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Non-Homogeneous Problem of Thermal Stresses in the Context of Cylindrical Surface Heat Source

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Abstract: *This paper is concerned with inverse quasi-static thermal stress analysis in a thin circular cylinder due to internal heat generation under transient temperature conditions. The internal heat generation is taken as a cylindrical surface heat source in the annular region of the linear length of the cylinder and is situated concentrically inside the cylinder releases heat spontaneously at the time* $t=\varsigma$. *The thin circular cylinder is subjected to known internal heat flux and the convection due to dissipation takes place through the upper surface and circular boundary surface. The unknown convective heat flux applied at the upper surface of cylinder are required to be determined. The integral transform methods are used for the analytical solution of non-homogeneous boundary value problem and illustrated graphically.*

Keywords: quasi-static , thermal stress , circular cylinder . heat generation, heat flux.

I. INTRODUCTION

Wankhede [9] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. **Khalsa , et al. [5]** have studied two-dimensional transient problem for a thick disc with internal heat sources . **Ghadle et al. [4]** have studied an inverse quasi-static thermoelastic problem of a thick circular plate. **Khobragade**, **et al. [6]** have studied , an inverse thermoelastic problem of finite length thick hollow cylinder with internal heat source .

Warsha K. Dange [1,2 ,3] has determined thermal stresses in a hollow Cylinder, thermal stresses in a hollow cylinder with internal heat generation and also thermal stresses in annular disc due to boundary conditions of radiation type. This paper is concerned with inverse quasi-static thermal stress analysis in a thin circular cylinder due to internal heat generation under transient temperature conditions. The internal heat generation is taken as a cylindrical surface heat source in the annular region of the linear length of the cylinder and is situated concentrically inside the cylinder releases heat spontaneously at the time $t=\varsigma$. The thin circular cylinder is subjected to known internal heat flux and the convection due to dissipation takes place through the upper surface and circular boundary surface. The unknown convective heat flux applied at the upper surface of cylinder are required to be determined. The integral transform methods are used for the analytical solution of non-homogeneous boundary value problem

MATHEMATICAL FORMULATION OF PROBLEM :

The heat source $g(r,z,t)$ is an instantaneous cylindrical surface heat source of strength g_i, in annular region of linear length of cylinder and is situated concentrically inside the cylinder releases heat spontaneously at time $t=\varsigma$ is given by

$$g(r,z,t)=\frac{g_i}{2\pi r}\delta(r-r_1)\delta(z-z_1)\delta(t-\varsigma) \quad (1)$$

where δ is the Dirac-delta function.

The governing partial differential equation of non-homogeneous boundary value problem,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g(r,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

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with the boundary conditions

$$\frac{\partial T}{\partial z} - h_1 T = U(r,t) \quad \text{(unknown)} \quad \text{at } z = -\frac{h}{2},\ 0 \le r \le a \qquad (3)$$

$$\frac{\partial T}{\partial z} = f(r,t) \quad \text{(known)} \quad \text{at } z = \xi,\ 0 \le r \le a \qquad (4)$$

$$\frac{\partial T}{\partial z} + h_2 T = 0 \quad \text{at } z = \frac{h}{2},\ 0 \le r \le a \qquad (5)$$

and

$$\frac{\partial T}{\partial z} + h_3 T = 0 \quad \text{at } r = a,\ -\frac{h}{2} \le z \le \frac{h}{2} \qquad (6)$$

$$T = 0 \quad \text{at } t = 0 \qquad (7)$$

where h_1, h_2, h_3 be the relative heat transfer coefficients on the lower, upper and circular surface of the thin circular cylinder respectively,

k is thermal conductivity of material of cylinder and

α is thermal diffusivity of material of cylinder.

