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A Linear Non-Fragile Controller Design for a Class of 4D Nonlinear Systems with Multiple Uncertainties

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Abstract: *This paper explores the linear non-fragile controller design for a class of 4D nonlinear control systems with multiple uncertainties. Using differential-integral inequalities, a non-fragile linear controller will be proposed to achieve global exponential stability for a class of 4D nonlinear control systems with multiple uncertainties. At the same time, the exponential convergence rate of the system will also be precisely calculated. Finally, two numerical simulation examples will be provided, along with the electronic circuit implementation diagram of the proposed linear controller, to demonstrate the design flow of the main theorem and verify its correctness.*

Keywords*:* multiple uncertainties, non-fragile control, nonlinear systems, linear control

I. INTRODUCTION

The models of nonlinear systems are very different from those of linear systems. Because the models of nonlinear systems are more complex, they are more difficult to analyze and design than linear systems. As is well known, chaotic systems are nonlinear systems, and the chaotic signals they generate are both disordered and unpredictable, making the analysis and controller design of chaotic systems increasingly difficult. In the past, some well-known chaotic systems have been proposed, analyzed, or studied, such as hyperchaotic Lu system, hyperchaotic Liu system, hyperchaotic Rabinovich system, and the modified hyperchaotic Pan system ; see, for example, [1]-[14] and their references.

As we know, linear controllers have the dual advantages of being cheaper and easier to implement than nonlinear controllers. However, finding a suitable controller to achieve the desired result for nonlinear control systems remains a challenge for most researchers. Furthermore, to avoid distortion caused by parameter variations in the components within the controller, using a non-fragile controller is a powerful tool for mitigating this issue. For various studies on non-fragile controllers, please refer to references [14]-[22]. It is obvious that although the design of non-fragile controllers is more difficult than that of traditional controllers, their performance on the system is significantly better than that of traditional controllers.

For the reasons mentioned above, our team intends to design a non-fragile linear controller for a class of non-linear control systems that encompass a variety of well-known chaotic systems, so as to enable the closed-loop system to achieve global exponential stability. Furthermore, the exponential convergence rate of the entire closed-loop control system will be precisely calculated. Finally, several numerical simulation results will be provided, along with the circuit architecture of the proposed linear controller, to demonstrate the design process and correctness of the main theorem.

For simplicity, in this paper, $|a|$ represents the modulus of a real number a, $\Re^n$ represents the n-dimensional Euclidean space, and $\lambda_{\min}(Q)$ represents the minimum eigenvalue of the matrix Q with real eigenvalues.

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II. DESCRIPTION AND MAIN RESULTS OF 4D NONLINEAR CONTROL SYSTEMS

In this paper, we explore the following uncertain 4D nonlinear control systems with multiple uncertainties:

$$\dot{x}_1 = \Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_4 + f_1(x_1, x_2, x_3, x_4), \quad \text{(1a)}$$

$$\dot{x}_2 = \Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + f_2(x_1, x_2, x_3, x_4) + u_1, \quad \text{(1b)}$$

$$\dot{x}_3 = \Delta d_7 x_2 + \Delta b x_3 + \Delta d_8 x_4 + f_3(x_1, x_2, x_3, x_4), \quad \text{(1c)}$$

$$\dot{x}_4 = \Delta d_9 x_1 + \Delta d_{10} x_2 + \Delta d_{11} x_3 + \Delta d_{12} x_4 + f_4(x_1, x_2, x_3, x_4) + u_2, \quad \text{(1d)}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \in \Re^{4\times 1}$ is the state vector, $u(t) := [u_1(t) \quad u_2(t)]^T \in \Re^{2\times 1}$ is a non-fragile controller as

$$u_i = (K_i + \Delta K_i) x_{2\times i}, \quad \forall\, i \in \{1, 2\}, \quad \text{(1e)}$$

where K_i is the control gain and ΔK_i represents the uncertainties, for every $i \in \{1, 2\}$. Furthermore, $\Delta a, \Delta b$, $\Delta d_i, \forall\, i \in \{1,2,3,\cdots,12\}$ are uncertain parameters, and f_i is nonlinear function with $f_i(0,0,0,0) = 0, \forall\, i \in \{1,2,3,4\}$. In order to ensure that the solution of dynamic system (1) exists, we assume that $f_1, f_2, f_3,$ and f_4 are all continuous functions.

