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Fixed Point Theorems in Partially Ordered Perturbed Metric Spaces

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Abstract: *Recently, Jleli and Samet (2025) introduced perturbed metric spaces, providing a novel framework for generalizing Banach's fixed point theorem. In this paper, we extend this framework to the setting of partially ordered perturbed metric spaces. By integrating order structures with perturbed distances, we generalize the fundamental results of Ran and Reurings (2003) and Nieto and Rodriguez-Lopez (2005) to this broader class of spaces.*

Keywords: fixed point; partial order; complete metric space; perturbed metric space

I. INTRODUCTION

The study of fixed point theory concerns results showing that, under certain conditions, a mapping $F:X\to X$ admits one or more fixed points, that is, points satisfying $Fx=x$. Fixed point theory is commonly grouped into three areas: metric fixed point theory, topological fixed point theory, and discrete fixed point theory. This classification is not rigid but depends largely on the type of hypotheses used in the fixed point theorems. Classical results in these areas include Banach's fixed point theorem [2], Brouwer's fixed point theorem [3], and Tarski's fixed point theorem [19].

Banach's fixed point theorem is notable for its simplicity and remains one of the most widely applied results in analysis. As a fundamental tool in nonlinear analysis, Banach's theorem has motivated many generalizations and extensions, either by weakening the contractive condition or by extending the structure of the underlying space ([1], [4]–[9], [13]–[18], and references therein). In 2003, Ran and Reurings [14] introduced a weakened contraction principle for complete metric spaces endowed with a partial order. Later, Nieto and Rodriguez-Lopez (2005) [13] refined this framework by replacing the continuity assumption with a weaker sequential condition. Since then, fixed point theory has increasingly incorporated partial order structures, showing that contractivity may only be required for comparable elements. This perspective allows the continuity condition to be relaxed through properties of partially ordered metric spaces ([7], [12], [15], [16]). Collectively, these developments underscore the continuing influence of Banach's theorem and its extensions in ordered metric spaces.

Recently, Jleli and Samet (2025) [10] introduced the notion of a perturbed metric space D on X, which need not satisfy the usual axioms of a metric space. In this framework, distances are determined not only by the standard metric rule but also by an additional perturbation term. This extension of the classical metric structure provides a more flexible setting for fixed point theory, particularly in cases where continuity, monotonicity, or convergence properties may be influenced by perturbations.

Building on this recent development, we study perturbed metric spaces endowed with a partial order and establish existence and uniqueness results for monotone mappings. In particular, we derive analogues of the results of Nieto and Rodriguez- Lopez [13] and Ran and Reurings [14], now formulated within the broader framework of perturbed metric spaces. In this way, the core ideas of ordered fixed point theory are preserved while being extended to a more general setting that accommodates perturbations in the distance structure.

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II. PRELIMINARIES

We begin by recalling several foundational definitions and results from the literature. First, we review key concepts and theorems in the context of partially ordered metric spaces. Then, we highlight the contributions of Jleli and Samet [10], whose work offers important results that form a basis for our study.

Throughout this paper, X denotes an arbitrary non-empty set, and $\mathbb{N}$ denotes the set of nonnegative integers.

Definition 2.1. A partially ordered set is a system $(X, \preceq)$ where X is a non-empty set and $\preceq$ is a binary relation on X satisfying, for all $x, y, z \in X$,

a. $x \preceq x$ (reflexivity)

b. if $x \preceq y$ and $y \preceq x$, then $x = y$ (antisymmetry)

c. if $x \preceq y$ and $y \preceq z$, then $x \preceq z$ (transitivity)

Definition 2.2. A non-empty set X together with a metric $d: X \times X \to \mathbb{R}^+ \cup \{0\}$ is called a metric space if the following conditions are satisfied by any $x, y, z \in X$:

a. $d(x, y) \geq 0$ and $d(x, y) = 0$ if and only if $x = y$

b. $d(x, y) = d(y, x)$

c. $d(x, z) \leq d(x, y) + d(y, z)$

Definition 2.3. Let (X, d) be a metric space. A sequence $\{x_n\} \in X$ is a Cauchy sequence if it has the property that given $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \epsilon$ whenever $n, m \geq N$. The metric (X, d) is complete if every Cauchy sequence in X is convergent.

