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A Study of Decimal Place Value System in Ancient Indian Mathematics

Vijay P Sangale[1] and Govardhan K Sanap[2]
Professor, Department of Mathematics[1,2]
R. B. Attal College, Georai, Beed, Maharashtra, India[1]
Sunderrao Solanke Mahavidyalaya, Majalgaon, Maharashtra, India[2]

Abstract: *The aim of this article is to overview of ancient Indian Mathematics in respect of decimal place value system. The base ten system is not much different than other number systems. It is called the Hindu Arabic system. The digits were started to appear in India a few centuries before Christ. The decimal system emerged as we know it in 825 in a book by Arabian mathematician al-Khwarizmi. Fibonacci popularized it in Europe around 1200. The earliest evidence of decimal place value system in India and south east Asia is from the middle of the first millennium CE. The written evidence in a decimal place value notation was found on the copper plate from Gujarat, India in the date 595 CE. The decimal numerals recording have also been found in stone inscriptions in Indonesia and Cambodia in the years 683 CE, where Indian cultural influence was substantial.*

Keywords: decimal place value system, duodecimal system, decimal fractions, Brahmi numerals, Vedic literature, centesimal system etc.

I. INTRODUCTION

There are various number systems used in ancient world, like Senary (base 6), Duodecimal (base 12), Hexadecimal (base 16), Vigesimal (base 20), Sexagesimal (base 60) etc. The Old Babylonians had a sexagesimal (base 60) system that included sexagesimal fractions like our decimal numbers. Mayans were using vigesimal number system with base 20. Languages using duodecimal number systems are uncommon. Languages in the Nigerian middle belt such as Janji, Gbiri-Niragu, Piti, Gwandara and the Chepang of Nepal are known to use duodecimal number system. The Dozenal Society of America and the Dozenal Society of Great Britain promoted widespread adoption of the base-twelve system [2]. They use the word 'dozenal system' instead of duodecimal system to avoid the more overtly base-ten terminology. Base-ten arithmetic has been used by humans for thousands of years, and seems to have arisen independently a few times, most likely because we have ten fingers, and they make a ready-made counting device to use simply.

Jim Al-Khalili stated that "Two of the most famous Baghdadi scholars, the philosopher Al-Kindi and the mathematician Al-Khawarizmi, were certainly the most influential in transferring the Hindu number system to the Muslim world [1, 3]. Both wrote books on the subject during al-Ma'mun's reign, and their work was translated into Latin and transmitted to the West, thus introducing Europeans to the decimal system, which was known in the Middle Ages only as Arabic numerals. But it would be many centuries before it was widely accepted in Europe. One reason for this was sociological, the decimal numbers were considered for a long time as symbols of the evil Muslim foe. Al-Uqlidisi developed a notation to represent decimal fractions in his text *The Arithmetic of Al-Uqlîdisî* . The numerals came to fame due to their use in the pivotal work of the mathematician Al-Khwarizmi whose book *On the Calculation with Hindu Numerals* was written about 825, and the Arab mathematician Al-Kindi, who wrote four volumes "On the Use of the Indian Numerals" (Ketab fi Isti'mal al-'Adad al-Hindi) about 830. They, amongst other works, contributed to the diffusion of the Indian system of numeration in the Middle-East and the West.

People have been counting in tens and hundreds since earliest antiquity, probably because we have ten fingers. The English, Latin, Greek, Russian, Sanskrit etc. words for ten and hundred are manifestly related, which shows that people used to count by tens and hundreds before the Indo-European languages separated, on the other hand, there is no common Indo-European word for 1000 (British 'thousand', Latin 'mille', Greek 'chilia' etc. are not so related), suggesting that speakers of primitive Indo-European did not push counting that far. The Syrian bishop Severus Sebokth wrote in the middle of seventh century CE about the 'nine signs' of the Indians for expressing numbers. However, how, when, and

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where the first decimal place value system was invented is not so clear. The earliest extant script used in India was the *kharoshti* script used in the *Gandhara* culture of the north-west India. It is thought to be of Aramaic origin and it was in use from the fourth century BC to the fourth century CE [5]. Almost contemporaneously, another script, the *Brahmi* script, appeared on much of the subcontinent, and would later become the foundation of many scripts of South Asia and South-east Asia. Both scripts had numeral symbols and number systems, which were initially not based on a place-value system. Its significance is stated by the great French mathematician, Laplace that ' The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and absolute value emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated'. As Laplace noted, the scheme is anything but 'trivial,' since it eluded the best minds of the ancient world, even super human geniuses such as Archimedes. The great mathematician Archimedes saw far beyond the mathematics of his time, even anticipating numerous important ideas of modern calculus and numerical analysis. He was also very skilled in applying mathematical principles to engineering and astronomy [3]. Nonetheless he used a cumbersome Greek numeral system for calculations. Archimedes' computation of pi, numerical interval analysis, was performed without either positional notation or trigonometry.