Regarding the above uncertain terms and nonlinear terms, we make the following appropriate assumptions.

(A1) There exist constants $\overline{a}, \underline{a}, \overline{b}, \underline{b}, \overline{K_i}$, and $\overline{d_i}$ that satisfy

$$-\overline{a} \le \Delta a \le -\underline{a} < 0, \quad -\overline{b} \le \Delta b \le -\underline{b} < 0, \quad |\Delta K_i| \le \overline{K_i}, \quad \forall\, i \in \{1,2\};$$

$$|\Delta d_j| \le \overline{d_j}, \quad \forall\, j \in \{1,2,3,\cdots,12\}.$$

(A2) There exist positive numbers $k_1, k_2, k_3,$ and k_4 that satisfy

$$\sum_{i=1}^{4} k_i^2 \cdot x_i \cdot f_i(x_1, x_2, x_3, x_4) = 0.$$

Remark 1. In the absence of input sources, i.e., $u_i = 0, \forall\, i \in \{1, 2\}$, some well-known chaotic systems are special cases of system (1) with (A1)-(A2); such as 4D chaotic laser system considered in [9], hyperchaotic Lu system [10], hyperchaotic Liu system [11], hyperchaotic Rabinovich system [12], and modified hyperchaotic Pan system mentioned in [13].

For the sake of simplicity, we define the following parameters in this paper:

$$\lambda_1 := \frac{k_1^2 \cdot \overline{d_1} + k_2^2 \cdot \overline{d_3}}{k_1 k_2}, \quad \lambda_2 := \frac{k_1^2 \cdot \overline{d_2} + k_4^2 \cdot \overline{d_9}}{k_1 k_4}, \quad \lambda_3 := \frac{k_2^2 \cdot \overline{d_5} + k_3^2 \cdot \overline{d_7}}{k_2 k_3}, \quad \text{(2a)}$$

$$\lambda_4 := \frac{k_3^2 \cdot \overline{d_8} + k_4^2 \cdot \overline{d_{11}}}{k_3 k_4}, \quad \lambda_5 := \frac{k_2^2 \cdot \overline{d_6} + k_4^2 \cdot \overline{d_{10}}}{k_2 k_4}. \quad \text{(2b)}$$

This paper intends to design a linear controller with the advantage of easy hardware implementation, so that system (1) can achieve the goal of global exponential stability, which is defined as follows.

Definition 1. The uncertain systems (1) with (A1) and (A2) is said to be globally exponentially stable if there exist a control u and positive numbers α and k, such that

$$|x_i(t)| \le k \cdot e^{-\alpha t}, \quad \forall\, t \ge 0, \; i \in \{1,2,3,4\}.$$

In such a case, the positive number α is called the exponential convergence rate.

This paper not only seeks a suitable non-fragile linear controller of (1e) that achieves global exponential stability, but also calculates the exponential convergence rate of the entire closed-loop control system. Below, we present the main result for the global exponential stabilization of uncertain systems (1) with (A1) and (A2).

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Theorem 1: The uncertain nonlinear systems (1) with (A1) and (A2), and supplemented with the control gains

$$K_1 = -\left(\overline{d_4} + h_1 + \overline{K_1}\right), \quad K_2 = -\left(\overline{d_{12}} + h_2 + \overline{K_2}\right), \qquad (3)$$

are globally exponentially stable, where h_1 and h_2 are any real numbers that satisfy

$$h_1 > \frac{\lambda_1^2 \underline{b} + \lambda_3^2 \underline{a}}{4\underline{a}\underline{b}}, \; h_2 > \frac{\left(4\lambda_2^2 \underline{b} + 4\lambda_4^2 \underline{a}\right)h_1 - \det(R)}{16\underline{a}\underline{b}h_1 - 4\lambda_1^2 \underline{b} - 4\lambda_3^2 \underline{a}}, \text{ with } (4)$$

$$R = \begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 0 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 0 \end{bmatrix}. \qquad (5)$$

In this situation, the guaranteed exponential convergence rate is given by

$$\alpha = \frac{1}{2} \times \lambda_{\min}\left(\begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 2h_1 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 2h_2 \end{bmatrix}\right). \qquad (6)$$