Definition 2.4. $(X, d, \preceq)$ is a partially ordered complete metric space if (X, d) is a complete metric space and $(X, \preceq)$ is a partially ordered set.

Theorem 2.5. *(Banach's Fixed Point Theorem (1922) [2]) Let (X, d) be a complete metric space and let $F: X \to X$ be a contraction mapping. Then F has a unique fixed point and for each $x \in X$, $\lim_{n\to\infty} F^n(x) = x_0$. Moreover,*

$$d(F^n(x), x_0) \leq \frac{k^n}{1-k} d\big(x, F(x)\big).$$

Theorem 2.6. *(Ran and Reurings Fixed Point Theorem (2003) [14]) Let $(X, \preceq)$ be a partially ordered set such that every pair $x, y \in X$ has a lower bound and an upper bound. Furthermore, let d be a metric on X such that (X, d) is a complete metric space. If F is a continuous monotone (either order-preserving or order-reversing) map from X into X such that*

a. $\exists\, 0 < c < 1 : d(Fx, Fy) \leq c\, d(x, y)$ *for all* $x \preceq y$

b. $\exists\, x_0 \in X: x_0 \preceq Fx_0$ *or* $Fx_0 \preceq x_0$

Then F has a unique fixed point x^*. Moreover, for every $x \in X$

$$\lim_{n\to\infty} F^n(x) = x^*.$$

Theorem 2.7. *(Nieto and Rodriguez-Lopez Fixed Point Theorem (2005) [13]) Let $(X, \preceq)$ be a partially ordered set such that every pair of elements of X has an upper bound or a lower bound, and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Let $F: X \to X$ be a nondecreasing (order- preserving) mapping such that*

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there exists $c \in (0, 1]$ with $d(Fx, Fy) \le cd(x, y)$ for all $x \preceq y \in X$. Assume that either F is continuous or X satisfies the condition that: if nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for $n \in \mathbb{N}$ and there exists $x_0 \in X$ with $x_0 \preceq Fx_0$. Then F has a unique fixed point $\bar{x}$.

For completeness, we briefly review the concept of perturbed metric spaces and highlight important results established in [10].

Definition 2.8 (Perturbed Metric Space**).** Let X be a non-empty set and let $D, P : X \times X \to [0, \infty)$ be two mappings. We say that D is a *perturbed metric* on X with respect to P if the function

$$(D - P)(x, y) = D(x, y) - P(x, y), (x, y) \in X \times X,$$

satisfies the following properties:

i. $(D - P)(x, y) \ge 0$;

ii. $(D - P)(x, y) = 0 \Leftrightarrow x = y$;

iii. $(D - P)(x, y) = (D - P)(y, x)$;

iv. $(D - P)(x, y) \le (D - P)(x, z) + (D - P)(z, y)$, for all $x, y, z \in X$

In this case, P is called a *perturbed mapping*, $d = D - P$ is called the *exact metric*, and the triple (X, D, P) is referred to as a *perturbed metric space*.

Remark 2.9. A perturbed metric on X is not necessarily a metric on X. Examples are given below to show this remark [10].

Example 2.10. Define the mapping $D : \mathbb{R} \times \mathbb{R} \to [0, \infty)$ by

$$D(x, y) = |x - y| + x^m y^n, \quad x, y \in \mathbb{R}; m, n \in \mathbb{N}$$

where $P: \mathbb{R} \times \mathbb{R} \to [0, \infty)$ is given by

$$P(x, y) = x^m y^n$$

while d is the exact metric $d: \mathbb{R} \times \mathbb{R} \to [0, \infty)$

$$d(x, y) = |x - y|, \ x, y \in \mathbb{R}.$$

Note that D is a perturbed metric on X but not a metric on X, since

$$D(1, 1) = 1 \ne 0$$

for any $m, n \in \mathbb{N}$.

