditions given in equations (2.1.3), (2.1.4) and using equation (2.2.7), one obtains

$$\overline{T}(\alpha_n,z)=(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]$$

$$\left(\frac{\alpha_n\cosh[\alpha_n(z+h)]+h_s\sinh[\alpha_n(z+h)]}{\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)}\right) \qquad (2.2.8)$$

On applying the inverse Hankel transform defined in equation (2.2.2), one obtain the expression for the temperature as

$$T=\sum_{n=1}^{\infty}\left(\frac{2T\cdot_0 r\cdot J_0(\alpha_n r)}{a^2 J_0{}^2(\alpha_n a)}\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]$$

$$\left(\frac{\alpha_n\cosh[\alpha_n(z+h)]+h_s\sinh[\alpha_n(z+h)]}{\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)}\right) \qquad (2.2.9)$$

III. THERMAL STRESS ANALYSIS

3.1 Development of Thermoelastic Equations

Following Noda et al [4], The Naviers equations for axisymmetric thermoelastic problems can be expressed as

$$\nabla^2 u_r-\frac{u_r}{r^2}+\left(\frac{1}{1-2\nu}\right)\frac{\partial e}{\partial r}-2\alpha\left(\frac{1+\nu}{1-2\nu}\right)\frac{\partial\tau}{\partial r}+\frac{2(1+\nu)}{E}F_r=0 \qquad (3.1.1)$$

$$\nabla^2 u_z+\left(\frac{1}{1-2\nu}\right)\frac{\partial e}{\partial z}-2\alpha\left(\frac{1+\nu}{1-2\nu}\right)\frac{\partial\tau}{\partial z}+\frac{2(1+\nu)}{E}F_z=0 \qquad (3.1.2)$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}.\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \qquad (3.1.3)$$

e - dilatation
E - Young's modulus
α - coefficient of linear thermal expansion
ν - Poisson ratio

The solution of Naviers equations (3.1.1) and (3.1.2) without body forces can be expressed by Goodiers thermoelastic displacement potential ϕ and Boussinesq harmonic functions φ and ψ under the axisymmetric conditions.

The Goodiers thermoelastic displacement potential ϕ must satisfy the governing equations

$$\nabla^2\phi=K\tau$$

i.e.
$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=K\tau \qquad (3.1.4)$$

where K *is* Restraient coefficient as

$$K=\frac{\beta}{\lambda+2\mu}=\left(\frac{1+\nu}{1-\nu}\right)\alpha$$

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where β - thermoelastic constant

λ & μ - Lames elastic constants.

Boussinesq harmonic functions φ and ψ must satisfy the governing equations

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{3.1.5}$$

and

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = 0 \tag{3.1.6}$$

when deformation in the cylindrical coordinate system are discussed, Michells function M instead of Boussinesq harmonic functions φ and ψ is often used.

Taking

$$M = -\int(\varphi + z\psi)dz \tag{3.1.7}$$

The Michell's function M must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{3.1.8}$$

The component of the displacement and stresses are represented by the thermoelastic displacement potential ϕ and Michell's function M as

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2 M}{\partial r dz} \tag{3.1.9}$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \tag{3.1.10}$$

$$\sigma_{rr} = 2G\left[\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right] \tag{3.1.11}$$

$$\sigma_{\theta\theta} = 2G\left[\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right] \tag{3.1.12}$$

$$\sigma_{zz} = 2G\left[\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{3.1.13}$$

and

$$\sigma_{rz} = 2G\left[\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{3.1.14}$$

For traction free surface the stress functions

$$\sigma_{rr} = \sigma_{rz} = 0 \quad \text{at } r = a \text{ and } r = b$$

$$\sigma_{zz} = \sigma_{rz} = 0 \quad \text{at } z = \pm h \tag{3.1.15}$$

The set of equations (3.1.1) to (3.1.15) constitute mathematical formulation for displacement and thermal stresses developed within solid due to temperature change.