Proof. From (A1) and (4)-(5), it can be readily obtained that $\det([2\underline{a}]) > 0$, $\det\left(\begin{bmatrix} 2\underline{a} & 0 \\ 0 & 2\underline{b} \end{bmatrix}\right) > 0$, $\det\left(\begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 \\ 0 & 2\underline{b} & -\lambda_3 \\ -\lambda_1 & -\lambda_3 & 2h_1 \end{bmatrix}\right) > 0$, and $\det\left(\begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 2h_1 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 2h_2 \end{bmatrix}\right) > 0$. This implies that the matrix of

$$Q := \begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 2h_1 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 2h_2 \end{bmatrix}$$ is positive definite. Let

$$V(x(t)) := k_1^2 \cdot x_1^2(t) + k_2^2 \cdot x_2^2(t) + k_3^2 \cdot x_3^2(t) + k_4^2 \cdot x_4^2(t). \qquad (7)$$

The time derivative of $V(x(t))$ along the trajectories of uncertain system (1) with (A1)-(A3) and (2)-(5), is given by

$$\begin{aligned} \dot{V}(x(t)) &= 2k_1^2 \cdot x_1\dot{x}_1 + 2k_2^2 \cdot x_2\dot{x}_2 + 2k_3^2 \cdot x_3\dot{x}_3 + 2k_4^2 \cdot x_4\dot{x}_4 \\ &= 2k_1^2 x_1\left(\Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_4 + f_1\right) \\ &\quad + 2k_2^2 x_2\left(\Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + f_2 + u_1\right) \\ &\quad + 2k_3^2 x_3\left(\Delta d_7 x_2 + \Delta b x_3 + \Delta d_8 x_4 + f_3\right) \\ &\quad + 2k_4^2 x_4\left(\Delta d_9 x_1 + \Delta d_{10} x_2 + \Delta d_{11} x_3 + \Delta d_{12} x_4 + f_4 + u_2\right) \end{aligned}$$

