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On the Relationship between Laplace Transform and New Integral Transform "Iman Transform"

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Abstract: *In this paper we discusses some relation ship between Laplace transform and the new integral transform called Iman transform, we solve first and second order ordinary differential equations with constant and non-constant coefficients , using both transforms, and showing Iman transform is closely connected with Laplace transform..*

Keywords: Iman transform, Laplace transform, Differential equations

I. INTRODUCTION

The integral transform method is an efficient method to solve, differential equations ,system of differential equations, integral equations ,system, of integral equations and so on. Recently, Iman Ahmed introduced a new transform and named as Iman transform , which is defined by the following formula.

$$\mathrm{I}[f(t)] = \frac{1}{v^2}\int_0^\infty f(t)e^{-tv^2}dt = G(v) \quad , \ t \geq 0 \qquad (1)$$

While Lapace transform is defined by the following formula

$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s) \quad , \ \mathrm{Re(s)}>0$$

The sufficient conditions for the existence of Iman transform are that $f(t)$ be piecewise continuous and of exponential order, this means that Tarig transform may or may not exist. Iman transform can certainly treat all problems that are usually treated by the well-known and extensively used Laplace transform.

Indeed as the next theorem shows Iman transform is closely connected with Laplace transform.

Theorem (1)

Let $f(t) \in A = \{f(t) : \exists M, k1, k2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}};\ t \in (-1)^j \times [0, \infty[\ \}$

With Laplace transform $F(s)$. Then : Iman transform $G(v)$ of $f(t)$ is given by

$$G(v) = v^2 F(v^4) \qquad (2)$$

Proof:

Let $f(t) \in A$

then for $\quad \mathrm{I}[f(t)] = \int_0^\infty f\left(\frac{t}{v^2}\right) e^{-tv^2}dt \ = G(\mathrm{v})$

Let $w = \frac{t}{v^2}$, then we have:

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$$G(v) = v^2 \int_0^\infty f(w) e^{-wv^4} dw$$

$$G(v) = v^2 F(v^4)$$

Also we have that $G(1) = F(1)$ so that both Iman and Laplace transform must coincide at $v = s = 1$.

Iman Transform of Derivatives and Integrals

Being restatement of the relation(2) will serve as our working definition, since Laplace transform of sin t is $\frac{1}{1+s^2}$

then view of (2) its Iman transform is $\frac{v^4}{1+v^8}$

This exemplifies the duality between those two transforms.

Theorem (2):

Let $F'(s)$ *and* $G'(v)$ be Laplace Iman transform of the derivative of $f(t)$. then:

(i) $G'(v) = v^4 G(v) - v^2 f(0)$

(ii) $G''(v) = v^8 G(v) - v^6 f(0) - v^2 f'(0)$

Proof:

(i) Since Laplace transform of the derivatives of $f(t)$ is

$F'(s) = s\,F(s) - f(0)$

Then

$$G'(v) = v^2 F'(v^4) = v^2\{v^4 F(v^4) - f(0)\} = v^6 F(v^4) - v^2 f(0)$$

$$= v^4 G(v) - v^2 f(0)$$

The generalization to nth order derivative in (ii) can be proved by using mathematical induction.

Theorem (3)

Let $G'(u)$ *and* $F'(s)$ denote Iman and Laplace transform of the definite integral of f(t)

h(t)= $\int_0^t f(\tau)\, d\tau$

$$G'(v) = I[h(t)] = \frac{1}{v^4} G(v)$$

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Proof:

By definition of Laplace transform.

$$F'(s) = \ell[h(t)] = \frac{F(s)}{s}$$

Hence

$$G'(v) = v^2 F'(v^4) = v^2 \left\{\frac{1}{v^4} F(v^4)\right\} = \frac{1}{v^2} F(v^4)$$

$$= v^4\, G(v)$$

Theorem (4):

Let $G(v)$ is Iman transform of $f(t)$ then:

$$I[tf(t)] = \frac{1}{2}\left[\frac{1}{v^6}\frac{d}{dv}G(v) + \frac{1}{v^4}G(v)\right]$$

Proof:

By definition of Iman transform we have:

$$\frac{d}{dv}G(v) = v^2 \int_0^\infty tf(t)e^{-tv^4}dt$$

then

$$I[tf(t)] = \frac{1}{2}\left[\frac{1}{v^6}\frac{d}{dv}G(v) + \frac{1}{v^4}G(v)\right]$$

Theorem (5):

Let $G(v)$ is Iman transform of $f(t)$ then:

(i) $\mathrm{I}[t\, f'(t)] = \frac{1}{2v^6}\frac{d}{dv}[v^4\, G(v) - v^2 f(0)] + \frac{1}{2v^4}[v^4\, G(v) - v^2 f(0)]$

(ii) $\mathrm{I}[t\, f''(t)] = \frac{1}{2v^6}\frac{d}{dv}[v^8\, G(v) - v^6 f(0) - v^2 f'(0)] + \frac{1}{2v^4}[v^8\, G(v) - v^6 f(0) - v^2 f'(0)]$

Proof:

(i) From theorem (4), we have:

$$\mathrm{I}[t\, f'(t)] = \frac{1}{2v^6}\frac{d}{dv}I\big(f'(t)\big) + \frac{1}{2v^4}I\big(f'(t)\big)$$

$$= \frac{1}{2v^6}\frac{d}{dv}[v^4\, G(v) - v^2 f(0)] + \frac{1}{2v^4}[v^4\, G(v) - v^2 f(0)]$$

The proof of (ii) is similar to the proof of (i).

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Theorem (6) (Convolution)

Let $f(t)$ *and* $g(t)$ *be in* A, having Laplace transform

$F(s)$ and $G(s)$, and Iman transform $M(v)$ and $N(v)$

Then:

$I[(f * g)(t)] = \frac{1}{v^2} M(v)\, N(v)$

Proof:

Firs recall that Laplace transforms of $(f * g)$ is given by

$\ell\,[(f * g)(t)] = F(s)\, G(s)$

Now, since, by the duality relation (2) we have,

$I[(f * g)(t)] = v^2 \ell\,[(f * g)(t)]$

and since

$M(v) = v^2 F(v^4)$

$$N(v) = v^2 G(v^4)$$

Iman transform of $(f * g)$ is obtained as follows:

$I[(f * g)(t)] = v^2 F(v^4)\, G(v^4)$

$= \frac{1}{v^2} M(v)\, N(v)$

Example :

Consider the first – order ordinary differential equation,

$$\frac{dx}{dt} + px = f(t), t >$$

$$x(0) = a \qquad (4)$$

Where p and a are constants and $f(t)$ is an external

input function so that its Laplace and Iman Transforms are exist.

First Solution by Laplace Transform

$$sX(s) - x(0) + pX(s) = F(s)$$

$$X(s) = \frac{a}{s+p} + \frac{F(s)}{s+p}$$

Where that $X(s)$ and F(s) are Laplace transform of $x(t)$

and $f(t)$.

Then

$$x(t) = ae^{-pt} + \ell^{-1}\left[\frac{F(s)}{s+p}\right]$$

In particular if $f(t) = c \equiv$ constant, then the Solution of (3) becomes:

$$x(t) = \frac{c}{p} + \left(a - \frac{c}{p}\right) e^{-pt}$$

Second Solution By Iman Transform:

Using Iman transform of equation (3) we get

$$v^4\, X(v) - v^2 X(0) + pX(v) = F(v)$$

$$v^4\, X(v) + pX(v) = F(v) + v^2 X(0)$$

$$(v^4 + p)X(v) = F(v) + av^2$$

$$X(v) = \frac{F(v) + av^2}{(v^4 + p)}$$

The inverse Iman transform leads to the solution in the form.

$x(t) = \frac{c}{p} + \left(a - \frac{c}{p}\right) e^{-pt}$, When $f(t) = c$

II. CONCLUSIONS

Iman transform is a convenient tool for solving differential equations in the time domain without the need for performing an inverse Iman transform and the connection of Iman transform with Laplace transform goes much deeper.

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Table of Functions and their Iman Transform

$f(t)$	$I[f(t)] = F(v)$
	$\frac{1}{v^4}$
t	$\frac{1}{v^6}$
t^2	$\frac{2!}{v^8}$
$^n \quad n \in N$	$\frac{n!}{v^{2n+4}}$
e^{at}	$\frac{1}{v^2(v^2 - a)}$
$\sin(at)$	$\frac{a}{v^2(v^4 + a^2)}$
$\cos(at)$	$\frac{1}{v^4 + a^2}$
$H(t - a)$	$\frac{1}{v^4} e^{-av^2}$
$\delta(t - a)$	$\frac{1}{v^2} e^{-av^2}$
$\sinh(at)$	$\frac{a}{v^2(v^4 - a^2)}$
$\cosh(at)$	$\frac{a}{v^4 - a^2}$
$t^{a-1}/\Gamma(a)$, a>0	$\left(\frac{1}{v^2}\right)^{a+1}$

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