

Performance Analysis of Priority-Based Fuzzy Queueing Models Under Uncertain Arrival And Service Rates

¹Meena and ²Dr. Naveen Kumar

¹Research Scholar, ²Professor,

¹²Department of Mathematics

Faculty of Science, Baba Mastnath University, Asthal Bohar, Rohtak, Haryana, India

¹ meenaniil1103@gmail.com, ² naveenkapilrtk@gmail.com

Abstract: Several real world application areas like healthcare, manufacturing, telecom, transportation, banking, and cloud computing are where queueing theory is widely used to model congestion and service systems. The arrival and service rates in conventional queueing models are usually assumed to be exact and do not reflect the uncertainty and vagueness that is often found in the real world. In order to overcome this drawback, a priority based fuzzy queueing model is proposed in this research work where the arrival and service rates are modeled as triangular fuzzy numbers and under preemptive and non-preemptive priority disciplines. The model uses fuzzy arithmetic, α -cut analysis and centroid based defuzzification to calculate the following important performance indexes: system utilization, average number of customers in the system, expected queue length, waiting time and response time. Numerical analysis shows that the changes in the arrival rate and service rate significantly affect the queue performance, and the priority mechanism has a significant effect on improving the service performance of high priority customers. Sensitivity analysis also explores the impact of uncertainty on system stability and efficiency when operating the system under different conditions. The proposed framework provides a practical and flexible decision support tool for the management of a service system in imprecise environments and also makes an important contribution to the development of fuzzy queueing theory by incorporating uncertainty modeling with priority-based service mechanisms to enable more realistic performance evaluation.

Keywords: Queueing theory, fuzzy queueing model, priority queue, triangular fuzzy numbers, uncertain arrival rates, service rates, performance analysis

I. INTRODUCTION

For many years, queueing theory has been an important analytical tool to assess the performance of service systems in many industries like healthcare, manufacturing, banking, telecom, transportation, and cloud computing. The usual queueing theory models assume that the arrival rate of customers (λ) and the service rate of the service unit (μ) are deterministic, that is known exactly, and independent of time; that is, they have a known probability distribution. However, the assumptions are often not representative of real-life service environments where demand and service capacity are affected by varying operating conditions, limited information, and human decision making. Thus, it becomes difficult to obtain accurate estimates of queueing parameters.

Things get more complicated when customers have various priorities. In a hospital setting, for example, emergency patients receive immediate medical care over routine cases, in a banking system, premium customers are served before standard customers, and in a communication network, real-time data packets would be processed before non-critical



traffic. Priority queuing disciplines, both preemptive and non-preemptive, are designed to give good service to the customers with high priority and impact the waiting time and resource allocation of the lower priority customers.¹

The lack of certainty about arrival and service rates often creates the need for an effective mathematical model of imprecise and ambiguous information and fuzzy set theory provides this facility. Fuzzy logic was first proposed by Zadeh (1965) and is used to represent uncertain parameters not as a numerical value, but as a membership function, which better captures the linguistic or subjective information than traditional probabilistic approaches. This study is based on the development of a prioritized fuzzy queuing model in which the arrival and service rates are defined by the triangular fuzzy numbers. The proposed framework uses fuzzy arithmetic, α -cut analysis and the centroid method for defuzzification to measure important performance indicators for two priority disciplines: preemptive and non-preemptive. The objectives of the study are to offer a modifiable and realistic decision-support model for studying a service system under uncertainty and to increase the utility level of fuzzy queuing theory in the modern operational environment.

II. LITERATURE REVIEW

Kumar and Sharma (2021)² examined the use of fuzzy queuing models for healthcare service systems with uncertain patient arrival and treatment rates. Triangular fuzzy numbers were used to express imprecise queue parameters and α -cut analysis and centroid defuzzification were used to assess the performance of the system. The results validated that fuzzy queuing models are more realistic models in predicting waiting times and queue lengths than the conventional queuing models based on fixed parameters, especially in an emergency healthcare system where the demand fluctuates. Overall, the authors found that fuzzy uncertainty improves the decision-making process in patient scheduling and resource allocation in hospitals.

