IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 2, July 2026
Impact Factor: 8.2

Generalized Fractional Sumudu–Fourier Transform and its Analytical Structure

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Abstract: *The Fractional Sumudu–Fourier Transform (FrSFT) is a useful mathematical tool that combines the features of the Sumudu Transform and the Fractional Fourier Transform. It provides a simple and effective way to study functions with fractional and oscillatory behaviour and has applications in mathematics, signal processing, and engineering. In this work, the analyticity of the Fractional Sumudu–Fourier Transform is established using testing function spaces and generalized functions. The obtained result strengthens the mathematical foundation of the transform and provides a basis for further studies and practical applications in applied mathematics.*

Keywords: Fractional Sumudu–Fourier Transform, Fractional Sumudu Transform, Fourier Transform, Analyticity Theorem, Testing Function Space, Generalized Functions.

I. INTRODUCTION

Integral transforms are important mathematical tools that help to simplify complex mathematical problems into forms that are easier to solve. They are widely used in mathematics, physics, engineering, and signal processing to solve differential equations and to study different types of functions [8]. Among these transforms, the Fourier Transform is one of the most widely used because it converts a function from the time domain to the frequency domain, making it easier to analyse signals and physical systems [33], [34], [35].

The Fractional Fourier Transform (FrFT) was introduced as a generalization of the classical Fourier Transform to overcome some of its limitations. It allows signal analysis in intermediate domains between the time and frequency domains by introducing a fractional order [1]. This additional flexibility has made the FrFT useful in optics, image processing, communication systems, and many other scientific applications [11]. Similarly, the Sumudu Transform has gained attention because of its simple operational properties and its ability to preserve the original units of a function. It has been successfully applied to solve many ordinary and partial differential equations arising in science and engineering [13], [14], [15].

In recent years, researchers have developed several generalized integral transforms by combining the advantages of existing transforms [2]. The Fractional Sumudu–Fourier Transform (FrSFT) is one such transform that combines the properties of the Fractional Fourier Transform and the Sumudu Transform. This transform provides a broader mathematical framework for studying functions with both fractional and oscillatory behaviour [17-22]. Like every integral transform, it is important to investigate its fundamental mathematical properties before it can be applied to practical problems [9].

One of the most important properties of any integral transform is its analyticity. The analyticity theorem shows that the transformed function is analytic under suitable conditions, which is essential for developing further theoretical results such as inversion formulas, convolution theorems, and operational properties [7]. The theories of generalized functions,

DOI: 10.48175/IJARSCT-37746

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distribution spaces, functional analysis, and fractional calculus provide the mathematical foundation for proving these results [35], [36].

The main aim of this work is to establish the Analyticity Theorem for the Fractional Sumudu–Fourier Transform in the framework of testing function spaces and generalized function [4]. The obtained results strengthen the mathematical foundation of the FrSFT and provide a basis for its future applications in fractional differential equations, signal processing, mathematical physics, and other areas of applied mathematics [19-24].

The organization of the paper is as follows. Section 2, introduced the basic definition and preliminary concept that required for the study. In section 3, suitable testing function space is constructed. Section 4, presents the Distributional Fractional Sumudu – Fourier Transform and discusses its formulation. In section 5, main body of this paper is represented i.e. Analyticity Theorem. And section 6, describe final conclusion. The notation and terminology as per A. H. Zemanian [9], [20].

II. DEFINITION

2.1 Fractional Sumudu Transform:

The Fractional Sumudu transform of a function $f(t)$ with parameter s is denoted by, $F_\alpha(s)$ and is defined by

$$FrST\{f(t)\} = F_\alpha(s) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}} \int_{-\infty}^{\infty} f(t)\; e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]} dt$$

where $s\ is\ a$ parameter > 0.

2.2 Fourier Transform:

The Fourier transform of a function $f(x)$ with parameter p is denoted by $F[f(x)] = F(p)$ and is defined as,

$$F(p) = \int_{-\infty}^{\infty} f(x)\, e^{-ipx} dx$$

for parameter $p > 0$.

