IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 1, March 2026
Impact Factor: 8.2

Innumerable Sequences of Hyperbolic Polynomial Diophantine Triples

G. Janaki[1] and R. Sarulatha[2]
Associate Professor, PG and Research Department of Mathematics[1]
Research Scholar, PG and Research Department of Mathematics[2]
Cauvery College for Women (Autonomous), Tiruchirappalli, India

Abstract: *This paper presents a novel construction of infinite sequence of Hyperbolic polynomial Diophantine triples using Vieta polynomials with the property D(m). Initial pairs formed by these polynomial-derived hyperbolic numbers are extended to triples through algebraic manipulations. A recursive application then chains these triples into infinite sequences, where each subsequent triple emerges from the pair of the previous hyperbolic triple construction*
2020 Mathematics Subject classification: 11D99.

Keywords: Diophantine triple, Hyperbolic number, Vieta-Fibonacci polynomial, Vieta-Lucas polynomial, Vieta-Pell polynomial, Vieta-Pell-Lucas polynomial, Integer sequence

I. INTRODUCTION

The classical Diophantine problem seeks set of integers where pairwise products plus a fixed integer or a polynomial with integer coefficient yield perfect square, a challenge originating with Diophantus's rational quadruple for $m = 1$. In modern number theory, Diophantine k- tuple with the property $D(m)$ is a set $\{a_1, a_2, a_3, \dots \dots \dots a_k\} \subset \mathbb{Z}\backslash\{0\}$ satisfying $a_i a_j + m = \alpha^2$ for all $i \neq j$. While $D(1)$ quadruples exist, extending beyond $k = 4$ proves difficult, spurring extensive research into this field [2-4,11-13].

Hyperbolic Diophantine triple extends this framework from Hyperbolic numbers, defined as ordered pairs of real numbers $P = p_1 + p_2 j$, where p_1, p_2 are real numbers and j is the Hyperbolic unit satisfying $j^2 = 1$ and $j \neq \pm 1$.

A set of Hyperbolic numbers $\{a_1, a_2, a_3\}$ forms a Hyperbolic Diophantine triple with the property $D(m)$ if, for all $1 \leq i < j \leq 3$,

$$a_i a_j + m = \alpha^2$$

Existing research constructs Diophantine triples, Gaussian triples and their extensions to quadruples using various figurate numbers and polynomials [1,5,7-10,14]. This investigation addresses the representation of Hyperbolic numbers through polynomials, enabling the construction of Polynomial Diophantine triples termed Hyperbolic Polynomial Diophantine Triples. These triples emerge systematically from Vieta polynomials [6], with the present work introducing their derivation from $D(m)$- Hyperbolic pairs constructed using Vieta-Fibonacci and Vieta-Lucas polynomial, Vieta-Pell and Vieta-Pell-Lucas polynomial. Pairs from each triple serve as the basis for next triple, generating infinite sequences while preserving the $D(m)$ property throughout the process.

II. PRELIMINARIES

The Vieta-Fibonacci polynomial is defined as,

$$v_k(y) = \sum_{s=0}^{\left[\frac{k-1}{2}\right]} (-1)^s \binom{k-s-1}{s} y^{k-2s-1}, \qquad v_0(y) = 0$$

DOI: 10.48175/IJARSCT-31425

The Vieta-Lucas polynomial is defined as,

$$V_k(y) = \sum_{s=0}^{\left[\frac{k}{2}\right]} (-1)^s \frac{k}{k-s}\binom{k-s}{s} y^{k-2s}, \qquad V_0(y) = 2$$

The Vieta-Pell polynomial is defined as

$$\Delta_n(y) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ 2y\Delta_{n-1}(y) - \Delta_{n-2}(y), & n \geq 2 \end{cases}$$

The Vieta-Pell Lucas polynomial is defined as

$$\nabla_n(y) = \begin{cases} 2, & n = 0 \\ 2y, & n = 1 \\ 2y\nabla_{n-1}(y) - \nabla_{n-2}(y), & n \geq 2 \end{cases}$$

III. SEQUENCES OF HYPERBOLIC VIETA POLYNOMIAL

This section exemplifies the formation of Hyperbolic pairs, formed using Vieta-Fibonacci and Vieta-Lucas polynomials, Vieta-Pell and Vieta-Pell Lucas polynomials, and their extension to Hyperbolic triples. The requisite additional polynomial is derived through algebraic manipulations, thereby constructing infinite sequences.