Example 2.11. Let $C([0, 1]) = \{f : [0, 1] \to \mathbb{R} : f$ is continuous on $[0, 1]\}$. Define the mapping $D : C([0, 1]) \times C([0, 1]) \to [0, \infty)$ by

$$D(f, g) = \int_0^1 |f(t) - g(t)|dt + \big(f(0) - g(0)\big)^2, \quad f, g \in C([0, 1]).$$

Then D is perturbed with respect to the perturbed mapping $P: C([0, 1]) \times C([0, 1]) \to [0, \infty)$ given by

$$P(f, g) = (f(0) - g(0))^2, \quad f, g \in C([0, 1]).$$

The exact metric $d : C([0, 1]) \times C([0, 1]) \to [0, \infty)$ is

$$d(f, g) = \int_0^1 |f(t) - g(t)|dt, f, g \in C([0, 1]).$$

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Note that D is symmetric and $D(f, g) = 0$ if and only if $f = g$. However, $D$is not a metric on$C([0, 1])$.

Theorem 2.12. *Let $D, P, Q : X \times X \to [0, \infty)$ be mappings and let $\alpha > 0$ be a constant. Then:*

If $(X, D, P)$and (X, D, Q) are perturbed metric spaces, then

$$\left(X, D, \frac{P+Q}{2}\right)$$

is also a perturbed metric space.

i. *If (X, D, P) is a perturbed metric space, then*

$$(X, \alpha D, \alpha P)$$

is a perturbed metric space.

Definition 2.13. Let (X, D, P) be a perturbed metric space, $\{z_n\}$ a sequence in X, and $T : X \to X$ a mapping. Then:

i. A sequence $\{z_n\}$ is called a *perturbed convergent sequence* in (X, D, P) if it is a convergent sequence in the metric space (X, d), where $d = D - P$ is the exact metric.
ii. A sequence $\{z_n\}$ is called a *perturbed Cauchy sequence* in (X, D, P) if it is a Cauchy sequence in the metric space (X, d).
iii. The space (X, D, P) is called a *complete perturbed metric space* if (X, d) is a complete metric space, or equivalently, every perturbed Cauchy sequence in (X, D, P) is a perturbed convergent sequence in (X, D, P).
iv. A mapping T is called a *perturbed continuous mapping* if T is continuous with respect to the exact metric d.

The main result of Jleli and Samet [X] is given in the theorem below.

Theorem 2.14. *Let (X, D, P) be a complete perturbed metric space and let $T: X \to X$ be a mapping. Assume that the following conditions hold:*

i. T is a perturbed continuous mapping;
ii. *There exists $\lambda \in (0, 1)$ such that*

$$D(Tu, Tv) \leq \lambda D(u, v), \qquad \forall u, v \in X.$$

Then T admits one and only one fixed point.

III. RESULTS AND DISCUSSION

This section focuses on fixed point theorems in partially ordered perturbed metric spaces. We first present analogues of the classical results of Ran and Reurings' theorem [14], establishing existence and uniqueness of fixed points, then prove a version of Nieto and Rodriguez-Lopez [13], replacing the continuity condition. These results extend and generalize earlier results in the literature.

For simplicity of notation, we use $(X, D, P, \preceq)$ to represent a complete perturbed metric space D with respect to P where the set X is equipped with the partial ordering $\preceq$.

Theorem 3.1. *Let* $(X, D, P, \preceq)$ *be a partially ordered complete perturbed metric space such that every pair* $x, y \in X$ *has a lower bound or an upper bound. Let* $F : X \to X$ *be a mapping such that the following conditions are satisfied:*

i. *There exists* $x_0 \in X$ *such that* $x_0 \preceq Fx_0$
ii. *For all* $x, y \in X$, *if* $x \preceq y$ *then* $Fx \preceq Fy$
iii. *F is a perturbed continuous mapping.*
iv. *There exists* $\alpha \in (0, 1)$ *such that* $D(Fx, Fy) \leq \alpha D(x, y)$ *for all* $x \preceq y$

Then F has a unique fixed point.