the expressions for displacement and thermal stresses for plane stress i.e. solid thin circular cylinder with radius *a* are

$$u = (1+\nu) a_t \left[\frac{1}{r}\int_a^r \tau r dr + \left(\frac{1-\nu}{1+\nu} \frac{r}{a^2} \right) \left(\int_0^a \tau r dr \right) \right] \qquad (8)$$

$$\sigma_{rr} = a_t E \left[\frac{-1}{r^2} \int_a^r \tau r dr + \left(\frac{1}{a^2} \int_0^a \tau r dr \right) \right] \qquad (9)$$

$$\sigma_{\theta\theta} = a_t E \left[\frac{1}{r^2} \int_a^r \tau r dr + \left(\frac{1}{a^2} \int_0^a \tau r dr - \tau \right) \right] \qquad (10)$$

$$\sigma_{zz} = \sigma_{r\theta} = 0 \qquad (11)$$

For long solid circular cylinder in plane strain the radial displacement and thermal stresses are given by

$$u = \left(\frac{1+\nu}{1-\nu} a_t \right) \left[\left(\frac{1}{r} \int_a^r \tau r dr \right) + \left((1-2\nu) \frac{r}{a^2} \right) \left(\int_0^a \tau r dr \right) \right] \quad \text{for } \varepsilon_{zz} = 0$$

and

$$u = \left(\frac{1+\nu}{1-\nu} a_t \right) \left[\left(\frac{1}{r} \int_a^r \tau r dr \right) + \left(\frac{1-3\nu}{1+\nu} \frac{r}{a^2} \right) \left(\int_0^a \tau r dr \right) \right] \quad \text{for } \varepsilon_{zz} = \varepsilon_0 \qquad (12)$$

$$\sigma_{rr} = \frac{a_t E}{1-\nu} \left[\frac{-1}{r^2} \int_a^r \tau r dr + \left(\frac{1}{a^2} \int_0^a \tau r dr \right) \right] \qquad (13)$$

$$\sigma_{\theta\theta} = \frac{a_t E}{1-\nu} \left[\frac{1}{r^2} \int_a^r \tau r dr + \left(\frac{1}{a^2} \int_0^a \tau r dr - \tau \right) \right] \qquad (14)$$

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$$\sigma_{zz}=\frac{a_t E}{1-\nu}\left(\frac{2\nu}{a^2}\int_0^a \tau r dr-\tau\right) \quad \text{for } \varepsilon_{zz}=0$$

and

$$\sigma_{zz}=\frac{a_t E}{1-\nu}\left(\frac{2}{a^2}\int_0^a \tau r dr-\tau\right) \quad \text{for } \varepsilon_{zz}=\varepsilon_0 \qquad (15)$$

The equations (1) to (15) constitute the mathematical formulation of the problem.

THE SOLUTION FOR UNKNOWN TEMPERATURE

The integral transform method has been used to find analytical solution of non-homogeneous boundary value problem (equations 1- 7).

The Hankel transform and its inverse developed to remove space variable *r,* defined as

$$\overline{T}(\alpha_n,z,t)=\int_0^a rJ_0(\alpha_n r)T(r,z,t)\,dr \qquad (16)$$

$$T(r,z,t)=\sum_{n=1}^{\infty}\frac{2J_0(\alpha_n r)\overline{T}(\alpha_n,z,t)}{a^2\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]} \qquad (17)$$

where α_n are positive roots of the transcendental equation

$$h_3 J_0(\alpha a)-\alpha J_1(\alpha a)=0.$$

and the finite Fourier transform and its inverse developed to remove space variable *z*, defined as

$$\overline{\overline{T}}(\alpha_n,\beta_m,t)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{\beta_m\cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]+h_1\sin\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\}\overline{T}(\alpha_n,z,t)\,dr \qquad (18)$$

$$\overline{T}(\alpha_n,z,t)=\sum_{n=1}^{\infty}\frac{1}{\sqrt{N}}\left\{\beta_m\cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]+h_1\sin\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\}\overline{\overline{T}}(\alpha_n,\beta_m,t) \qquad (19)$$

where *N* is normality constant which can be determined by orthogonality of circular functions as

$$\sqrt{N}=\frac{4}{2\beta_m^2\left(\xi+\frac{h}{2}\right)h+\beta_m\sin\left[2\beta_m\left(\xi+\frac{h}{2}\right)\right]}$$

and β_m are positive roots of transcendental equation

$$\beta h_2\cos(\beta h)-\beta^2\sin(\beta h)=0.$$

On applying Hankel transform as defined in equation (16) and Finite Fourier transform as defined in equation(18) and their corresponding inverse transforms, one obtain the temperature distribution within circular cylinder as

$$T(r,z,t)=\left(\frac{8}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)+J_1^2(\alpha_n a)}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right) \tag{20}$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}$$

Since $T=0$ at $t=0$

$\Rightarrow$ The temperature change $\tau=T-T_i=T$ (21)

