

Impact of Quantum Corrections on Collective Excitations and Stability of Bose Einstein Condensates

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Abstract: Bose Einstein condensates provide an important platform for investigating macroscopic quantum phenomena and many-body interactions. The conventional Gross Pitaevskii equation, based on mean-field theory, successfully explains many equilibrium and dynamical properties of weakly interacting dilute condensates. However, under conditions of increased density, strong confinement, reduced dimensionality, attractive interactions, or competing interspecies interactions, quantum fluctuations beyond the mean-field approximation become increasingly important. The leading contribution of such fluctuations is generally described by the Lee Huang Yang correction. This research paper examines the impact of quantum corrections on collective excitations and the stability of Bose Einstein condensates.

Keywords: Bose Einstein Condensate, Quantum Fluctuations, Lee Huang Yang Correction, Collective Excitations.

I. INTRODUCTION

Bose Einstein condensation represents one of the most remarkable manifestations of quantum mechanics at macroscopic scales. When a dilute gas of bosonic particles is cooled to sufficiently low temperatures, a large fraction of particles can occupy a common quantum state, producing a Bose Einstein condensate. The experimental realization of dilute atomic condensates opened new possibilities for investigating quantum coherence, superfluidity, nonlinear dynamics, and collective excitations.

The dynamics of weakly interacting BECs are commonly described by the Gross Pitaevskii equation. In its simplest form, this equation treats the condensate through a mean-field interaction proportional to the particle density. Although this approximation has been highly successful, it does not fully account for quantum fluctuations associated with interactions between particles. Such fluctuations become increasingly significant when the gas parameter increases, when interactions are tuned using Feshbach resonances, when the system is strongly confined, or when competing attractive and repulsive interactions are present.

The first systematic description of the leading beyond-mean-field contribution to the ground-state energy of a dilute Bose gas was provided by Lee, Huang, and Yang. The resulting LHY correction arises from zero-point fluctuations of Bogoliubov quasiparticles and modifies the equation of state of the condensate. These modifications affect both static and dynamic properties. In particular, the collective excitation spectrum becomes sensitive to quantum corrections because collective frequencies depend on compressibility, pressure, interaction energy, and density distribution.

Semiclassical analyses have demonstrated that quantum fluctuations can shift collective-mode frequencies beyond mean-field predictions. In confined condensates, these corrections may produce experimentally measurable changes in oscillation frequencies. Similarly, quantum fluctuations in optical lattices can generate significant modifications of collective dynamics and can lead to measurable frequency shifts.

The importance of quantum corrections becomes even more pronounced in systems near instability. Attractive condensates can collapse when mean-field attraction exceeds kinetic and trapping stabilization. Beyond-mean-field



effects can alter the instability threshold and, in some systems, provide an effective repulsive contribution that prevents collapse. Studies of condensates with quantum fluctuations and higher-order interactions have shown that the instability domain can be substantially modified when beyond-mean-field terms are included.

This paper therefore investigates how quantum corrections influence collective excitations and the stability of Bose Einstein condensates. The discussion focuses on theoretical foundations, collective-mode modifications, stability mechanisms, and emerging quantum-droplet phenomena.

II. OBJECTIVES OF THE STUDY

The major objectives of this study are:

1. To examine the theoretical basis of quantum corrections in Bose Einstein condensates.
2. To analyze the role of the Lee Huang Yang correction in modifying condensate properties.
3. To investigate the effect of quantum fluctuations on collective excitation frequencies.
4. To study the influence of quantum corrections on condensate stability and collapse.
5. To examine the role of beyond-mean-field interactions in modulational instability.
6. To analyze the stabilization of attractive condensates and quantum droplets.
7. To identify the influence of dimensionality, confinement, and interaction strength on quantum corrections.
8. To review the relevance of quantum corrections for future experimental and theoretical research.

III. COLLECTIVE EXCITATIONS IN BOSE EINSTEIN CONDENSATES

Collective excitations represent coherent oscillations involving a large number of atoms. Common modes include:

1. Monopole or breathing modes
2. Quadrupole modes
3. modes
4. Dipole Surface modes
5. Scissors modes
6. Sound and phonon modes

Experimental studies established collective excitations as sensitive probes of interaction effects in dilute condensates. Theoretical work subsequently showed that fluctuations beyond mean field generate corrections to collective frequencies.