2.3 Fractional Sumudu – Fourier Transform:

The Fractional Sumudu – Fourier transform of $f(t,x)$ is given by

$$FrSFT\ \{f(t,x)\} = F_\alpha(s,p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x)\ K_\alpha(t,x,s,p)\, dtdx$$

$$FrSFT\{f(t,x)\} = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x)\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}\, e^{-ipx}\, dtdx$$

where,

$$K_\alpha(t,x,s,p) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}\, e^{-ipx}$$

is the kernel of Fractional Sumudu – Fourier transform such that $0 < \alpha < \frac{\pi}{2}$.

$K_\alpha(t,x,s,p)$ belongs to the testing function space and $f(t,x)$ lies in its dual space.

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III. TESTING FUNCTION SPACE $E(R^n)$

An infinitely differentiable complex valued smooth function $\phi(t,x)$ on R^n belongs to $E(R^n)$, if for each compact set $I \subset S_{a,b}$ such that,

$$S_{a,b} = \{t, x: t, x \in R^n, |t| \le a, |x| \le b, a > 0, b > 0\}$$

and the seminorm, $\gamma_{E,l,n}(\phi) = \sup_{t,x\in I} \left|D_t^l D_x^n \phi(t,x)\right| < \infty$ *where* $l,n \ge 0$

Hence, $E(R^n) = \{\phi \in R^n : \gamma_{E,l,n}(\phi) < \infty, \forall\, l,n \ge 0\}$

Moreover, $E(R^n) = \{\phi \in R^n : supp(\phi) \subseteq S_{a,b}\}$

where $supp\ (\phi)$ denotes the support of function ϕ.

Finally, the dual space of $E(R^n)$ is defined by,

$$E^*(R^n) = \{T: E(R^n) \to C; T \text{ is the contineous linear functional}\}.$$

A generalized function f is said to be Fractional Sumudu–Fourier transformable if $f \in E^*(R^n)$.

IV. DISTRIBUTIONAL FRACTIONAL SUMUDU-FOURIER TRANSFORM

The distributional Fractional Sumudu – Fourier transform of $f(t,x) \in E(R^n)$ defined by,

$$FrSFT\{f(t,x)\} = F_\alpha(s,p) = \langle f(t,x), K_\alpha(t,x,s,p)\rangle$$

where, $K_\alpha(t,x,s,p) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}\,coseca\right]}\, e^{-ipx}$

for each fixed $t\ (-\infty < t < \infty), x\ (-\infty < x < \infty), s > 0,\ p > 0$ *and* $0 < \alpha < \frac{\pi}{2}$.

Also denotes $A_\alpha = \sqrt{\frac{1-i\,cot\alpha}{2\pi i}}$ and $B_\alpha = \frac{1}{2\,sin\alpha}$

Therefore, the above kernel can be written as,

$$K_\alpha(t,x,s,p) = \frac{1}{s} A_\alpha\, e^{B_\alpha\left[\left(t^2+\frac{1}{s^2}\right)cos\alpha - \frac{2t}{s}\right]}\, e^{-ipx}$$

V. ANALYTICITY THEOREM

5.1 Lemma:

Statement: Let $K_\alpha(t,x,s,p) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}\,coseca\right]}\, e^{-ipx}$

then, $D_t^l K_\alpha(t,x,s,p) = A_\alpha \sum_{k=0}^{\left[\frac{l}{2}\right]} \frac{l!}{2^k\, k!(l-2k)!}\,(cot\alpha)^k \left(t\,cot\alpha - \frac{1}{s} coseca\right)^{l-2k}$

where, $A_\alpha = K_\alpha(t,x,s,p)$.

Proof: Let $K_\alpha(t,x,s,p) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}\,coseca\right]}\, e^{-ipx}$

Define $A = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{-ipx}$ which is independent of t.