Proof. Let $x_0 \in X$ such that $x_0 \preceq Fx_0 = x_1$. If $x_0 = x_1$ then x_0 is a fixed point of F, and we are done.

Suppose that $x_0 \neq x_1$, by condition *ii*, $Fx_0 \preceq Fx_1$. Let $x_2 = Fx_1$ such that $x_1 \preceq x_2$ and by condition *iv*,

$$D(x_1, x_2) = D(Fx_0, Fx_1) \leq \alpha D(x_0, x_1)$$

Again,fromconditioniiandiv,$Fx_1 \preceq Fx_2$,and there exists $x_3 = Fx_2$ such that $x_2 \preceq x_3$and

$$D(x_2, x_3) = D(Fx_1, Fx_2)$$
$$\leq \alpha D(x_1, x_2)$$
$$\leq \alpha^2 D(x_0, x_1)$$

Continuing this process, by induction, we obtain a non decreasing sequence $\{x_n\}$ such that $x_n = Fx_{n-1}$ and

$$D(x_n, x_{n+1}) = D(Fx_{n-1}, Fx_n)$$
$$\leq \alpha^n D(x_0, x_1), \; n \in \mathbb{N}$$

Let d defined as $d = D - P$ be the exact metric. Since $P(x, y) \geq 0$ for $x, y \in X$, it follows that

$$d(x_n, x_{n+1}) + P(x_n, x_{n+1}) \leq \alpha^n D(x_0, x_1)$$
$$d(x_n, x_{n+1}) \leq \alpha^n D(x_0, x_1), \; n \in \mathbb{N}$$

Next, we show that $\{x_n\}$ is a Cauchy sequence in the metric space (X, d). Let $N \in \mathbb{N}$, and $m, n \geq N$ such that $m > n$, then,

$$d(x_n, x_m) \leq D(x_n, x_{n+1}) + D(x_{n+1}, x_{n+2}) + \ldots + D(x_{m-1}, x_m)$$
$$\leq \alpha^n D(x_0, x_1) + \alpha^{n+1} D(x_0, x_1) + \ldots + \alpha^{m-1} D(x_0, x_1)$$
$$= \alpha^n D(x_0, x_1)(1 + \alpha + \ldots + \alpha^{m-1-n})$$
$$\leq \alpha^n D(x_0, x_1) \frac{1}{1-\alpha}$$

As $n \to \infty$, $d(x_n, x_m) \to 0$, and this implies that $\{x_n\}$ is a Cauchy sequence in (X, d), thus $\{x_n\}$ is a perturbed Cauchy sequence in (X, D, P).

By the completeness of (X, D, P), (X, d) is a complete metric space. Hence, there exists $x \in X$such that

$$\lim_{n\to\infty} d(x_n, x) = 0$$

which implies that $\{x_n\}$ is a perturbed convergent sequence and it converges to x.

Now, we show that x is a fixed point of F. Since F is a perturbed continuous mapping, then F is continuous with respect to the exact metric d. It follows that

$$\lim_{n\to\infty} Fx_n = F\left(\lim_{n\to\infty} x_n\right) \Longrightarrow \lim_{n\to\infty} x_{n+1} = Fx$$

The limit in (X, d) is unique; thus, $x = Fx$. This confirms that x is a fixed point of F.

Next, we show the uniqueness of the fixed point. By sense of contradiction, assume that there are two fixed points of F, say x and y. For the first case, assume that x and y are comparable, that is, $x \preceq y$. Then by condition *iv*,

$$D(x,y) = D(Fx,Fy) \le \alpha D(x,y)$$

$$D(x,y) \le \alpha D(x,y)$$

$$d(x,y) + P(x,y) \le \alpha(d(x,y) + P(x,y))$$

However, $x \neq y$, thus $d(x, y) + P(x, y) \neq 0$ and $\alpha \geq 1$, which is a contradiction to condition *iv*. Hence, if x and y are comparable, F has a unique fixed point.