Hyperbolic pair formation: Consider the polynomials defined as $v_4(y) = y^3 - 2y$ and $V_3(y) = y^3 - 3y$. Their hyperbolic representations are given by $a(y) = y^3 - 2y + 7kj$ and $b(y) = y^3 - 3y + 7kj$. These polynomials constitute a Hyperbolic pair, as their product satisfies

$$a(y)b(y) - 5y^4 + 19y^2 - 35ykj = \alpha^2$$

$(y^3 - 2y + 7kj, y^3 - 3y + 7kj)$ is a Hyperbolic pair with the property $D(-5y^4 + 19y^2 - 35ykj)$.

Similarly, consider the polynomials $\Delta_4(y) = 8y^3 - 4y$ and $\nabla_3(y) = 8y^3 - 6y$, with hyperbolic forms $r(y) = 8y^3 - 4y + 7kj$ and $s(y) = 8y^3 - 6y + 7kj$, form a Hyperbolic pair $(8y^3 - 4y + 7kj, 8y^3 - 6y + 7kj)$ since their product added to y^2 is a perfect square.

Hyperbolic pair I extension:

For the pair $(y^3 - 2y + 7kj, y^3 - 3y + 7kj)$, the third polynomial $c(y)$ is determined as

$$a(y)c(y) - 5y^4 + 19y^2 - 35ykj = \beta^2$$
$$b(y)c(y) - 5y^4 + 19y^2 - 35ykj = \gamma^2$$

Assume $\beta = a(y) + \alpha$ and $\gamma = b(y) + \alpha$. Substituting these linear conversions above and deducting leads to

$(a - b)c = (a + \alpha)^2 - (b + \alpha)^2$

Upon commutating, the value of $c(y)$ is identified as,

$$c(y) = 4y^3 - 15y + 28kj$$
$$(y^3 - 2y + 7kj, \quad y^3 - 3y + 7kj, \quad 4y^3 - 15y + 28kj)$$

is a Hyperbolic polynomial Diophantine triple with the property $D(-5y^4 + 19y^2 - 35ykj)$.

To construct the hyperbolic sequence, the established pair $(y^3 - 3y + 7kj,\ 4y^3 - 15y + 28kj)$ is utilized, with the third polynomial incorporated as $d(y)$

$$b(y)d(y) - 5y^4 + 19y^2 - 35ykj = \beta^2$$
$$c(y)d(y) - 5y^4 + 19y^2 - 35ykj = \gamma^2$$

Applying the linear conversion $\beta = b(y) + \alpha$; $\gamma = c(y) + \alpha$ and performing the algebraic manipulations yield

$$d(y) = 9y^3 - 34y + 63kj$$

$(y^3 - 3y + 7kj, 4y^3 - 15y + 28kj, 9y^3 - 34y + 63kj)$ is the second Hyperbolic polynomial Diophantine triple with the property $D(-5y^4 + 19y^2 - 35ykj)$.

IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 1, March 2026
Impact Factor: 8.2

Continuing with the pair $(4y^3 - 15y + 28kj, 9y^3 - 34y + 63kj)$, the third polynomial $e(y)$ is determined as $25y^3 - 95y + 175kj$.

Thus, a Hyperbolic polynomial Diophantine triples sequence

$$\{(a(y), b(y), c(y)), (b(y), c(y), d(y)), (c(y), d(y), e(y)), \dots\dots.., \dots\dots\}$$

exists preserving the defined Diophantine property $D(-5y^4 + 19y^2 - 35ykj)$ across all triples.

Few sequences are tabulated below for $k = 1$ and suitable y

TABLE 1

k	y	$\{(a(y), b(y), c(y)), (b(y), c(y), d(y)), (c(y), d(y), e(y)), \dots\dots.., \dots\dots\}$
1	2	$\{(4 + 7j, 2 + 7j, 2 + 28j), (2 + 7j, 2 + 28j, 4 + 63j),$ $(2 + 28j, 4 + 63j, 10 + 175j), \dots., \dots..\}$
1	3	$\{(21 + 7j, 18 + 7j, 63 + 28j), (18 + 7j, 63 + 28j, 141 + 63j),$ $(63 + 28j, 141 + 63j, 390 + 175j), \dots.., \dots\}$
1	4	$\{(56 + 7j, 52 + 7j, 196 + 28j), (52 + 7j, 196 + 28j, 440 + 63j),$ $(196 + 28j, 440 + 63j, 1220 + 175j), \dots., \dots.\}$

Hyperbolic pair II extension:

Now consider the Hyperbolic pair $(8y^3 - 4y + 7kj, 8y^3 - 6y + 7kj)$. The algebraic procedures outlined in the preceding subsection are executed, yielding the third polynomial $t(y)$ as $32y^3 - 20y + 28kj$ completing the Hyperbolic triple $(8y^3 - 4y + 7kj, 8y^3 - 6y + 7kj, 32y^3 - 20y + 28kj)$.