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$$
\begin{aligned}
&\le -2k_1^2\underline{a}x_1^2 + 2k_1^2\overline{d_1}|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_4| \\
&\quad + 2k_2^2\overline{d_3}|x_1||x_2| + 2k_2^2\overline{d_4}x_2^2 + 2k_2^2\overline{d_5}|x_2||x_3| \\
&\quad + 2k_2^2\overline{d_6}|x_2||x_4| + 2k_3^2\overline{d_7}|x_2||x_3| - 2k_3^2\underline{b}x_3^2 \\
&\quad + 2k_3^2\overline{d_8}|x_3||x_4| + 2k_4^2\overline{d_9}|x_1||x_4| + 2k_4^2\overline{d_{10}}|x_2||x_4| \\
&\quad + 2k_4^2\overline{d_{11}}|x_3||x_4| + 2k_4^2\overline{d_{12}}x_4^2 \\
&\quad + 2\left(k_1^2x_1f_1 + k_2^2x_2f_2 + k_3^2x_3f_3 + k_4^2x_4f_4\right) \\
&\quad + 2k_2^2x_2u_1 + 2k_4^2x_4u_2 \\
&= -2k_1^2\underline{a}x_1^2 + 2\left(k_1^2\overline{d_1} + k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2} + k_4^2\overline{d_9}\right)|x_1||x_4| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5} + k_3^2\overline{d_7}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6} + k_4^2\overline{d_{10}}\right)|x_2||x_4| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2\left(k_3^2\overline{d_8} + k_4^2\overline{d_{11}}\right)|x_3||x_4| + 2k_4^2\overline{d_{12}}x_4^2 \\
&\quad + 2k_2^2x_2u_1 + 2k_4^2x_4u_2 \\
&= -2k_1^2\underline{a}x_1^2 + 2\left(k_1^2\overline{d_1} + k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2} + k_4^2\overline{d_9}\right)|x_1||x_4| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5} + k_3^2\overline{d_7}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6} + k_4^2\overline{d_{10}}\right)|x_2||x_4| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2\left(k_3^2\overline{d_8} + k_4^2\overline{d_{11}}\right)|x_3||x_4| + 2k_4^2\overline{d_{12}}x_4^2 \\
&\quad + 2k_2^2\left(K_1 + \Delta K_1\right)x_2^2 + 2k_4^2\left(K_2 + \Delta K_2\right)x_4^2 \\
&\le -2k_1^2\underline{a}x_1^2 + 2\left(k_1^2\overline{d_1} + k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2} + k_4^2\overline{d_9}\right)|x_1||x_4| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5} + k_3^2\overline{d_7}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6} + k_4^2\overline{d_{10}}\right)|x_2||x_4| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2\left(k_3^2\overline{d_8} + k_4^2\overline{d_{11}}\right)|x_3||x_4| + 2k_4^2\overline{d_{12}}x_4^2 \\
&\quad - 2k_2^2\left(\overline{d_4} + h_1 + \overline{K_1}\right)x_2^2 + 2k_2^2\overline{K_1}x_2^2 - 2k_4^2\left(\overline{d_{12}} + h_2 + \overline{K_2}\right)x_4^2 + 2k_4^2\overline{K_2}x_2^2 \\
&= -2k_1^2\underline{a}x_1^2 + 2\left(k_1^2\overline{d_1} + k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2} + k_4^2\overline{d_9}\right)|x_1||x_4| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5} + k_3^2\overline{d_7}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6} + k_4^2\overline{d_{10}}\right)|x_2||x_4| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2\left(k_3^2\overline{d_8} + k_4^2\overline{d_{11}}\right)|x_3||x_4| + 2k_4^2\overline{d_{12}}x_4^2 \\
&\quad - 2k_2^2\left(\overline{d_4} + h_1\right)x_2^2 - 2k_4^2\left(\overline{d_{12}} + h_2\right)x_4^2 \\
&= -2k_1^2\underline{a}x_1^2 + 2\left(k_1^2\overline{d_1} + k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2} + k_4^2\overline{d_9}\right)|x_1||x_4| \\
&\quad + 2\left(k_2^2\overline{d_5} + k_3^2\overline{d_7}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6} + k_4^2\overline{d_{10}}\right)|x_2||x_4| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2\left(k_3^2\overline{d_8} + k_4^2\overline{d_{11}}\right)|x_3||x_4| - 2k_2^2h_1x_2^2 - 2k_4^2h_2x_4^2 \\
&= -2k_1^2\underline{a}x_1^2 + 2\lambda_1k_1k_2|x_1||x_2| + 2\lambda_2k_1k_4|x_1||x_4| \\
&\quad + 2\lambda_3k_2k_3|x_2||x_3| + 2\lambda_5k_2k_4|x_2||x_4| - 2k_3^2\underline{b}x_3^2 + 2\lambda_4k_3k_4|x_3||x_4| \\
&\quad - 2k_2^2h_1x_2^2 - 2k_4^2h_2x_4^2 \\
&= -\left[k_1|x_1| \quad k_3|x_3| \quad k_2|x_2| \quad k_4|x_4|\right]Q\left[k_1|x_1| \quad k_3|x_3| \quad k_2|x_2| \quad k_4|x_4|\right]^T \\
&\le -\left[\lambda_{\min}(Q)\right]\cdot\left\|\left[k_1|x_1| \quad k_3|x_3| \quad k_2|x_2| \quad k_4|x_4|\right]\right\|^2 \\
&= -\left[\lambda_{\min}(Q)\right]\cdot V, \quad \forall\, t \ge 0.
\end{aligned}
$$

Thus, we have

$$e^{[\lambda_{\min}(Q)]t}\cdot \dot{V} + e^{[\lambda_{\min}(Q)]t}\cdot[\lambda_{\min}(Q)]\cdot V = \frac{d}{dt}\left[e^{[\lambda_{\min}(Q)]t}\cdot V\right]\le 0,\quad \forall\, t\ge 0.$$

It results that

$$\int_0^t \frac{d}{d\tau}\left[e^{[\lambda_{\min}(Q)]\tau}\cdot V(x(t))\right]d\tau = e^{[\lambda_{\min}(Q)]t}\cdot V(x(t)) - V(x(0)) \le \int_0^t 0\, d\tau = 0,\quad \forall\, t\ge 0. \qquad (8)$$

From (7) and (8), it can be easily deduced that

$$\begin{aligned} k_i^2 x_i^2(t) &\le k_1^2 x_1^2(t) + k_2^2 x_2^2(t) + k_3^2 x_3^2(t) + k_4^2 x_4^2(t) \\ &= V(x(t)) \le e^{-[\lambda_{\min}(Q)]t} V(x(0)),\ \ \forall\, t\ge 0,\ \ i\in\{1,2,3,4\}. \end{aligned}$$