II. DECIMAL ENUMERATION IN THE ṚIGVEDA

During the Vedic period (1500–500 BC), motivated by geometric construction of the fire altars and astronomy, the use of a numerical system and of basic mathematical operations developed in northern India. Hindu cosmology required the mastery of very large numbers such as the *kalpa* (the lifetime of the universe) said to be 4,320,000,000 years and the "orbit of the heaven" said to be 18,712,069,200,000,000 *yojanas*. Numbers were expressed using a "named place-value notation", using names for the powers of like 10, *dasa, shatha, sahasra, ayuta, niyuta, prayuta, arbuda, nyarbuda, samudra, Madhya, anat, parardha* etc. the last of these being the name for a trillion. For example, the number 48,572 was expressed as '4 *ayuta*, 8 *sahasra*, 5 *shatha*, 7 *dasa*, 2.' In the Buddhist text *Lalitvistara* the *Buddha* is said to have narrated a scheme of numbers up to 10^{53}.

A decimal system representation was employed in a verse composition technique, later labeled *Bhuta-Sankhya* used by early Sanskrit authors of technical books. Since many early technical works were composed in verse, numbers were often represented by objects in the natural or religious world that correspondence to them; this allowed a many-to-one correspondence for each number and made verse composition easier [5]. According to Plofker, the number 4, for example, could be represented by the word Veda (since there were four of these religious texts), the number 32 by the word "teeth" (since a full set consists of 32), and the number 1 by "moon" (since there is only one moon). So, Veda/teeth/moon would correspond to the decimal numeral 1324, as the convention for numbers was to enumerate their digits from right to left.

The form of numerals in Ashoka's inscriptions in the Brahmi script (middle of the third century BC) involved separate signs for the numbers 1 to 9, 10 to 90, 100 and 1000. A multiple of 100 or 1000 was represented by a modification of the sign for the number using the sign for the multiplier number. Such enciphered numerals directly represented the named place-value numerals used verbally [6]. They continued to be used in inscriptions until the end of the 9th century. The first Brahmi numerals ancestors of Hindu-Arabic numerals, used by Ashoka in his Edicts of Ashoka circa 250 BC. In his seminal text of 499 CE, Aryabhata devised a novel positional number system, using Sanskrit consonants for small numbers and vowels for powers of 10. Using the system, numbers up to a billion could be expressed using short phrases, e. g., *khyu-ghṛ* representing the number 4,320,000. The system did not catch on because it produced quite unpronounceable phrases, but it might have driven home the principle of positional number system (called *dasa-gunottara*, exponents of 10) to later mathematicians. A more elegant *katapayadi* scheme was devised in later centuries representing a place-value system including zero.

The principles of decimal enumeration had already been mastered by the time of the Ṛigveda, the oldest layer of Vedic literature. The Sanskrit terms for the nine numerals occur several times in the Ṛigveda. Among powers of ten, the Ṛigveda frequently uses das´a (ten), s´ata (hundred) and sahasra (thousand); ayuta (ten thousand) too is mentioned in a few hymns. For compound numbers, the names for nine numerals and powers of ten are combined as in our verbal decimal terminology; e.g., "seven hundred and twenty" is expressed as sapta s´atāni viṁs´atiḥ in Ṛigveda (I.164.11). In a detailed study of decimal enumeration in the Ṛgveda, B. Bavare and P. P. Divakaran have shown that this combination is actually

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an application of the more general grammatical rules of nominal composition followed in Sanskrit from the Vedic time (at least) and formulated later by Pāṇini. Thus, the verbal decimal terminology – the precursor to, and a certain manifestation of, the abstract place value principle – is an offshoot of the grammatical principles of Vedic Sanskrit.

III. DECIMAL PLACE VALUE SYSTEM IN LITERATURE OF ARYABHATTA

Indian mathematicians Aryabhatta stated that '*sthānam sthānam daśa guṇam*' meaning 'From place to place, ten times in value'. Indian mathematicians and astronomers also developed Sanskrit positional number words to describe astronomical facts or algorithms using poetic sutras. A system called *bhūta-sankhya* ("object numbers" or "concrete numbers") was employed for representing numerals in Sanskrit verses, by using a concept representing a digit to stand for the digit itself. The Jain text entitled Lokavibhaga dated 458 CE, mentions the objectified numeral '*panchabhyah khalu shunyebhyah param dve sapta chambaram ekam trini cha rupam cha*'meaning, "five voids, then two and seven, the sky, one and three and the form", i.e., the number 13107200000. Such objectified numbers were used extensively from the 6th century onward, especially after Varahamihira (575 CE). Zero is explicitly represented in such numbers as "the void" (*sunya*) or the "heaven-space" (*ambara akasha*). Correspondingly, the dot used in place of zero in written numerals was referred to as a *sunya-bindu*.