The unknown convective heat flux applied on the lower surface of cylinder is given by

$$U(r,t)=\left(\frac{8}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)+J_1^2(\alpha_n a)}\right)\left(\frac{-h_1\beta_m}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{22}$$

DISPLACEMENT AND THERMAL STRESSES IN THIN SOLID CIRCULAR CYLINDER

The expressions for displacement and thermal stresses for plane stress i.e. solid thin circular cylinder can be obtained by using equations (8) to (11) as

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{a(1+\nu)J_1(\alpha_n r)+r(1-\nu)J_1(\alpha_n a)}{\alpha_n a\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)$$

$$\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{23}$$

$$\sigma_{rr}(r,z,t)=\left(\frac{8a_tE}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{rJ_1(\alpha_n a)-aJ_1(\alpha_n r)}{\alpha_n a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{24}$$

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$$\sigma_{\theta\theta}(r,z,t)=\left(\frac{8a_tE}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+rJ_1(\alpha_n a)-a\cdot rJ_0(\alpha_n a)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \quad (25)$$

$$\sigma_{zz}=\sigma_{rz}=0 \quad (26)$$

DISPLACEMENT AND THERMAL STRESSES IN LONG SOLID CIRCULAR CYLINDER

For long solid circular cylinder in plane strain the radial displacement and thermal stresses can be obtained by using equations (12) to (15) as

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+(1-2\nu)rJ_1(\alpha_n a)}{\alpha_n a\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \text{ for } \varepsilon_{zz}=0 \quad (27)$$

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{a(1+\nu)J_1(\alpha_n r)+(1-3\nu)rJ_1(\alpha_n a)}{\alpha_n\cdot a\cdot(1+\nu)\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}$$

$$\text{for } \varepsilon_{zz}=\varepsilon_0 \quad (28)$$

$$\sigma_{rr}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{rJ_1(\alpha_n a)-aJ_1(\alpha_n r)}{\alpha_n a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \quad (29)$$

$$\sigma_{\theta\theta}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+rJ_1(\alpha_n a)-a\cdot r\cdot\alpha_n\cdot J_0(\alpha_n a)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\bar{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t \bar{\bar{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}$$

(30) $$\sigma_{zz}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{2\nu\cdot J_1(\alpha_n a)-aJ_0(\alpha_n r)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\bar{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t \bar{\bar{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}\text{ for }\varepsilon_{zz}=0 \quad (31)$$

$$\sigma_{zz}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{2\cdot J_1(\alpha_n a)-a\cdot\alpha_n\cdot J_0(\alpha_n r)}{\alpha_n\cdot a\cdot\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\bar{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t \bar{\bar{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}$$

for $\varepsilon_{zz}=\varepsilon_0$ (32)

NUMERICAL CALCULATION

SPECIAL CASE:

Setting $f(r)=\left(r^2-a^2\right)^2$

$$g(r,z,t)=\frac{g_i}{2\pi r}\delta(r-r_1)\delta(z-0)\delta(t-\varsigma)$$

One have,

$$\bar{f}(\alpha_n)=\frac{8a}{\alpha_n^5}\left[\left(8-a^2\alpha_n^2\right)J_1(\alpha_n a)-49\alpha_n J_0(\alpha_n a)\right]$$

$$\bar{\bar{g}}(\alpha_n,\beta_m,\tau)=\frac{g_i}{2\pi r}r_1J_0(\alpha_n r_1)\left[\beta_m\cos\left(\beta_m\frac{h}{2}\right)+h_1\sin\left(\beta_m\frac{h}{2}\right)\right]\delta(t-\varsigma)$$

where r_1 is the radius of annular region situated $0<r_1<r$ and δ is the Dirac-delta function.

The heat source $g(r,z,t)$ is an instantaneous cylindrical heat source of strength $g_i=50Btu/hr.ft$, situated at the center of the circular cylinder along radial direction and releases its heat instantaneously at the time $t=\varsigma=2hr$.