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3.2 The Solution for Displacement and Thermal Stresses

Assuming the displacement function $\phi(r,z)$ in the form

$$\phi(r,z)=\sum_{n=1}^{\infty} D_n\left\{J_0(\alpha_n r)(z+h)\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle\right\} \tag{3.2.1}$$

and using ϕ in (3.1.4), one obtains

$$D_n=\frac{K\left(T_0\cdot r\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{\alpha_n a^2 J_0{}^2(\alpha_n a)\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}$$

Thus equation (25) becomes

$$\phi(r,z)=\left(K\right)\sum_{n=1}^{\infty}\left(T_0\cdot r\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]$$

$$\left(\frac{J_0(\alpha_n r)}{\alpha_n a^2 J_0{}^2(\alpha_n a)}\right)\left\{\frac{(z+h)\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle}{\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_{s2}\cosh(2\alpha_n h)}\right\} \tag{3.2.2}$$

Now suitable form of M satisfying (3.1.8) is given by

$$M=\left(K\right)\sum_{n=1}^{\infty}\left(\frac{2J_0(\alpha_n r)\left(T_0\cdot r\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2 J_0{}^2(\alpha_n a)}\right)$$

$$\left\{B_n\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle\right.$$

$$\left.+\mathrm{C}_n\alpha_n\left(z+h\right)\left\langle h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right\rangle\right\} \tag{3.2.3}$$

where B_n and C_n are arbitrary functions.

Now using equations (2.2.9), (3.2.2), (3.2.3) and (26) in (3.1.9) to (3.1.14), one obtains the expressions for displacements and stresses, respectively as

$$u_r=\left(K\right)\sum_{n=1}^{\infty}\left(\frac{2\left(T_0\cdot r\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2 J_0{}^2(\alpha_n a)}\right)$$

$$\left(J_1(\alpha_n r)\right)\times\left\{\left[\frac{-(z+h)\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}\right]\right.$$

$$+B_n\alpha_n^2\left\langle h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right\rangle$$

$$+\mathrm{C}_n\alpha_n^2\left\langle\left(h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right)\right.$$

$$\left.\left.+\ \alpha_n\left(z+h\right)\left(h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right)\right\rangle\right\} \tag{3.2.4}$$

$$u_z=\left(K\right)\sum_{n=1}^{\infty}\left(\frac{2\left(T_0\cdot r\right)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2 J_0{}^2(\alpha_n a)}\right)$$

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$$\left(J_0(\alpha_n r)\right)\times\left\{\left[\left[\frac{\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\alpha_n\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}\right]\right.\right.$$

$$+\left[\frac{(z+h)\left\langle h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}\right]$$

$$-B_n\alpha_n^2\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle$$

$$+C_n\alpha_n^2\left\langle 2(1-2v)\left(h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right)\right.$$

$$\left.\left.-\alpha_n(z+h)\left(h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right)\right\rangle\right\} \tag{3.2.5}$$

$$\sigma_{rr}=(2GK)\sum_{n=1}^{\infty}\left(\frac{2(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2J_0^{\,2}(\alpha_n a)}\right)$$

$$\left\{\left(\frac{J_1(\alpha_n r)}{r}-\alpha_n J_0(\alpha_n r)\right)\times\left[\frac{-(z+h)\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}\right]\right.$$

$$-\left(J_0(\alpha_n r)\right)\left[\frac{\alpha_n\cosh\left[\alpha_n(z+h)\right]+h_s\sinh\left[\alpha_n(z+h)\right]}{\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)}\right]$$

$$+B_n\alpha_n^2\left(\frac{J_1(\alpha_n r)}{r}-\alpha_n J_0(\alpha_n r)\right)\left\langle h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right\rangle$$

$$\left.+C_n\alpha_n^2\left[\begin{array}{l}2v\alpha_n\left(J_0(\alpha_n r)\right)\left\langle h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right\rangle\\+\left(\dfrac{J_1(\alpha_n r)}{r}-\alpha_n J_0(\alpha_n r)\right)\left\langle\begin{array}{l}\left(h_s\sinh\left[\alpha_n(z+h)\right]+\alpha_n\cosh\left[\alpha_n(z+h)\right]\right)\\+\alpha_n(z+h)\left(h_{s2}\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right)\end{array}\right\rangle\end{array}\right]\right\}$$