The idea of *sthāna* as a denominational place is taught in the *Gaṇita* (mathematics) chapter of the fifth century treatise Āryabhaṭīya (499 CE) of Āryabhaṭa. After an invocation in the first verse, the chapter narrates (in Verse two) the first ten notational places: eka, das´a, s´ata (hundred), sahasra (thousand), ayuta (ten thousand), niyuta (hundred thousand), prayuta (million), koṭi (ten million), arbuda (hundred million) and vṛnda (thousand million).Subsequent authors list more places (usually 18, sometimes 24) [2]. In an earlier millennium, Vedic authors like Medhātithi had already enunciated the "powers-of-ten" principle "each ten times the preceding". In the above verse of Āryabhaṭa, we see a clear formulation of the next crucial step for decimal place value notation – visualising each power of ten as a sthāna (place). The verse is only an exposition and not an announcement of a new concept. In the preceding verse (Verse one of *Gaṇita*), Āryabhaṭa acknowledges that he is recording the "knowledge revered at Kusumapura" (Pāṭaliputra, near modern Patna). Āryabhaṭa's methods in *Gaṇita* for extraction of the square root and the cube root of a number, which are slight variants of the modern methods, make intricate uses of the concepts of place value and zero [4]. The fact that these methods are briefly stated without elaboration indicates that the decimal place value notation with zero had already become firmly entrenched in Indian arithmetic. His alphabetical coding of large numbers (Verse two of the Gī tikā chapter of Āryabhaṭīya), using a centesimal system in which the consonants of the Sanskrit alphabet act as digits and the vowels play the role of place value , reiterate how strong was his mastery over all the ideas encapsulated in the decimal notation.

V. DECIMAL PLACE VALUE SYSTEM IN LITERATURE OF BRAHMAGUPTA

The work regarding decimal place value system was most likely to have been mentioned in Brahmagupta's Brahma Sphuta Siddhanta (The Opening of the Universe) which was written in 628 . Irrespective of whether Ifrah is right, since all Indian texts after Aryabhata's *Aryabhatiya* used the Indian number system, certainly from this time the Arabs had a translation of a text written in the Indian number system. Now as for the decimal system, there should be a unique representation of any number possible. Thus making the number system created by Brahmagupta with the addition of rules for zero by Aryabhata the first decimal system. Brahmagupta defined zero as the result of subtracting a number from itself, postulated negative numbers and discussed their properties under arithmetical operations [3]. His word for zero was *shunya* (void), the same term previously used for the empty spot in 9-digit place-value system. This provided a new perspective on the *shunya-bindu* as a numeral and paved the way for the eventual evolution of a zero digit. The dot continued to be used for at least 100 years afterwards, and transmitted to Southeast Asia and Arabia. Kashmir's *Sharada* script has retained the dot for zero until this day.

By the end of the seventh century, decimal numbers begin to appear in inscriptions in Southeast Asia as well as in India. Some scholars hold that they appeared even earlier. A sixth century copper-plate grant at *Mankani* bearing the numeral 346 (corresponding to 594 CE) is often cited. But its reliability is subject to dispute. The first indisputable occurrence of 0 in an inscription occurs at Gwalior in 876 CE, containing a numeral "270" in a notation surprisingly similar to ours. Throughout the 8th and 9th centuries, both the old Brahmi numerals and the new decimal numerals were used, sometimes appearing in the same inscriptions. In some documents, a transition is seen to occur around 866 CE. For clarity in our

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understanding of the development of zero in mathematics, a distinction is made between two of its important features: (i) the notational innovation of the zero symbol as a placeholder in a place value numeral system; and (ii) the abstract conception of the integer zero in a number system amenable to arithmetic operations. Aspect (ii) is central in modern mathematics with its emphasis on algebraic structures. The profound step (ii) of elevating zero to an algebraic number (saṅkhyā) was taken in India. In a section of Chapter XVIII of his treatise Brāhma Sphuṭa Siddhānta, Brahmagupta defines zero as and gives an elaborate treatment of the four fundamental operations involving numbers – positive, negative and zero – treating all on the same footing. His rules amount to the sophisticated idea of giving integers a ring structure with zero as the additive identity. This comprehensive treatment of zero and negative numbers was an important pillar for the foundations of algebra established by Brahmagupta in the above-mentioned chapter [1,4]. The use or mention of zero as a number in arithmetic can be glimpsed earlier in the *Bakṣālī* Manuscript, in the sixth-century treatise *Pan˜casiddhāntikā* of Varāhamihira and in the early seventh-century commentary on Āryabhaṭīya by Bhāskara I. That step (ii) needs a conceptual leap from (i) can be seen from the fact that although the decimal notation requires ten symbols, ancient and medieval scholars often refer to it as a system with 'nine figures' indicating that the placeholder zero symbol did not always receive the dignity accorded to the other nine numerals. We mention here that a partial use of the placeholder principle (i) had also emerged in some form among late Babylonians during 700–400 BCE; the Mayan culture (fourth century CE) too used zero as a placeholder and conceived of zero as an ordinal preceding one – the first day of their Haab calendar was day 'zero'.

VI. CONCLUSION

It would seem highly probable under the circumstances that the discovery of the decimal place value system were unique to the Indian civilization. As the Brahmi notation of the first nine whole numbers was autochthonous and free of any outside civilization influence, there can be no doubt that our decimal place-value system was invented in India and was the achievement of Indian civilization alone. The decimal numeral systems was first used in India, South east and Central Asia. This system was neither developed by Greeks nor by the Arabs, despite the fact that this numeral system is commonly called the Arabic numerals in Europe. But today we use decimal numeral system, with its powers of ten place-value, is a relatively modern invention.

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