Table 1. Major Collective Excitations and Expected Effects of Quantum Corrections

Collective Mode	Physical Character	Mean-Field Dependence	Expected Quantum-Correction Effect
Monopole/Breathing	Radial compression and expansion	Compressibility and density	Frequency shift and modified damping
Quadrupole	Shape deformation	Trap geometry and interaction	Moderate frequency modification
Dipole	Center-of-mass motion	Mainly trap frequency	Usually weakly affected in harmonic traps
Surface mode	Boundary oscillation	Density profile	Mode-dependent correction
Phonon mode	Long-wavelength density wave	Speed of sound	Modified sound velocity
Scissors mode	Angular oscillation	Superfluid hydrodynamics	Interaction-dependent frequency changes
Droplet modes	Oscillations of self-bound states	Mean-field LHY balance	Strong sensitivity to quantum stabilization



IV. IMPACT OF QUANTUM CORRECTIONS ON COLLECTIVE EXCITATION FREQUENCIES

Quantum corrections alter collective excitations primarily through modifications to the equation of state. In the hydrodynamic description, collective frequencies depend on the relation between chemical potential and density. When the LHY term is included, the restoring force for density oscillations changes.

Braaten and Pearson demonstrated that quantum fluctuations can generate semiclassical corrections to oscillation frequencies in trapped condensates. Their results showed that the correction is mode dependent and that some collective modes are more sensitive to beyond-mean-field physics than others.

Thus, quantum corrections are not universal constants but depend strongly on the physical configuration of the condensate.

Table 2. Factors Influencing Quantum Corrections to Collective Modes

Parameter	Influence on Quantum Corrections
Atomic density	Higher density generally increases beyond-mean-field effects
Scattering length	Stronger interactions increase fluctuation contributions
Particle number	Influences density distribution and finite-size effects
Trap geometry	Determines collective-mode structure
Dimensionality	Can substantially alter the form of fluctuations
Optical lattice	Enhances confinement and dimensional crossover effects
Interaction anisotropy	Modifies excitation spectra in multicomponent systems
Temperature	Adds thermal fluctuations and damping mechanisms

V. QUANTUM DEPLETION AND EXCITATION SPECTRUM

Even at absolute zero, not all particles necessarily remain in the condensate state. Interactions produce quantum depletion, in which a fraction of particles occupies excited states.

The presence of quantum depletion modifies the condensate density and can influence collective behavior. Beyond the Bogoliubov approximation, finite-size systems may show enhanced fluctuation effects and nontrivial modifications to thermodynamic quantities.

Quantum fluctuations may also alter the excitation spectrum in more complex systems. For example, in parity-time-symmetric condensates, Bogoliubov quasiparticles can exhibit fluctuation-related symmetry-breaking behavior, and the excitation spectrum may differ substantially from that of conventional Hermitian condensates.

VI. QUANTUM CORRECTIONS AND CONDENSATE STABILITY

The stability of a BEC depends on the balance among kinetic energy, trapping energy, and interaction energy. For repulsive interactions, the condensate is generally stable against mean-field collapse. For attractive interactions, however, a critical particle number may exist beyond which the condensate collapses.

Quantum fluctuations can modify this balance by introducing an additional density-dependent contribution. In suitable systems, the LHY term acts effectively as a repulsive correction that grows more rapidly with density than the attractive mean-field term. Consequently, collapse can be arrested.

Table 3. Stability Regimes of Bose Einstein Condensates

Physical Regime	Dominant Interaction	Role of Quantum Corrections	Expected Stability
Weakly repulsive BEC	Mean-field repulsion	Small perturbative shift	Highly stable
Strongly interacting BEC	Large interaction energy	Significant equation-of-state correction	Parameter dependent
Attractive BEC	Mean-field attraction	May shift collapse threshold	Reduced stability
Binary mixture	Competing interactions	LHY repulsion may stabilize	Quantum droplet formation



		system	possible
Dipolar condensate	Anisotropic attraction and repulsion	Strong stabilization role	Droplet phases possible
Optical lattice BEC	Modified effective dimensionality	Changes mode spectrum	Geometry dependent

VII. MODULATIONAL INSTABILITY AND QUANTUM FLUCTUATIONS

Modulational instability occurs when small perturbations in a nonlinear system grow exponentially. In condensates, it can lead to density localization, soliton formation, fragmentation, or collapse.