DIMENSION:

Radius of a circular cylinder $a = 1ft,$

Central circular path of cylinder $r_1 = 0.5ft$,

Thickness of a circular cylinder $z = 0.1ft,$

Level surface where known heat flux is situated inside circular cylinder $\xi = 0.05ft.$

MATERIAL PROPERTIES:

The numerical calculation has been carried out for an Aluminum (Pure) hollow circular cylinder with the material properties as,

Density $\rho = 169lb/ft^3$,

Specific heat $c_p = 0.208Btu/lb^0F$,

Thermal conductivity $k = 117Btu/\left(hr \cdot ft \cdot {}^0F\right)$,

Thermal diffusivity $\alpha = 3.33ft^2/hr$,

Poisson ratio $\nu = 0.35$,

Coefficient of linear thermal expansion $a_t = 12.84 \times 10^{-6} \dfrac{1}{F}$,

Lamé constant $\mu = 26.67$

Young's modulus $E = 130GPa.$

ROOTS OF TRANSCENDENTAL EQUATION:

The $\alpha_1 = 0.471448, \alpha_2 = 1.07908, \alpha_3 = 1.696798, \alpha_4 = 2.312416, \alpha_5 = 2.928661$ are the positive roots of transcendental equation $h_3 J_0\left(\alpha a\right) - \alpha J_1\left(\alpha a\right) = 0$ and the $\beta_1 = 2.627675, \beta_2 = 5.307325, \beta_3 = 8.067136, \beta_4 = 10.90871, \beta_5 = 13.819192,$ heat transfer coefficient $h_1 = h_2 = h_3 = 10$ are the positive roots of transcendental equation

$\beta h_2 \cos\left(\beta h\right) - \beta^2 \sin\left(\beta h\right) = 0$.

The numerical calculation has been carried out by setting for convenience,

$$\chi = \frac{8}{ka^2},\ \gamma = \left(\frac{16\left(1+\nu\right)\alpha}{ka^2}\right),\ \lambda = \left(\frac{8\alpha E}{ka^2}\right) \text{ and } \xi = \left(\frac{8\alpha E}{ka^2\left(1-\nu\right)}\right).$$

II. CONCLUDING REMARKS

In this problem a circular cylinder is considered under transient temperature field and determined the expressions for unknown temperature, displacement and thermal stress functions due to convective heat transfer and internal heat generation as a cylindrical surface heat source. The application of convective heat transfer causes temperature changes followed by displacement and thermal stresses within solid thin circular cylinder.

From figure 1: The temperature changes due to convective heat transfer with time and the convection due to dissipation through circular boundary surface is observed.

From figure 2: The temperature changes cause's heat transfer within circular cylinder which result displacement at circular boundary surface.

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From figure 3: The radial thermal stresses are zero at centre and at circular boundary. For long circular cylinder the radial thermal stresses are less as compare to thin cylinder. The radial stresses develop tensile stresses in both thin and long cylinder.

From figure 4: The development of compressive angular thermal stresses is observed. The angular thermal stresses are zero at centre.

From figure 5: The compressive axial thermal stresses seen around the centre where internal cylindrical surface heat source is situated.

So expansion is observed in radial as well as in axial direction which result into bending of thin circular cylinder whereas the thermal stresses developed in long circular cylinder is absorb along thickness and hence displacement is inversely proportional to length of cylinder.

The thermoelastic problems due to surface heat source has wide scope of applications in engineering problems like surface coatings, dielectric barriers, anti reflective coatings of glass and solar cells etc. Any particular case of special interest is derived by analytical expressions for temperature, displacement and thermal stresses.

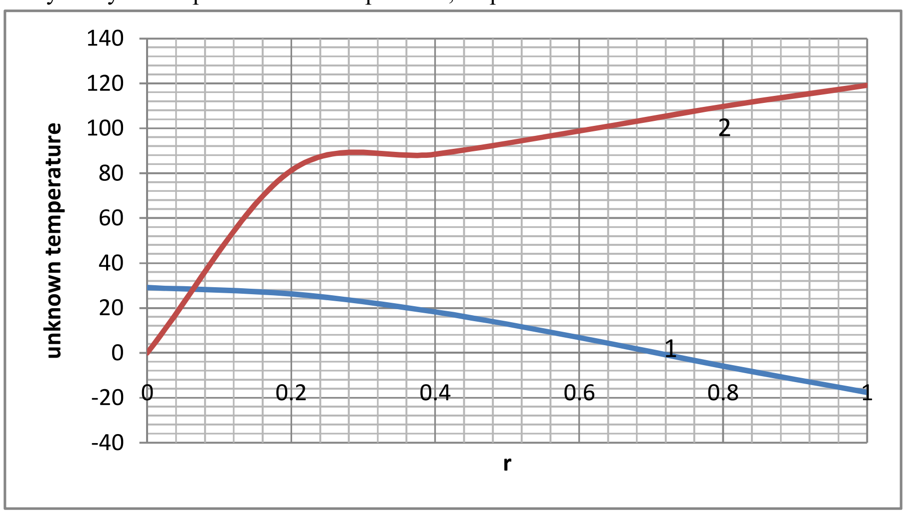

Figure 1: Unknown Temperature distribution $\frac{U(r,z,t)}{\chi}$ along radial direction.