(3.2.6)

$$\sigma_{\theta\theta}=(2GK)\sum_{n=1}^{\infty}\left(\frac{2(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2J_0^{\,2}(\alpha_n a)}\right)$$

$$\left\{\left(\frac{-J_1(\alpha_n r)}{r}\right)\times\left[\frac{(z+h)\left\langle h_s\cosh\left[\alpha_n(z+h)\right]+\alpha_n\sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2\sinh(2\alpha_n h)+\alpha_n h_s\cosh(2\alpha_n h)\right]}\right]\right.$$

$$-\left(J_0(\alpha_n r)\right)\left[\frac{\alpha_n \cosh\left[\alpha_n(z+h)\right]+h_s \sinh\left[\alpha_n(z+h)\right]}{\alpha_n^2 \sinh(2\alpha_n h)+\alpha_n h_s \cosh(2\alpha_n h)}\right]$$

$$+B_n\alpha_n^2\left(\frac{J_1(\alpha_n r)}{r}\right)\left\langle h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right\rangle$$

$$+C_n\alpha_n^2\left[\begin{array}{l}2v\alpha_n\left(J_0(\alpha_n r)\right)\left\langle h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right\rangle \\ +\left(\frac{J_1(\alpha_n r)}{r}\right)\left\langle\begin{array}{l}\left(h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right) \\ +\alpha_n(z+h)\left(h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right)\end{array}\right\rangle\end{array}\right]\Bigg\} \quad (3.2.7)$$

$$\sigma_{zz}=(2GK)\sum_{n=1}^{\infty}\left(\frac{2(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2 J_0^{\,2}(\alpha_n a)}\right)\left(J_0(\alpha_n r)\right)$$

$$\left(J_0(\alpha_n r)\right)\times\left\{\left[\frac{\alpha_n(z+h)\left\langle h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2 \sinh(2\alpha_n h)+\alpha_n h_s \cosh(2\alpha_n h)\right]}\right]\right.$$

$$-B_n\alpha_n^3\left\langle h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right\rangle$$

$$\left.+C_n\alpha_n^3\left\langle\begin{array}{l}(1-2v)\left(h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right) \\ -\alpha_n(z+h)\left(h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right)\end{array}\right\rangle\right\} \quad (3.2.8)$$

and

$$\sigma_{rz}=(2GK)\sum_{n=1}^{\infty}\left(\frac{2(T_0\cdot r)\left[J_0(\alpha_n r_1)+J_0(\alpha_n r_2)+J_0(\alpha_n r_3)+J_0(\alpha_n r_4)\right]}{a^2 J_0^{\,2}(\alpha_n a)}\right)$$

$$\left(J_1(\alpha_n r)\right)\times\left\{\left[\frac{-\left\langle h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2 \sinh(2\alpha_n h)+\alpha_n h_s \cosh(2\alpha_n h)\right]}\right]\right.$$

$$-\left[\frac{\alpha_n(z+h)\left\langle h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right\rangle}{2\left[\alpha_n^2 \sinh(2\alpha_n h)+\alpha_n h_s \cosh(2\alpha_n h)\right]}\right]$$

$$+B_n\alpha_n^3\left\langle h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right\rangle$$

$$\left.+C_n\alpha_n^3\left\langle\begin{array}{l}2v\left(h_s \cosh\left[\alpha_n(z+h)\right]+\alpha_n \sinh\left[\alpha_n(z+h)\right]\right) \\ +\alpha_n(z+h)\left(h_s \sinh\left[\alpha_n(z+h)\right]+\alpha_n \cosh\left[\alpha_n(z+h)\right]\right)\end{array}\right\rangle\right\} \quad (3.2.9)$$