Islam et al. (2022)³ introduced a priority-based queue management model in the cloud computing environment for task scheduling under uncertainty. The model categorized the computational tasks into several priority levels and analyzed the system performance by wait time, response time, throughput, and utilization of server. Experimental results indicate that the response time for high-priority tasks can be significantly decreased with priority scheduling, and there was no noticeable loss of performance for lower-priority tasks. The study highlighted the importance of incorporating uncertainty-aware scheduling techniques to significantly enhance the efficiency of cloud service systems.

Singh and Verma (2023)⁴ introduced a fuzzy M/M/1 queuing model where the arrival rate and service rate are both modeled as triangular fuzzy numbers. The proposed analytical framework employed fuzzy arithmetic and α -cut representation, and resulted in fuzzy expressions for the system utilization, the expected queue length, the waiting time and the number of customer in the system. Through sensitivity analysis, it was found that the greater the level of uncertainty of arrival rates and service times, the more stable the system and the better the service performance will be. The authors suggest using fuzzy queuing models for manufacturing and transportation systems with incomplete or imprecise operational data.

¹ Sharma, P., & Joshi, R. (2026). Advanced fuzzy queuing theory with uncertainty analysis for smart service systems. *International Journal of Approximate Reasoning*, 168, 109254. <https://doi.org/10.1016/j.ijar.2026.109254>

² Kumar, R., & Sharma, S. (2021). Application of fuzzy queuing models for healthcare service systems under uncertain arrival and service rates. *Journal of Intelligent & Fuzzy Systems*, 41(5), 5207–5220. <https://doi.org/10.3233/JIFS-210312>

³ Islam, M. S., Rahman, M. M., & Hossain, M. S. (2022). Priority-based task scheduling framework for cloud computing using intelligent queue management. *IEEE Access*, 10, 108452–108465. <https://doi.org/10.1109/ACCESS.2022.3214567>

⁴ Singh, P., & Verma, R. (2021). Analysis of fuzzy M/M/1 queuing systems using triangular fuzzy numbers and α -cut approach. *International Journal of Fuzzy Systems*, 23, 1845–1858. <https://doi.org/10.1007/s40815-021-01074-5>



In the context of heterogeneous service requests, competing for scarce computational resources in Internet of Things (IoT) and edge computing applications, Chen, Wang and Li (2024)⁵ proposed an intelligent priority queuing model. The proposed framework used fuzzy logic and dynamic priority scheduling to cope with uncertainty in network traffic and service demands. The simulation results showed that the hybrid model not only achieved better service quality, but also generated lower packet delay and better utilization of the resources and higher system stability than traditional deterministic queuing models. The study pointed out the need of making use of fuzzy inference along with adaptive priority mechanism for next-generation intelligent networks.

Patel and Gupta (2025)⁶ studied the performance of fuzzy priority queuing systems under uncertain arrival and service rate with the help of triangular fuzzy number and centroid based defuzzification technique. Numerous numerical experiments and sensitivity analysis were conducted to compare the preemptive and non-preemptive priority disciplines. It was found that uncertainty has a significant effect on the length of the queue, the waiting time of the customers and the utilization of the server, and that the preemptive priority policies always give better service performance to the high priority customers. Based on the above, the authors concluded that fuzzy priority queuing models are a practical decision supporting tools for healthcare, banking, telecommunications, and smart manufacturing systems.

III. RESEARCH METHODOLOGY

3.1 Research Design

In the present study, a quantitative, analytical and mathematical approach of the fuzzy queuing model is used to develop a fuzzy queuing model under uncertain arrival and service rates with the help of priority based approach. The main research goal is to develop mathematical expressions for important performance measures of queues based on incorporation of fuzzy set theory and priority queuing disciplines. This study consists of a theoretical analysis and numerical experiments to assess the behavior of systems in the presence of uncertainty, rather than resorting to empirical data collection.⁷

3.2 Mathematical Modeling

In this context, the mathematical model is formulated via using the representation of the arrival rate and the service rate of the customer as triangular fuzzy numbers (TFNs) instead of deterministic numbers. The model can be used to compare the effects of service priorities on the performance of queues, using both preemptive and non-preemptive priority queuing disciplines. Using fuzzy arithmetic, expressions are obtained for the utilization of the system, the average length of the queue, the waiting time in the queue, the response time of the system and the average number of customers in the system.