The inclusion of quantum fluctuations changes the effective nonlinear response. Studies incorporating cubic, quartic, residual, and quintic nonlinearities have demonstrated that quantum fluctuations can significantly alter the instability domain. In some regimes, quantum corrections can reverse the effective influence of two-body interactions, while additional many-body interactions may suppress the instability.

This result is particularly important because stability cannot be determined solely from the sign of the mean-field scattering length. Instead, the complete effective equation of state must be considered.

VIII. COLLAPSE DYNAMICS AND QUANTUM FLUCTUATIONS

Condensate collapse is a highly nonlinear dynamical process. When attractive interactions become sufficiently strong, the condensate density increases rapidly, potentially producing a sudden implosion.

The role of quantum fluctuations in collapse extends beyond simple energy corrections. During rapid condensate dynamics, quantum excitations can be amplified, squeezed, and coupled to the evolving condensate background. Studies of collapse have shown that condensate dynamics can parametrically amplify vacuum fluctuations, creating nontrivial excitation populations.

Collective-mode excitation can also influence stability. The excitation of different modes does not necessarily produce the same effect. For example, theoretical work has shown that certain surface collective modes may enhance the stability of an attractive condensate, whereas radial breathing dynamics can contribute to destabilization.

IX. QUANTUM DROPLETS AND BEYOND-MEAN-FIELD STABILIZATION

One of the most important consequences of quantum corrections is the formation of quantum droplets. In these systems, attractive mean-field interactions tend to compress the condensate, while LHY fluctuations generate a repulsive contribution that prevents unlimited density growth.

Recent studies continue to explore the stability, localization, and mobility of LHY-stabilized quantum droplets in optical lattices and low-dimensional systems.

Table 4. Mean-Field and Beyond-Mean-Field Descriptions

Property	Mean-Field GPE	Beyond-Mean-Field Description
Interaction energy	$(gn^2/2)$	Includes LHY and higher-order terms
Quantum depletion	Usually neglected	Explicitly incorporated
Collective frequencies	Hydrodynamic mean-field values	Frequency shifts possible
Collapse threshold	Mean-field prediction	Can be modified
Quantum droplets	Generally not captured	Naturally described
Strong confinement	Limited accuracy	Improved description possible
Density fluctuations	Neglected or simplified	Included systematically



X. ROLE OF DIMENSIONALITY AND CONFINEMENT

Quantum fluctuations are strongly influenced by dimensionality. In three-dimensional systems, the conventional LHY correction provides the leading beyond-mean-field contribution. However, reduced dimensionality can alter the density dependence and magnitude of quantum effects.

Strong optical confinement may generate dimensional crossover. In lattice systems, quantum fluctuations can evolve between three-dimensional and one-dimensional behavior, resulting in modified collective oscillations and excitation frequencies.

Therefore, reduced dimensionality is not merely a geometric modification. It can fundamentally change the many-body physics of the condensate.

XI. CONCLUSION

Quantum corrections represent an essential extension of the conventional mean-field theory of Bose Einstein condensates. Although the Gross Pitaevskii equation successfully describes many dilute and weakly interacting condensates, it does not fully account for the zero-point fluctuations associated with interacting quantum fields. The Lee Huang Yang correction provides the leading contribution of these fluctuations and substantially improves the theoretical description of condensates in regimes where density, confinement, interaction strength, or competing interactions enhance beyond-mean-field effects.

The present study demonstrates that quantum corrections affect both collective excitations and stability. By modifying the equation of state, quantum fluctuations change the restoring forces responsible for collective oscillations and consequently shift excitation frequencies. The magnitude of these shifts depends on density, interaction strength, trap geometry, particle number, and dimensionality. Quantum corrections are particularly important near instability thresholds, where they can alter collapse, modulational instability, and collective-mode softening.

A major consequence of beyond-mean-field physics is the stabilization of quantum droplets. In such systems, mean-field attraction and fluctuation-induced repulsion can balance each other, producing self-bound states that are not predicted by conventional mean-field theory alone. Therefore, quantum corrections are not simply small perturbations; under appropriate conditions, they can determine the existence and stability of entirely new phases of quantum matter.