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In order to satisfy the boundary conditions given in the equation (3.1.15) use equations (3.2.6), (3.2.8) and (3.2.9) for B_n and C_n one obtains,

$$B_n = \frac{(1-2\nu)}{2\alpha_n^3\left[\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)\right]} \tag{3.2.10}$$

and

$$C_n = \frac{1}{2\alpha_n^3\left[\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)\right]} \tag{3.2.11}$$

Using these values of B_n and C_n in equations (3.2.4) to (3.2.9) one obtains the expressions for displacements and stresses as

$$u_r = K(1-\nu)\sum_{n=1}^{\infty}\left(\frac{2(T_0 \cdot r)\left[J_0(\alpha_n r_1) + J_0(\alpha_n r_2) + J_0(\alpha_n r_3) + J_0(\alpha_n r_4)\right]J_1(\alpha_n r)}{a^2 \alpha_n J_0^{\,2}(\alpha_n a)}\right) \times\left[\frac{h_s \sinh\left[\alpha_n(z+h)\right] + \alpha_n \cosh\left[\alpha_n(z+h)\right]}{\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)}\right] \tag{3.2.12}$$

$$u_z = K(1-\nu)\sum_{n=1}^{\infty}\left(\frac{2(T_0 \cdot r)\left[J_0(\alpha_n r_1) + J_0(\alpha_n r_2) + J_0(\alpha_n r_3) + J_0(\alpha_n r_4)\right]J_0(\alpha_n r)}{a^2 \alpha_n J_0^{\,2}(\alpha_n a)}\right) \times\left[\frac{h_s \cosh\left[\alpha_n(z+h)\right] + \alpha_n \sinh\left[\alpha_n(z+h)\right]}{\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)}\right] \tag{3.2.13}$$

$$\sigma_{rr} = -2GK(1-\nu)\sum_{n=1}^{\infty}\left(\frac{2(T_0 \cdot r)\left[J_0(\alpha_n r_1) + J_0(\alpha_n r_2) + J_0(\alpha_n r_3) + J_0(\alpha_n r_4)\right]J_1(\alpha_n r)}{a^2 r \alpha_n J_0^{\,2}(\alpha_n a)}\right) \times\left[\frac{h_s \sinh\left[\alpha_n(z+h)\right] + \alpha_n \cosh\left[\alpha_n(z+h)\right]}{\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)}\right] \tag{3.2.14}$$

$$\sigma_{\theta\theta} = 2GK(1-\nu)\sum_{n=1}^{\infty}\left(\frac{2(T_0 \cdot r)\left[J_0(\alpha_n r_1) + J_0(\alpha_n r_2) + J_0(\alpha_n r_3) + J_0(\alpha_n r_4)\right]}{a^2 J_0^{\,2}(\alpha_n a)}\right) \left\{\left[\left(\frac{J_1(\alpha_n r)}{r\alpha_n} - J_0(\alpha_n r)\right)\right]\times\left[\frac{h_s \sinh\left[\alpha_n(z+h)\right] + \alpha_n \cosh\left[\alpha_n(z+h)\right]}{\alpha_n^2 \sinh(2\alpha_n h) + \alpha_n h_s \cosh(2\alpha_n h)}\right]\right\} \tag{3.2.15}$$

$$\sigma_{zz} = 0 \tag{3.2.16}$$

and

$$\sigma_{rz} = 0 \tag{3.2.17}$$

IV. NUMERICAL CALCULATIONS

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4.1 Dimensions

Radius of plate $a = 1m$,

Thickness (Height) of plate $h = 2m$.

4.2 Thermoelastic Constants

Poisson ratio $\nu = 0.35$,

Lamé constant $\mu = 26.67$.

Young's modulus $E = 130\ GPa$,

Strength of Impulse heating $T_0 = 100^0 C$,

Relative heat transfer coefficients $h_{s1} = 10$.