Therefore, $K_\alpha(t,x,s,p) = A\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}\,coseca\right]}$

Differentiate w. r. t. t, we get

$$D_t K_\alpha = A\, D_t\left[e^{\phi(t)}\right]$$

Where, $\phi(t) = \frac{1}{2}\left[\left(t^2 + \frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}\ coseca\right]$

using the chain rule, we get

$$D_t e^{\phi(t)} = \phi'(t) e^{\phi(t)} = \frac{1}{2}\left(2t\ cot\alpha - \frac{2}{s}\ coseca\right) e^{\phi(t)} = \left(tcot\alpha - \frac{coseca}{s}\right) e^{\phi(t)}$$

$$D_t K_\alpha = \left(t\ cot\alpha - \frac{1}{s}\ coseca\right) K_\alpha(t,x,s,p)$$

Differentiating again, we get

$$D_t^2 K_\alpha = D_t\left[\left(t\,cot\alpha - \frac{1}{s}\,cosec\alpha\right) K_\alpha\right] = (cot\alpha)K_\alpha + \left(t\,cot\alpha - \frac{1}{s}\,cosec\alpha\right) D_t K_\alpha$$

$$D_t^2 K_\alpha = \left[cot\alpha + \left(t\,cot\alpha - \frac{1}{s}\,cosec\alpha\right)^2\right] K_\alpha$$

similarly,

$$D_t^3 K_\alpha = D_t\{[cot\alpha + Q^2]K_\alpha\}$$

since $Q = t\,cot\alpha - \frac{1}{s}cosec\alpha$ and $Q' = cot\alpha$

$$D_t^3 K_\alpha = [3Q\,cot\alpha + Q^3]K_\alpha$$

again

$$D_t^4 K_\alpha = [3\,(cot\alpha)^2 + 6Q^2\,cot\alpha + Q^4]K_\alpha$$

from the above derivative we observe the pattern

$$D_t K_\alpha = Q K_\alpha$$
$$D_t^2 K_\alpha = (Q^2 + \ cot\alpha)K_\alpha$$
$$D_t^3 K_\alpha = (Q^3 + 3Q\,cot\alpha)K_\alpha$$
$$D_t^4 K_\alpha = [Q^4 + 6Q^2\,cot\alpha + 3(cot\alpha)^2]K_\alpha$$

this sequence coincides with the generalized Hermite polynomial expansion.
Hence

$$D_t^l K_\alpha(t,x,s,p) = K_\alpha(t,x,s,p)\sum_{k=0}^{\left\lfloor\frac{l}{2}\right\rfloor}\frac{l!}{2^k\,k!\,(l-2k)!}(cot\alpha)^k\left(t\,cot\alpha - \frac{1}{s}cosec\alpha\right)^{l-2k}$$

5.2 Analyticity Theorem

Statement: Let $f(t,x) \in E^*(R^n)$ and its Fractional Sumudu – Fourier Transformation be defined as,

$$FrSFT\{f(t,x)\} = F_\alpha(s,p) = \langle f(t,x), K_\alpha(t,x,s,p)\rangle$$

then $F_\alpha(s,p)$ is analytic on C^n if the $supp\ f \subset S_{a,b}$ such that

$$S_{a,b} = \{t,x : t,x \in R^n, |t| \le a, |x| \le b, a>0, b>0\}$$

moreover $FrSFT\{f(t,x)\} = F_\alpha(s,p)$ is differentiable and

$$D_t^l D_x^n[F_\alpha(s,p)] = \langle f(t,x), D_t^l D_x^n K_\alpha(t,x,s,p)\rangle$$

Where, $K_\alpha(t,x,s,p) = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\ e^{\frac{i}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}\ e^{-ipx}$

Proof: Let $s = (s_1, s_2, s_3, \dots, s_i, \dots, s_n) \in C^n$ and $p = (p_1, p_2, p_3, \dots, p_i, \dots, p_n) \in C^n$ be arbitrary but fixed point. Here we first prove that

$$\frac{\partial}{\partial s_i}\{F_\alpha(s,p)\} = \langle f(t,x), \frac{\partial}{\partial s_i}K_\alpha(t,x,s,p)\rangle$$

for fixed $s_i \ne 0$, choose two concentric circles $c'\,and\,c''$ with centre at s_i with radii r and r_1 respectively such that $0 < r < r_1 < |s_i|$

Let Δs_i be a complete increment satisfying $0 < |\Delta s_i| < r$

Consider,

$$\frac{F_\alpha(s_i+\Delta s_i, p) - F_\alpha(s_i,p)}{\Delta s_i} - \langle f(t,x), \frac{\partial}{\partial s_i}K_\alpha(t,x,s,p)\rangle = \langle f(t,x), \psi_{\Delta s_i}(t,x)\rangle \quad \text{………. (5.1)}$$

therefore,

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ISSN: 2581-9429 Volume 6, Issue 2, July 2026 Impact Factor: 8.2