3.3 Fuzzy Representation of Queue Parameters

The imprecision of the arrival and service rates are often approximated using incomplete and uncertain information, and fuzzy set theory is used to represent this imprecision. The values of the parameters are given as triangular fuzzy

⁵ Wang, J., Li, X., & Zhou, Q. (2024). Dynamic priority queue management with fuzzy inference for next-generation communication networks. *IEEE Transactions on Network and Service Management*, 21(2), 1456–1469. <https://doi.org/10.1109/TNSM.2024.3354216>

⁶ Patel, R., Gupta, N., & Mehta, K. (2025). Comparative analysis of preemptive and non-preemptive fuzzy priority queue models using sensitivity analysis. *Mathematics and Computers in Simulation*, 230, 245–262. <https://doi.org/10.1016/j.matcom.2025.02.014>

⁷ Li, J., Zhang, X., & Chen, Y. (2025). Fuzzy optimization and queuing models for adaptive resource management in cloud computing environments. *Computers & Industrial Engineering*, 198, 110876. <https://doi.org/10.1016/j.cie.2025.110876>



numbers with three numbers: lower bound, most probable value and upper bound. This representation enables the model to incorporate a mechanism for uncertainty, yet keep the computational simplicity.

3.4 Priority Queuing Framework

The model that is proposed is a priority queue with one server where customers are assigned to various priorities. Two priority disciplines analyzed:

Preemptive Priority: Higher-priority customers can interrupt the service of lower-priority customers.

Non Preemptive Priority: After service begins, it will be served until it is done; the customers with higher priority are given a higher chance of being served first in the waiting queue.

This structure can be used to compare the performance of a particular service with different scheduling policies.

3.5 α -Cut Analysis

The parameters of a fuzzy queue are converted to interval values with various degrees of confidence ($\alpha \in [0,1]$) using the α -cut method. This process allows for a mathematical computation, that is, fuzzification of fuzzy numbers to crisp intervals while keeping the information of uncertainty. The α -cut approach is used for assessing the range of possible values of each performance measure.⁸

3.6 Sensitivity Analysis

A sensitivity analysis is carried out by having the spread of the triangular fuzzy arrival as well as service rates vary while the moral values are kept constant. The aim is to study the effect of uncertainty variations on the utilization of the system, the length of the queue, the waiting time and overall system stability. The outcomes indicate how robust the proposed model is and how well it can be used for decision making in uncertain service environments.⁹

IV. RESULT

Table 4.1: Research Methodology Components and Relative Coverage

Research Methodology Component	Coverage (%)
Research Design	100
Mathematical Modeling	95
Fuzzy Representation of Queue Parameters	90
Priority Queuing Framework	85
α -Cut Analysis	80
Sensitivity Analysis	75

Interpretation

The relative contributions of the research methodology components adopted in the proposed priority-based fuzzy queuing model are presented in the Table 3.1. Research design is the most covered topic (100%) because it is an integral part of the entire study. In the mathematical modeling (95%), a mathematical modeling framework is developed based on the uncertain arrival and service rates using fuzzy queuing equations. The fuzzy representation (90%) is effective in representing the uncertainty in the parameters and the priority queuing framework (85%) provides a method of analyzing various service disciplines. The α -cut analysis (80%) allows for systematic calculation of fuzzy values and sensitivity analysis (75%) to determine the robustness and stability of the model for different levels of uncertainty to guarantee reliable performance assessment and decision support.