Line 1 represents Unknown Temperature distribution $\frac{U(r,z,t)}{\chi}$ for thin solid circular cylinder.

Line 2 represents Unknown Temperature distribution $\frac{U(r,z,t)}{\chi}$ for long solid circular cylinder.

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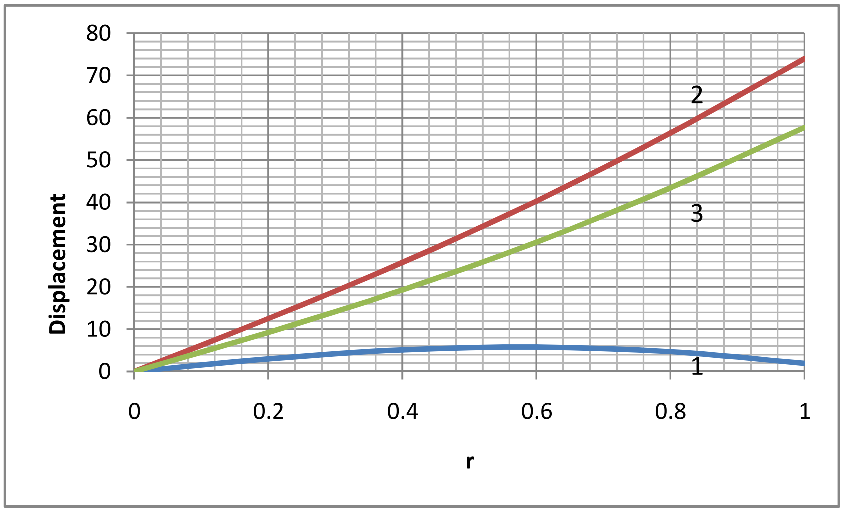

Figure 2 :Displacement function $\dfrac{u(r,z,t)}{\gamma}$ along radial direction.

Line 1 represents displacement function $\dfrac{u(r,z,t)}{\gamma}$ for thin solid circular cylinder.

Line 2 represent displacement function $\dfrac{u(r,z,t)}{\gamma}$ for long solid circular cylinder when $\varepsilon_{zz} = 0$.

Line 3 represent displacement function $\dfrac{u(r,z,t)}{\gamma}$ for long solid circular cylinder when $\varepsilon_{zz} = \varepsilon_0$.

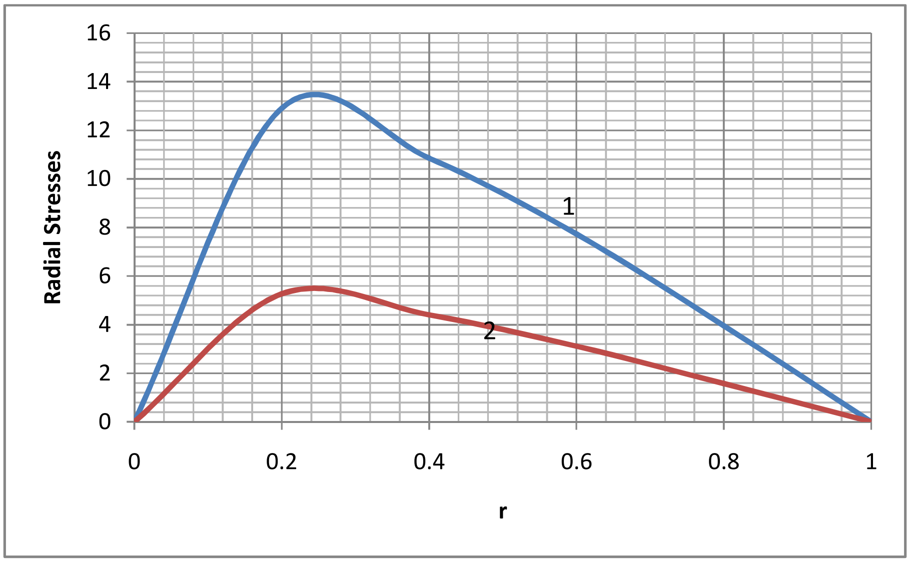

Figure 3: Radial stress function $\dfrac{\sigma_{rr}(r,z,t)}{\lambda}$ along radial direction.