4.3 Roots of transcendental equation

Let $\alpha_1 = 3.8317,\ \alpha_2 = 7.0156,\ \alpha_3 = 10.1735,\ \alpha_4 = 13.3237,\ \alpha_5 = 16.470,$

$\alpha_6 = 19.6159,\ \alpha_7 = 22.7601,\ \alpha_8 = 25.9037,\ \alpha_9 = 29.0468,\ \alpha_{10} = 32.18$

are the roots of transcendental equation $J_1(\alpha a) = 0$.

4.4 Material Properties

Property	Aluminum Pure	Copper Pure	Iron Pure
Thermal diffusivity α $(m^2 s^{-1})$	84.18×10^{-6}	112.34×10^{-6}	20.34×10^{-6}
Thermal conductivity k (W/mk)	204.2	386	72.7
Density ρ (kg/m^3)	2707	8954	7897
Specific heat c_p (J/kgK)	896	383	452
Coefficient of linear thermal expansion a_t $(1/K)$	23.1×10^{-6}	16.5×10^{-6}	11.8×10^{-6}

For convenience setting $A = K(1-\nu),\ B = 2GK(1-\nu)$. in the expressions (3.2.12) to (3.2.15). The numerical expressions for temperature, displacement and stress components are obtained by equations (2.2.9) and (3.2.12) to (3.2.17).

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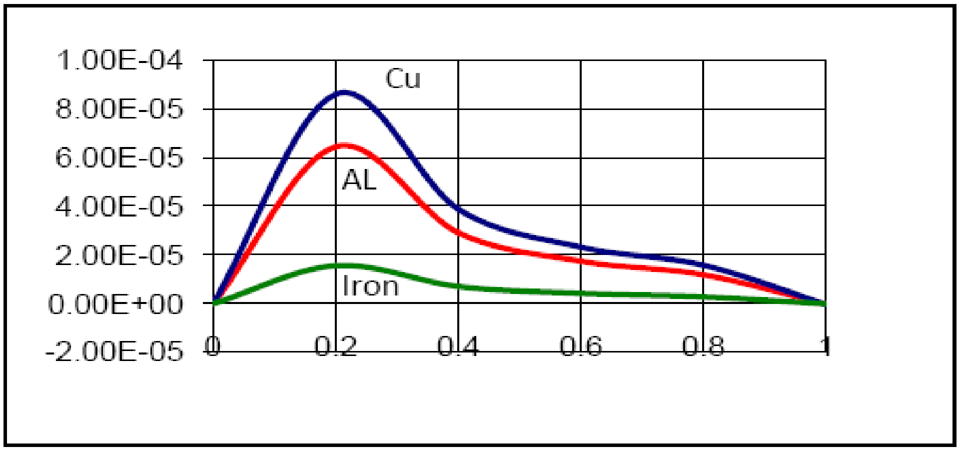

Figure1: The radial displacement function u_r/A in radial direction.

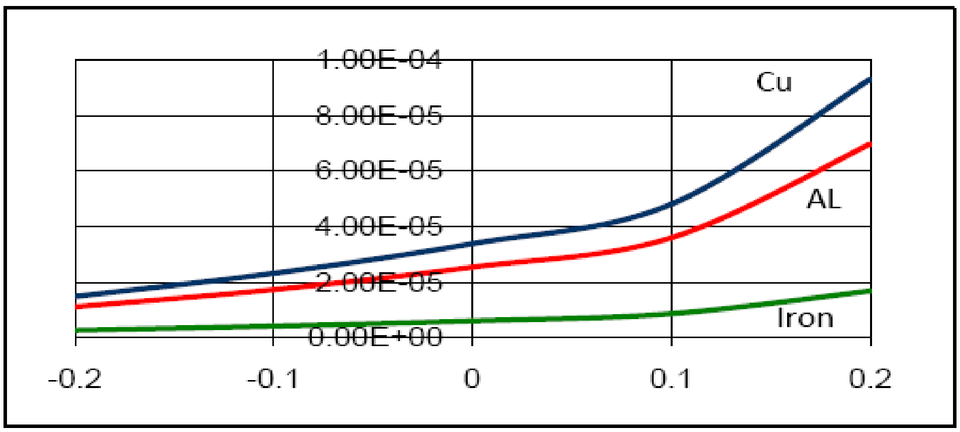

Figure2: The radial displacement function u_r/A in axial direction.