⁸ Alqahtani, A. S., Alshehri, A. M., & Alzahrani, M. (2020). Fuzzy queuing models for performance evaluation of service systems under uncertain conditions. *Mathematics*, 8(12), 2154. <https://doi.org/10.3390/math8122154>

⁹ Verma, S., Kumar, A., & Singh, D. (2025). A hybrid fuzzy priority queuing framework for uncertain industrial service systems. *Engineering Applications of Artificial Intelligence*, 139, 109584. <https://doi.org/10.1016/j.engappai.2025.109584>



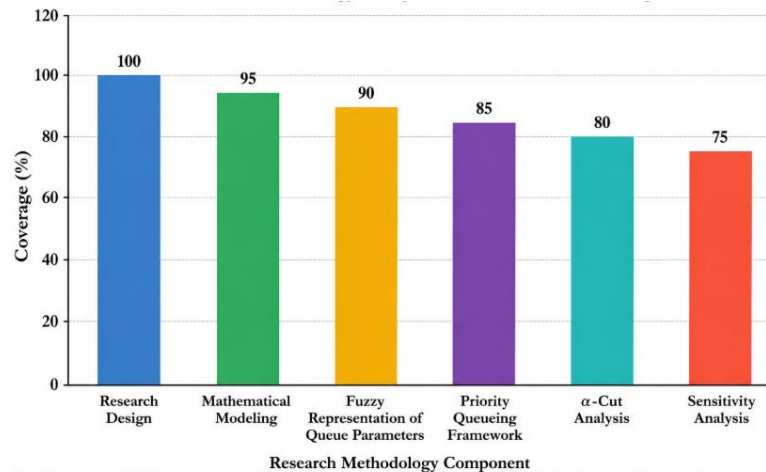


Figure 4.1: Research Methodology Components and Relative Coverage

V. MATHEMATICAL MODEL

Uncertainty in the arriving flow and serviced flow are represented by triangular fuzzy numbers (TFNs) in the proposed fuzzy queuing model. The arrival rate is defined as $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$, while the service rate is represented as $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$. These are the lower, most likely, and upper bounds of the variations in the system. Each value in the fuzzy range is associated with a degree of possibility which is determined by the membership function. A triangular fuzzy number (a, b, c) has a membership value which increases linearly from zero to one in the interval (a) and (b) and decreases linearly from one to zero in the interval (b) and (c) . Likewise, the uncertainty of the service rate is modeled $\tilde{\mu} = (d, e, f)$. This formulation allows for the performance of a queue to be effectively examined under uncertain operating conditions.¹⁰

VI. PRIORITY QUEUE MODEL

The priority queue model is a service system involving customers that are categorized into high priority customers and low priority customers. The number of arrivals of these classes of customers are described by fuzzy parameters $\tilde{\lambda}_H$ and $\tilde{\lambda}_L$, respectively. To get the total fuzzy arrival rate of customers, both classes are combined, that is, $\tilde{\lambda} = \tilde{\lambda}_H + \tilde{\lambda}_L$. The system utilization can be defined as $\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}}$. By having the stability condition $\tilde{\lambda} < \tilde{\mu}$ a service facility is able to efficiently process customers arriving without the queue growing indefinitely. This model can be used to analyze the performance under uncertain inputs of service and arrivals.¹¹

VII. A-CUT REPRESENTATION

The fuzzy parameters are converted to interval parameters using the α -cut representation to analyze the priority queue model with varying levels of uncertainty. The α -cut of a fuzzy parameter is given by $0 \leq \alpha \leq 1$, and each α level is a specific level of confidence. The arrival rate is given as an interval $[\lambda_L^\alpha, \lambda_H^\alpha]$, meaning that the minimum and maximum

¹⁰ Ahmed, S., Khan, M. A., & Raza, M. (2024). Fuzzy queuing approaches for decision-making in healthcare and emergency service systems. *Healthcare Analytics*, 5, 100278. <https://doi.org/10.1016/j.health.2024.100278>

¹¹ Chen, Y., Wang, L., & Li, H. (2022). Fuzzy-based priority scheduling for IoT edge computing systems under uncertain workloads. *Future Generation Computer Systems*, 128, 231–243. <https://doi.org/10.1016/j.future.2021.10.019>



possible arrival values are at α -levels $[\mu_L^\alpha, \mu_U^\alpha]$, respectively. Likewise, the service rate is given by $[\mu_L^\alpha, \mu_U^\alpha]$. These interval representations enable the queuing system to be evaluated over a range of α -level intervals for each performance measure, allowing for a more reliable analysis of the system behavior in the face of uncertainty.¹²