Line 1 represents radial stress function $\frac{\sigma_{rr}(r,z,t)}{\lambda}$ for thin solid circular cylinder.

Line 2 represents radial stress function $\frac{\sigma_{rr}(r,z,t)}{\lambda}$ for long solid circular cylinder which is almost zero.

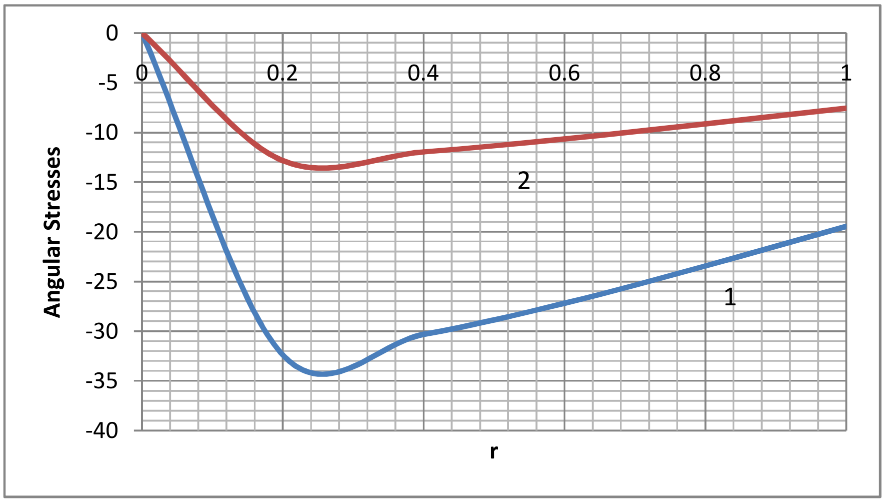

Figure 4: Angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ along radial direction.

Line 1 represents angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ for thin solid circular cylinder.

Line 2 represents angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ for long solid circular cylinder which is almost zero.

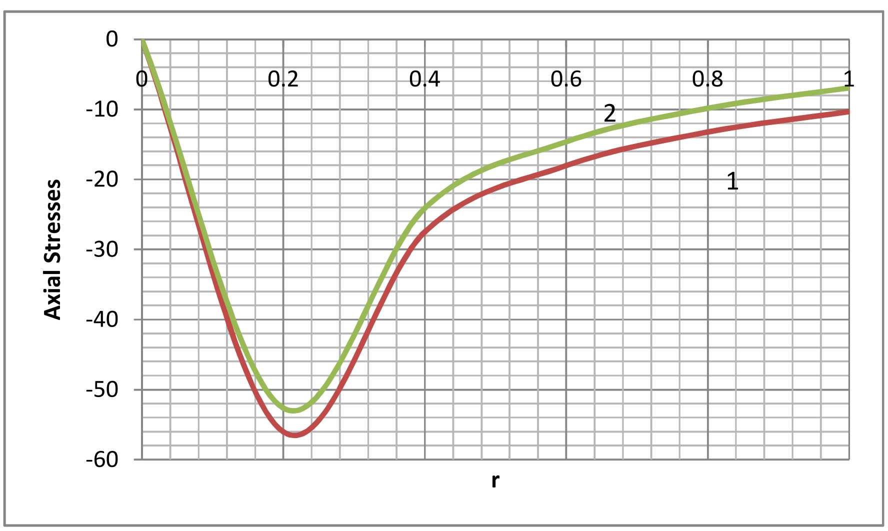

Figure 5: Axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ along radial direction.

The axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ for thin solid circular cylinder in plane state is zero.

Line 1 represent axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ for long solid circular cylinder when $\varepsilon_{zz} = 0$.

Line 2 represent axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ for long solid circular cylinder when $\varepsilon_{zz} = \varepsilon_0$.

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