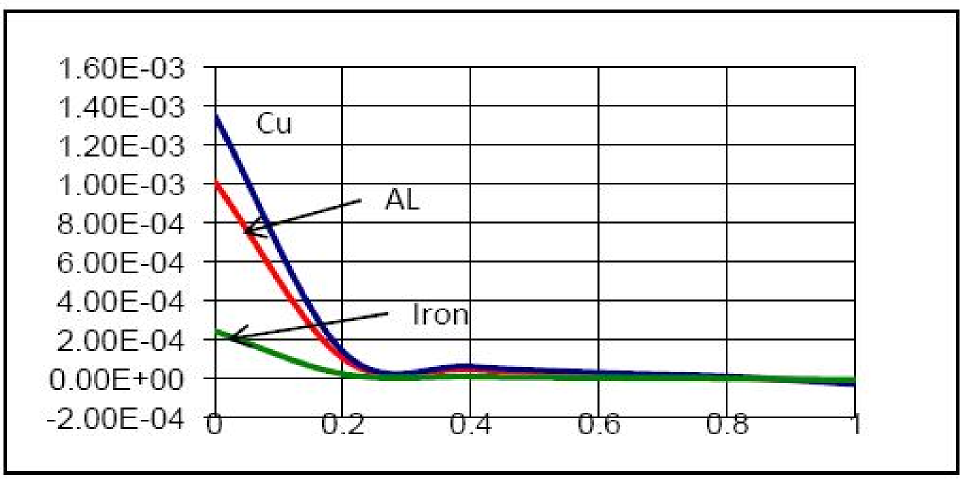

Figure3: The axial displacement function u_z /A in radial direction.

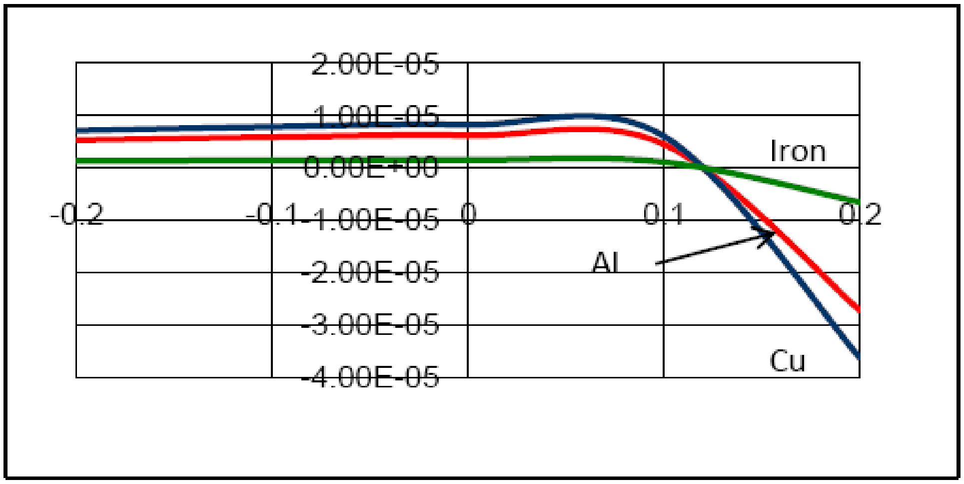

Figure 4 : The axial displacement function u_z/A in axial direction.