Likewise, Hyperbolic triples $(s(y), t(y), u(y))$ and $(t(y), u(y), v(y))$ emerges from hyperbolic pairs $(s(y), t(y))$ and $(t(y), u(y))$ with,

$$u(y) = 72y^3 - 48y + 63kj$$
$$v(y) = 200y^3 - 130y + 175kj$$

This systematic derivation preserves Diophantine property $D(y^2)$ across all triple structure.

Resultantly this procedure generates an infinite sequence of Hyperbolic polynomial Diophantine triples $\{(r(y), s(y), t(y)), (s(y), t(y), u(y)), (t(y), u(y), v(y)), \dots., \dots.\}$

Few sequences are tabulated below for $k = 1$ and suitable y

TABLE II

k	y	$\{(r(y), s(y), t(y)), (s(y), t(y), u(y)), (t(y), u(y), v(y)), \dots., \dots.\}$
1	2	$\{(56 + 7j, 52 + 7j, 216 + 28j), (52 + 7j, 216 + 28j, 480 + 63j),$ $(52 + 28j, 480 + 63j, 1340 + 175j), \dots., \dots..\}$
1	3	$\{(204 + 7j, 198 + 7j, 804 + 28j), (198 + 7j, 804 + 28j, 1800 + 63j),$ $(804 + 28j, 1800 + 63j, 5010 + 175j), \dots., \dots..\}$
1	4	$\{(496 + 7j, 488 + 7j, 1968 + 28j), (488 + 7j, 1968 + 28j, 4416 + 63j),$ $(1968 + 28j, 4416 + 63j, 12280 + 175j), \dots., \dots..\}$

Remarkable Observations:

Each triple in the sequences is noted as regular and non-extendable.

IV. CONCLUSION

This work establishes a systematic framework for generating innumerable sequences of Hyperbolic Polynomial Diophantine Triples from Hyperbolic pairs constructed using Vieta polynomials derived hyperbolic numbers. Future investigations may explore on higher tuples of Hyperbolic triples.

REFERENCES

[1]. Adžaga, Nikola, Alan Filipin, and Zrinka Franušić. "On the Extensions of the Diophantine Triples in Gaussian Integers." Monatshefte für Mathematik, vol. 197, no. 4, 2022, pp. 535–563.

[2]. Apostol, Tom M. *Introduction to Analytic Number Theory*. Springer Science & Business Media, 2013.

[3]. Dickson, Leonard Eugene. *History of the Theory of Numbers*. Vol. 1, University of Pennsylvania Press, 1999.

[4]. Dujella, Andrej. *Diophantine m-tuples and Elliptic Curves*. Vol. 79. Cham: Springer, 2024.

[5]. Geetha, V., and B. Priyanka. "Research on Some Non-Extendability of Gaussian Diophantine Quadruples." Research Updates in Mathematics and Computer Science, vol. 2, 2024, pp. 89–107.

[6]. Horadam, A. F. "Vieta Polynomials." The Fibonacci Quarterly, vol. 40, no. 3, 2002, pp. 223–232.

[7]. Janaki, G., and R. Sarulatha. "On Sequences of Geophine Triples Involving Padovan and Bernstein Polynomial with Propitious Property." Indian Journal of Science and Technology, vol. 17, no. 16, 2024, pp. 1690-1694.

[8]. Janaki, G., and R. Sarulatha. "Special Diophantine Quadruple for Gnomonic Number with D (2)." *Explorations in Diophantine Equations*, 2023, pp. 94-100.

[9]. Janaki, G., and R. Sarulatha. "Enticing Sequences of Polynomial Diophantine Triple." Bodhi International Journal of Research in Humanities, Arts and Science, vol. 9, no. 3, 2025, pp. 284-286.

[10]. Janaki, G., and R. Sarulatha. "Innumerable Sequences of Vieta Polynomial Diophantine Triples." Pandian Journal of Mathematical Sciences, vol. 4, no. 1, 2025, pp. 1-5.

[11]. Lang, Serge. *Algebraic number theory*. Vol. 110. Springer Science & Business Media, 1994.

[12]. Nagell, Trygve. Introduction to Number Theory. Vol. 163, American Mathematical Society, 2021.

[13]. Park, Jinseo. "Complete Solution of Diophantine Pairs Induced by Some Fibonacci Formula." Algebra Colloquium, vol. 30, no. 1, World Scientific Publishing Company, 2022.

[14]. Saranya, S., and V. Pandichelvi. "Perpetuation of Eminent Forms of 2-Tuples into 3-Tuples Interlacing Some k-Polygonal Numbers with Appropriate Properties." International Journal of Mathematics Trends and Technology, vol. 68, no. 8, 2022, pp. 167–176.