$\langle f(t,x), \psi_{\Delta s_i}(t,x)\rangle =$

$\frac{1}{\Delta s_i}\{\langle f(t,x), K_\alpha(t,x,s_i+\Delta s_i,p)\rangle - \langle f(t,x), K_\alpha(t,x,s,p)\rangle\} - \langle f(t,x), \frac{\partial}{\partial s_i}K_\alpha(t,x,s,p)\rangle$

$$= \langle f(t,x), \frac{1}{\Delta s_i}\{K_\alpha(t,x,s_i+\Delta s_i,p) - K_\alpha(t,x,s,p)\} - \frac{\partial}{\partial s_i}K_\alpha(t,x,s,p)\rangle$$

where, $\psi_{\Delta s_i}(t,x) = \frac{1}{\Delta s_i}\{K_\alpha(t,x,s_i+\Delta s_i,p) - K_\alpha(t,x,s,p)\} - \frac{\partial}{\partial s_i}K_\alpha(t,x,s,p)$

for any fixed $t, x \in R^n$ and fixed integer $l = (l_1, l_2, l_3, \dots, l_n)$

$$D_t^l\{K_\alpha(t,x,s,p)\} = D_t^l\left\{\frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}\, e^{-ipx}\right\}$$

now separating the constant part we get,

$$D_t^l\{K_\alpha(t,x,s,p)\} = D_t^l\{A_\alpha\, e^{\phi(t)}\}$$

where, $A_\alpha = \frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{-ipx}$ and $e^{\phi(t)} = e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}$

since the exponent is quadratic in t, the higher derivative follows the Gaussian derivative formula. Thus, we can write,

$$D_t^l\{K_\alpha(t,x,s,p)\} = \left\{\sum_{k=0}^{\left\lfloor\frac{l}{2}\right\rfloor}\frac{l!}{2^k\, k!(l-2k)!}(cot\alpha)^k\left(t\,cot\alpha - \frac{1}{s}cosec\alpha\right)^{l-2k}\right\} \times \left\{\frac{1}{s}\sqrt{\frac{1-i\,cot\alpha}{2\pi i}}\, e^{\frac{1}{2}\left[\left(t^2+\frac{1}{s^2}\right)cot\alpha - \frac{2t}{s}cosec\alpha\right]}\, e^{-ipx}\right\}$$

$$D_t^l K_\alpha(t,x,s,p) = \left\{\sum_{k=0}^{\left\lfloor\frac{l}{2}\right\rfloor}\frac{l!}{2^k\, k!\,(l-2k)!}(cot\alpha)^k\left(t\,cot\alpha - \frac{1}{s}cosec\alpha\right)^{l-2k}\right\} K_\alpha(t,x,s,p) \quad \text{………. (5.2)}$$

since for any fixed $t \in R^n$ and fixed l with $0 < \alpha \le \frac{\pi}{2}$.

$D_t^l K_\alpha(t,x,s,p)$ is analytic inside and on C'.

We have Cauchy integral formula,

$$D_t^l\psi_{\Delta s_i}(t,x) = \frac{1}{2\pi i}D_t^l\int_{c'}K_\alpha(t,x,\hat{s},p)\left[\frac{1}{\Delta s_i}\left(\frac{1}{z-s_i-\Delta s_i} - \frac{1}{z-s_i}\right) - \frac{1}{(z-s_i)^2}\right]dz$$

where $\hat{s} = (s_1, s_2, s_3, \dots, s_{i-1}, z, s_{i+1}, \dots, s_n)$

$$D_t^l\psi_{\Delta s_i}(t,x) = \frac{1}{2\pi i}D_t^l\int_{c'}K_\alpha\,(t,x,\hat{s},p)\left\{\frac{1}{\Delta s_i}\left[\frac{\Delta S_i}{(z-s_i-\Delta s_i)(z-s_i)}\right] - \frac{1}{(z-s_i)^2}\right\}dz$$