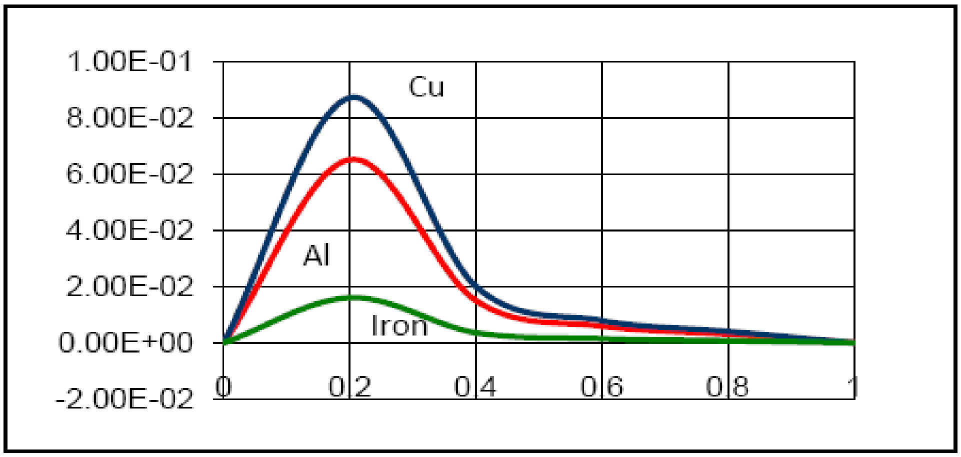

Figure 5 : The radial stress function σ_{rr}/B in radial direction.

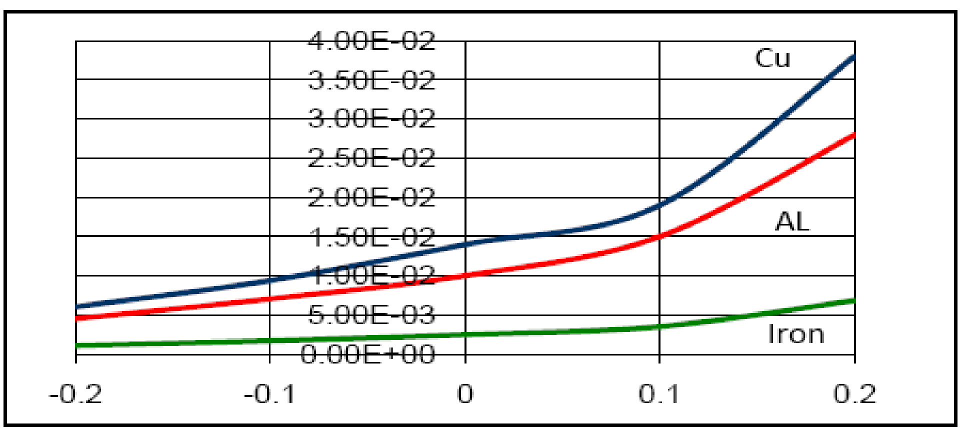

Figure 6 : The radial stress function σ_{rr}/B in axial direction.

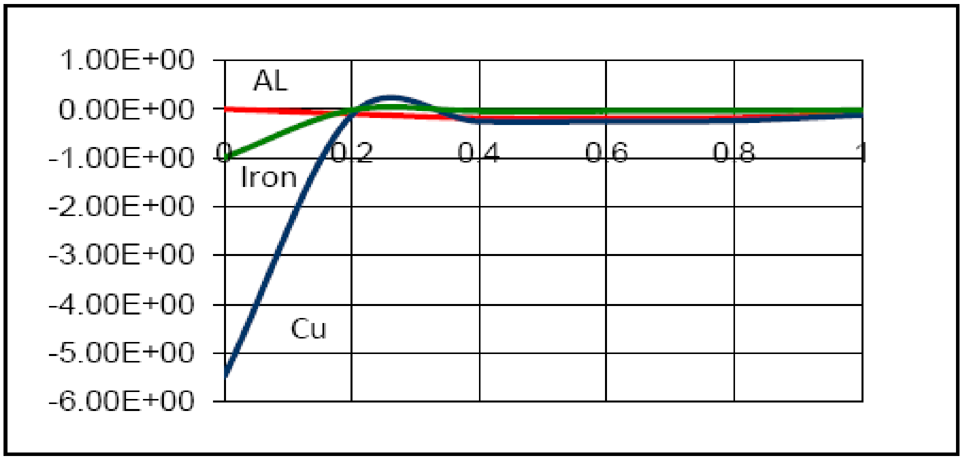

Figure 7 : The stress function $\sigma_{\theta\theta}/B$ in radial direction.