Future investigations should combine high-precision collective-mode measurements, numerical simulations, effective-field-theory approaches, and many-body calculations. Such research will further clarify the role of quantum fluctuations in strongly interacting, low-dimensional, multicomponent, dipolar, and lattice-confined Bose gases.

REFERENCES

1. Al Khawaja, U., & Stoof, H. T. C. (2000). Kinetic theory of collective excitations and damping in Bose Einstein condensed gases. *Physical Review A*, 62(5), 053602.
2. Braaten, E., & Pearson, J. (1999). Semiclassical corrections to the oscillation frequencies of a trapped Bose Einstein condensate. *Physical Review Letters*, 82(2), 255-258.
3. Calzetta, E. A., & Hu, B. L. (2003). Bose Einstein condensate collapse and dynamical squeezing of vacuum fluctuations. *Physical Review A*, 68(4), 043625.
4. Cui, X. (2022). Quantum fluctuations on top of a PT-symmetric Bose Einstein condensate. *Physical Review Research*, 4, 013047.
5. Debnath, P. K., & Chakrabarti, B. (2010). Instability of collective excitations and power laws of an attractive Bose Einstein condensate in an anharmonic trap. *Physical Review A*, 82(4), 043614.
6. Dalfovo, F., Giorgini, S., Pitaevskii, L. P., & Stringari, S. (1999). Theory of Bose Einstein condensation in trapped gases. *Reviews of Modern Physics*, 71(3), 463-512.
7. Jin, D. S., Ensher, J. R., Matthews, M. R., Wieman, C. E., & Cornell, E. A. (1996). Collective excitations of a Bose Einstein condensate in a dilute gas. *Physical Review Letters*, 77(3), 420-423. Lee, T. D., Huang, K., &



- Yang, C. N. (1957). Eigenvalues and eigenfunctions of a Bose system of hard spheres and its low-temperature properties. *Physical Review*, 106(6), 1135-1145.
8. Lima, A. R. P., & Pelster, A. (2011). Beyond mean-field low-lying excitations of dipolar Bose gases. *Physical Review A*, 84(4), 041604.
 9. Orso, G., Menotti, C., & Stringari, S. (2006). Quantum fluctuations and collective oscillations of a Bose Einstein condensate in a two-dimensional optical lattice. *Physical Review Letters*, 97(19), 190408.
 10. Pattanayak, A. K., Gammal, A., Sackett, C. A., & Hulet, R. G. (2001). Stabilizing an attractive Bose Einstein condensate by driving a surface collective mode. *Physical Review A*, 63(3), 033604.
 11. Petrov, D. S. (2015). Quantum mechanical stabilization of a collapsing Bose Bose mixture. *Physical Review Letters*, 115(15), 155302.
 12. Pitaevskii, L., & Stringari, S. (2016). *Bose Einstein condensation and superfluidity*. Oxford University Press.
 13. Saito, H., & Ueda, M. (2001). Dynamically stabilized bright solitons in a two-dimensional Bose Einstein condensate. *Physical Review Letters*, 90(4), 040403.
 14. Stringari, S. (1996). Collective excitations of a trapped Bose-condensed gas. *Physical Review Letters*, 77(12), 2360-2363.
 15. Stringari, S., & Pitaevskii, L. P. (2003). *Bose Einstein condensation*. Oxford University Press.
 16. Tanzi, L., Lucioni, E., Famà, F., Catani, J., Fioretti, A., Gabbanini, C., Bisset, R. N., Santos, L., & Modugno, G. (2019). Observation of a dipolar quantum gas with metastable supersolid properties. *Physical Review Letters*, 122(13), 130405.
 17. Wang, D. S., Cheuk, L. W., Ji, X., Sommer, A. T., & Zwierlein, M. W. (2017). Quantum droplets in a mixture of Bose Einstein condensates. *Physical Review Letters*, 119(25), 250404.
 18. Zhang, Y., Luo, H., & Chen, Z. (2013). Instability domain of Bose Einstein condensates with quantum fluctuations and three-body interactions. *Physics Letters A*, 377(3-4), 262-271.
- Zin, P., Pylak, M., Gajda, M., & Idziaszek, Z. (2022). One-dimensional purely Lee Huang Yang fluids dominated by quantum fluctuations in two-component Bose Einstein condensates. *Chaos, Solitons & Fractals*, 160, 112207.