For the second case, assume that x and y are not comparable, however, take note that every pair $x, y \in X$ has a lower bound or an upper bound. Without loss of generality, assume $z \in X$ is a common upper bound, that is, $x \preceq z$ and $y \preceq z$. Let $z_0 = z$, since $x \preceq z$ then $Fx \preceq Fz_0$. Let $z_1 = Fz_0$, then $x \preceq z_1$. By induction, $x \preceq z_{n-1}$ then $Fx \preceq Fz_{n-1}$ which implies that $x \preceq z_n$ for all $n \in \mathbb{N}$.

Since $x \preceq z_n$, then by condition *iv*,

$$D(x, z_n) = D(Fx, Fz_{n-1}) \le \alpha\, D(x, z_{n-1}) \text{ for all } n \in \mathbb{N}.$$

Applying this inequality recursively n times,

$$d(x, z_n) \le D(x, z_n) \le \alpha^n D(x, z_0)$$

Since $\alpha \in (0, 1)$, as $n \to \infty$, $\alpha^n \to 0$. Therefore

$$\lim_{n\to\infty} d(x, z_n) = 0 \implies z_n \to x$$

Using the same exact logic and process for y, we obtain,

$$d(y, z_n) \le D(y, z_n) \le \alpha^n D(y, z_0)$$

and as $n \to \infty$, $\alpha^n \to 0$. Therefore,

$$\lim_{n\to\infty} d(y, z_n) = 0 \implies z_n \to y$$

In a metric space, the limit of a convergent sequence is unique. Since the sequence $\{z_n\}$ converges to both x and y in the metric space (X, d), it must be that $x = y$. Therefore, in both cases, the fixed point is unique.

A dual result of Theorem 3.1 can be obtained with some modifications in conditions *i* and *ii* given in Theorem 3.2.

Theorem 3.2. *Let $(X, D, P, \preceq)$ be a partially ordered complete perturbed metric space such that every pair $x, y \in X$ has a lower bound or an upper bound. Let $F : X \to X$ be a mapping such that the following conditions are satisfied:*

i. *There exists $x_0 \in X$ such that $Fx_0 \preceq x_0$*
ii. *For all $x, y \in X$, if $x \preceq y$ then $Fy \preceq Fx$*
iii. *F is a perturbed continuous mapping*

iv. *There exists* $\alpha \in (0, 1)$ *such that* $D(Fx, Fy) \le \alpha D(x, y)$ *for all* $x \preceq y$

Then F has a unique fixed point.

Proof. The proof follows from Theorem 3.1 by considering $x_0 \in X$ such that $Fx_0 \preceq x_0$ and consequently obtaining a nonincreasing sequence $\{x_n\}$ such that $Fx_{n-1} \preceq x_n$. □

Remark 3.3. Note that Theorems 3.1 and 3.2 generalize the fixed point theorems of Ran and Reurings within the framework of perturbed metric spaces. By setting $D = d$ and $P = 0$ (i.e. $P(x, y) = 0$ for all $x \preceq y$), we obtain the original results of Ran and Reurings (2003).

Remark 3.4. The method of Ran and Reurings is an order-basedapproach that identifies fixed points by iterating along elements that are comparable under a partial order. Its main feature is that the contraction condition, $D(Fx, Fy) \le \alpha D(x, y)$, only needs to hold for pairs of elements where $x \preceq y$. In this setting, the partial order ensures that the iterative sequence $\{x_n\}$ remains monotone $x_n \preceq x_{n+1}$ (or $x_{n+1} \preceq x_n$), even if perturbations affect the space D. However, this approach depends critically on starting with an initial point x_0 satisfying $x_0 \preceq Fx_0$ (or $Fx_0 \preceq x_0$) because without such a starting element, convergence is not guaranteed.

In contrast, the framework introduced by Jleli and Samet (2025) adopts a globalapproach based on a generalized Banach contraction. Here, the contraction inequality applies to all x, y in the space, independent of any ordering. The mapping's inherent contractive property ensures that the sequence converges to a fixed point regardless of the initial choice x_0, making the approach more flexible and computationally straightforward. Perturbations in the space are effectively managed by the global contraction, so no special starting conditions are necessary to guarantee convergence.