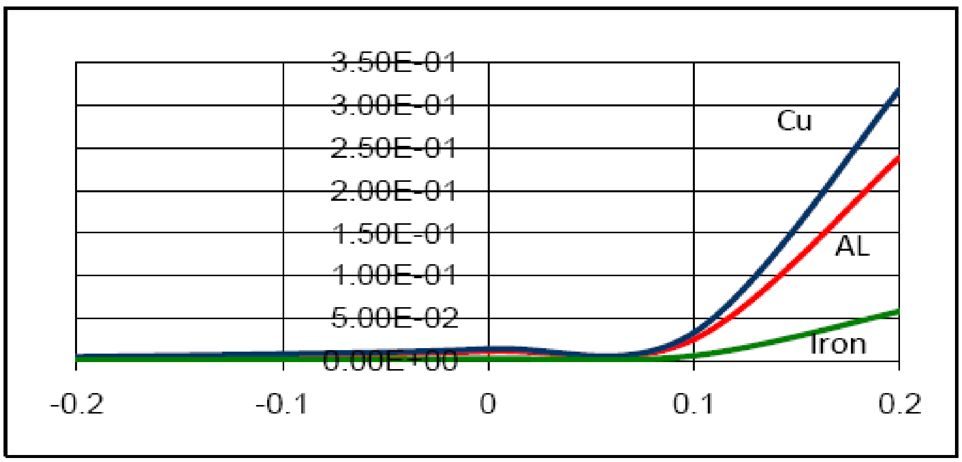

Figure 8 : The stress function $\sigma_{\theta\theta}/B$ in axial direction.

V. CONCLUDING REMARKS

In this paper a thick circular plate is considered under steady temperature field and determined the expressions for temperature, displacement and thermal stress functions due to impulse heating on the upper surface. Heat dissipates by convection from the lower boundary surface into the surrounding where as the exchange of heat stopped by insulating outer circular surface. The Dirac-delta function has been used to describe impulse heating. As a special case mathematical model is constructed for different metals viz Copper (Pure), Aluminium (Pure) and Iron (Pure) thick circular plate with the material properties specified as above.

From figure 1 and 2, radial displacement function u_r shows sinusoidal fluctuations from centre to outer circular surface in radial direction where as in axial direction it is decreases from upper surface to lower surface.

From figure 3 and 4, axial displacement function u_z is maximum at centre and decreases towards outer circular surface where as in axial direction it increases from upper surface to lower surface.

From figure 5 and 6, the radial stress function σ_{rr} develops tensile stresses in radial direction within circular region $0 \le r \le 1$ and it decreases from upper surface to lower surface in axial direction. Also it can be observed that the radial stress function σ_{rr} satisfies the equilibrium conditions i.e. $\sigma_{rr} = 0$ at outer traction free surface $r = 1$.

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From figure 7 and 8, the angular stress function $\sigma_{\theta\theta}$ develops compressive stresses near centre in radial and axial direction it develops tensile stresses on the upper surface where as it is almost zero at the lower surface.

Due to applying the impulse heat supply on the upper surface on the plate, the radial and axial displacements occurs near the centre and plate expands towards the lower direction. The axial stress component and resultant stress component are zero due to exchange of heat through lower boundary into surrounding medium. It means we may find out that displacement and stress components occurs near heat source. The numerical values of the displacements and stresses for the plates of metals iron, aluminum and copper are in the proportion and follows relation $Iron \le Alu\min ium \le Copper$. It means these values are directly proportional to their thermal diffusivity. The results presented here will be more useful in engineering problem particularly in the impulse heat sealer.

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