$$D_t^l\psi_{\Delta s_i}(t,x) = \frac{1}{2\pi i}D_t^l\int_{c'}K_\alpha\,(t,x,\hat{s},p)\left[\frac{\Delta s_i}{(z-s_i-\Delta s_i)(z-s_i)^2}\right]dz$$

$$D_t^l\psi_{\Delta s_i}(t,x) = \frac{\Delta s_i}{2\pi i}\int_{c'}\left[\frac{D_t^l K_\alpha(t,x,\hat{s},p)}{(z-s_i-\Delta s_i)(z-s_i)^2}\right]dz$$

$$D_t^l\psi_{\Delta s_i}(t,x) = \frac{\Delta s_i}{2\pi i}\int_{c'}\left[\frac{Q(t,x,\hat{s},p)}{(z-s_i-\Delta s_i)(z-s_i)^2}\right]dz \quad \text{………. (5.3)}$$

But for all $z \in c'$ and t is restricted to a compact subset of R^n with $0 < \alpha \le \frac{\pi}{2}$.

$Q(t,x,\hat{s},p) = D_t^l K_\alpha(t,x,\hat{s},p)$ as in (5.3) bounded by constant L.

Moreover, $|z - s_i - \Delta s_i| > r_1 - r > 0$ and $|z - s_i| = r_1$, therefore we have

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Volume 6, Issue 2, July 2026
Impact Factor: 8.2

$$
\begin{aligned}
|D_t^l \psi_{\Delta s_i}(t,x)| &= \left|\frac{\Delta s_i}{2\pi i}\int_{c'}\left[\frac{Q(t,x,\hat{s},p)}{(z-s_i-\Delta s_i)(z-s_i)^2}\right]dz\right| \\
&\le \frac{|\Delta s_i|}{2\pi}\int_{c'}\frac{L}{(r_1-r)r^2}|dz| \\
&\le \frac{|\Delta s_i|}{2\pi}\frac{L}{(r_1-r)r_1^2}2\pi r_1 \\
&\le \frac{|\Delta s_i| L}{(r_1-r)r_1}
\end{aligned}
$$

Similarly,

$$|D_x^n \psi_{\Delta p_i}(t,x)| \le \frac{|\Delta p_i| M}{(r_1-r)r_1}$$

Where, $R(t,x,s,\hat{p}) = D_x^n K_\alpha(t,x,s,\hat{p})$ is bounded by a constant M. Thus, as $|\Delta s_i| \to 0$ and $|\Delta p_i| \to 0$. $|D_t^l \psi_{\Delta s_i}(t,x)|$ and $|D_x^n \psi_{\Delta p_i}(t,x)|$tends to zero uniformly on the compact subset of R^n, therefore it follows that $\psi_{\Delta s_i}(t,x)$ and $\psi_{\Delta p_i}(t,x)$ converges in $E(R^n)$ to zero. Since $f(t,x) \in E^*$, we conclude that equation (5.1) also tends to zero. Therefore $F_\alpha(s,p)$ is differentiable with respect to s_i and p_i. But this is true for all $i = 1,2,3,\dots,n$. Hence $F_\alpha(s,p)$ is analytic on C^n and $D_t^l D_x^n[F_\alpha(s,p)] = \langle f(t,x), D_t^l D_x^n K_\alpha(t,x,s,p)\rangle$.
Hence proved.

VI. CONCLUSION

This study explored the analyticity of the Fractional Sumudu – Fourier transform (FrSFT) and examined its important analytical behaviour in suitable testing function. The finding shows that the transform has a strong mathematical foundation and behaves effectively under the appropriate condition. By combining the characteristics of both the Sumudu transform and the Fourier transform, FrSFT provides a broader approach for studying function that involves fractional behaviour and oscillatory patterns.

The study also confirm that the analyticity and the convergence of the transform can be established under the specific assumption making it reliable for mathematical analysis. In addition, the Fractional Sumudu – Fourier transform may be useful in solving fractional differential equation and problem related to signal processing, mathematical physics and engineering. Therefore, the FrSFT can be considered an effective and promising transform in advance analytical studies.

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International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429 Volume 6, Issue 2, July 2026 Impact Factor: 8.2

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ISSN: 2581-9429
Volume 6, Issue 2, July 2026
Impact Factor: 8.2

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