In the next theorem, we replace condition *iii* with: If $\{x_n\} \to x$ is a nondecreasing sequence in X, then $x_n \preceq x$ for all $n \in \mathrm{N}$ and $\lim_{n\to\infty} P(x_n, x) = 0$.

Theorem 3.5. *Let* $(X, D, P, \preceq)$ *be a partially ordered complete perturbed metric space such that every pair* x, $y \in X$ *has a lower bound or an upper bound. Let* $F\colon X \to X$ *be a mapping such that the following conditions are satisfied:*

i. *There exists* $x_0 \in X$ *such that* $x_0 \preceq Fx_0$
ii. *For all* $x, y \in X$, *if* $x \preceq y$ *then* $Fx \preceq Fy$
iii. *If* $\{x_n\} \to x$ *is a nondecreasing sequence in* (X, d), *then* $x_n \preceq x$ *for all* $n \in \mathrm{N}$ *and* $\lim_{n\to\infty} P(x_n, x) = 0$.
iv. *There exists* $\alpha \in (0, 1)$ *such that* $D(Fx, Fy) \le \alpha D(x, y)$ *for all* $x \preceq y$

Then F has a unique fixed point.

Proof. Arguing exactly as Theorem 3.1, we have a Cauchy sequence $\{x_n\}$ in (X, d), where $\lim_{n\to\infty} d(x_n, x) = 0$. Using condition *iii*, $x_n \preceq x$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} P(x_n, x) = 0$.

By condition *iv*,

$$D(Fx_n, Fx) \le \alpha D(x_n, x)$$

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$$\Rightarrow \quad D(x_{n+1}, Fx) \leq \alpha D(x_n, x)$$

$$\Rightarrow \quad d(x_{n+1}, Fx) \leq \alpha D(x_n, x)$$

Since $D = d + P$, then

$$d(x_{n+1}, Fx) \leq \alpha[d(x_n, x) + P(x_n, x)]$$

Taking the limit both sides as $n \to \infty$, we deduce that $x = Fx$. Therefore, F has a fixed point.

The proof for uniqueness is similar to Theorem 3.1.

A dual of Theorem 3.5 is given below. □

Theorem 3.6. *Let* $(X, D, P, \preceq)$ *be a partially ordered complete perturbed metric space such that every pair* $x, y \in X$ *has a lower bound or an upper bound. Let* $F : X \to X$ *be a mapping such that the following conditions are satisfied:*

i. *There exists* $x_0 \in X$ *such that* $Fx_0 \preceq x_0$

For all $x, y \in X$, *if* $x \preceq y$ *then* $Fy \preceq Fx$

i. *If* $\{x_n\} \to x$ *is a nonincreasing sequence in* (X, d), *then* $x \preceq x_n$ *for all* $n \in \mathbb{N}$ *and* $\lim_{n\to\infty} P(x_n, x) = 0$

ii. *There exists* $\alpha \in (0, 1)$ *such that* $D(Fx, Fy) \leq \alpha D(x, y)$ *for all* $x \preceq y$

Then F *has a unique fixed point.*

Remark 3.7. Theorems 3.5 and 3.6 generalize the results of Nieto and Rodriguez- Lopez in the context of perturbed metric spaces. Their original results can be deduced by letting $D = d$ and $P = 0$, i.e. $P(x, y) = 0$ for $x \preceq y$.

IV. CONCLUSION

In this paper, we extended the classical fixed point results to the setting of partially ordered perturbed metric spaces. By generalizing the theorems of Ran and Reurings (2003) as well as Nieto and Rodriguez-Lopez (2005), we established existence and uniqueness results for monotone mappings under more flexible conditions that incorporate perturbations in the distance function. These results not only preserve the essential features of ordered fixed point theory but also open new avenues for applications in areas where perturbations or irregularities are present. Future work may explore further generalizations, computational methods, and potential applications to differential equations, integral equations, and other problems in applied mathematics.

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