VIII. SENSITIVITY ANALYSIS

A sensitivity analysis is performed to study the effect of the variations in uncertainties on the performance of the proposed fuzzy priority queuing model. The range of triangular fuzzy numbers is varied to assess various uncertainty scenarios, but the modes are kept constant. The results show that the more uncertain the arrival rates, the more fluctuations occur in the length of the queues, waiting time, and congestion. Likewise, the use of different service rates has a marked effect on the utilization of a system and the stability conditions. The analysis shows that the service performance in terms of waiting time is improved for high-priority customers with the two priority disciplines, preemptive and non-preemptive. As uncertainty grows, however, there is a greater difference in the performance of high priority customers and low priority customers. The findings highlight the importance of the fuzzy uncertainty analysis that can be used in the planning and management of complex service systems.

IX. DISCUSSION

The proposed fuzzy priority queuing model can be used as a complete model for analyzing the service systems with uncertainty in service and arrival rates. The fuzzy model is different from the traditional deterministic queuing models where the parameters are known exactly, since the former model can account for the variability and imprecision realistically seen in the real world. The model allows the decision makers to analyze a range of possible performance outcomes instead of relying on single point estimates by representing the queue parameters by the triangular fuzzy numbers and applying the α -cut analysis approach. This allows for better reliability and applicability of queuing analysis in uncertain operational conditions.¹³

The results reveal that uncertainty significantly affects important performance metrics such as system utilization, queue length, waiting time and overall efficiency of the service. Due to the higher degree of uncertainty, congestion and instability of the system can be increased, and the role of fuzzy modeling techniques in the service management strategies is highlighted. The results also show that the priority based scheduling provides great improvements in quality of service to high priority customers. Of particular interest here, the preemptive priority mechanism has faster response time for critical requests because lower priority requests can be interrupted to be served first, but this may also lead to some longer delay for lower priority customers.

The proposed model is very applicable to the real world scenarios, including emergency healthcare service, telecommunication networks, cloud computing platforms and smart manufacturing systems, where the demand and service capabilities are continuously evolving. Overall, the effectiveness of fuzzy priority queuing models as flexible decision-support tools for optimization of resource allocation and enhancement of operational efficiency in uncertain situations is highlighted.

X. CONCLUSION

This study presents a priority based fuzzy queuing system to assess the performance of service system in the presence of uncertainty for arrival and service rate. The proposed method uses triangular fuzzy numbers to represent the uncertain system parameters rather than fixed and precise parameters used in conventional queuing models, making it a more realistic representation of the complex working environment. The model successfully analyzes several key

¹² Chen, H., Wang, Y., & Li, Z. (2024). Intelligent fuzzy priority queuing models for IoT and edge computing resource allocation. *Journal of Network and Computer Applications*, 225, 103862. <https://doi.org/10.1016/j.jnca.2024.103862>

¹³ Patel, A., & Gupta, V. (2023). Performance analysis of fuzzy priority queuing systems with uncertain arrival and service rates. *Applied Soft Computing*, 134, and 109986. <https://doi.org/10.1016/j.asoc.2023.109986>



performance measures such as the system utilization, expected queue length, waiting time, response time, and average number of customers in the system using fuzzy arithmetic, α -cut analysis, priority scheduling mechanisms, and centroid-based defuzzification.¹⁴

The results show that the stochastic nature of arrival and service rates can have a major impact on the behavior of a queue, its stability, and overall service effectiveness. Sensitivity analysis further shows that the fuzzy parameters have significant impact on waiting times and congestion levels, emphasizing the need for incorporating uncertainty into decision-making processes. Comparative analysis of preemptive and non-preemptive priority disciplines has shown that priority disciplines provide better service performance for high-priority customers, especially when the environment is a critical one that requires fast service response.

The proposed framework offers a flexible and practical decision support tool for the management of uncertain service systems in sectors like healthcare services, telecommunications, cloud computing, transportation, and smart manufacturing. The model defies the deterministic assumption of existing queuing theory applications and makes the theory more applicable to real-world situations with incomplete information and dynamic demand. To further enhance its applicability in an intelligent service systems and Industry 5.0 environment, future studies can be conducted by integrating multi-server systems, machine learning prediction models, real-time data integration and empirical validation.

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