Therefore, we conclude that

$$|x_i(t)| \le \frac{\sqrt{V(x(0))}}{k_i}\cdot e^{\frac{-1}{2}\times[\lambda_{\min}(Q)]t} \le \frac{\sqrt{V(x(0))}}{\min_i\{k_1,k_2,k_3,k_4\}}\cdot e^{\frac{-1}{2}\times[\lambda_{\min}(Q)]t},\quad \forall\, t\ge 0,\ \ i\in\{1,2,3,4\}.$$

This means that the entire closed-loop system is globally exponentially stable and the guaranteed exponential convergence rate is $\frac{1}{2}\times\lambda_{\min}(Q)$. This proof is completed. □

III. COMPUTER SIMULATION RESULTS AND CIRCUIT IMPLEMENTATION

This section provides two examples to illustrate the design process of linear controllers and their numerical simulation results.

Example 1. Consider the uncertain systems (1) with

$$f_2 = -x_1x_3,\quad f_3 = x_1x_2,\quad f_1 = f_4 = 0, \qquad (9a)$$

$$-11\le \Delta a\le -10,\quad -3.1\le \Delta b\le -2.1,\quad -1\le \Delta d_i\le 1,\quad \forall\, i\in\{2,4,5,6,7,8,9,11\}, \qquad (9b)$$

$$|\Delta d_1|\le 12,\quad |\Delta d_3|\le 28,\quad |\Delta d_{10}|\le 10,\quad |\Delta d_{12}|\le 0.2, \qquad (9c)$$

$$-2\le \overline{\Delta K_j}\le 2,\quad \forall\, j\in\{1,2\}. \qquad (9d)$$

By selecting the parameters $\underline{a}=10,\ \underline{b}=2.1,\ \overline{d_i}=1,\ \forall\, i\in\{2,4,5,6,7,8,9,11\},\ \ \overline{d_1}=12,\ \overline{d_3}=28,\ \overline{d_{10}}=10,\ \overline{d_{12}}=0.2,$ $\overline{K_j}=2,\ \forall\, j\in\{1,2\}$, and $k_1=k_2=k_3=k_4=1,$ (A1) and (A2) are evidently satisfied. From (2a) and (2b), one has $\lambda_1=40,\ \lambda_2=\lambda_3=\lambda_4=2,$ and $\lambda_5=11$. Thus, from (4) and (5), we select $h_1=41$ and $h_2=122.8$, in view of $\frac{\lambda_1^2\underline{b}+\lambda_3^2\underline{a}}{4\underline{a}\underline{b}}\approx 40.48,\ \ \det(R)=-13540$, and $\frac{(4\lambda_2^2\underline{b}+4\lambda_4^2\underline{a})h_1-\det(R)}{16\underline{a}\underline{b}\,h_1-4\lambda_1^2\underline{b}-4\lambda_3^2\underline{a}}\approx 122.03$. Therefore, by Theorem 1, we conclude that the uncertain systems (1) and (9), combined with control gains $K_1=-44$ and $K_2=-125$, are globally exponentially stable. Besides, according to (6), the guaranteed exponential convergence rate can be precisely calculated as

$$\alpha = \frac{1}{2}\times\lambda_{\min}\left(\begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 2h_1 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 2h_2 \end{bmatrix}\right) = 0.0006.$$

The time-domain signal trajectory diagrams of the uncontrolled system and the feedback-controlled system are displayed in Figure 1 and Figure 2, respectively. In addition, the input signal trajectory diagram and the electronic circuit that implements this control law are shown in Figure 3 and Figure 4, respectively.

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Example 2. Consider the uncertain system (1) with

$$f_1 = x_2x_3,\quad f_2 = -x_1x_3,\quad f_3 = x_1x_2,\quad f_4 = 0, \qquad (10a)$$

$$-5 \le \Delta a \le -4,\quad -2 \le \Delta b \le -1,\quad -1 \le \Delta d_i \le 1,\quad \forall\, i \in \{2,4,5,6,7,8,9,11,12\}, \qquad (10b)$$

$$|\Delta d_1| \le 6.75,\quad |\Delta d_3| \le 6.75,\quad |\Delta d_{10}| \le 2, \qquad (10c)$$

$$-2 \le \overline{\Delta K_j} \le 2,\quad \forall\, j \in \{1,2\}. \qquad (10d)$$

By selecting the parameters $\underline{a} = 4,\ \underline{b} = 1,\ \overline{d_i} = 1,\ \forall\, i \in \{2,4,5,6,7,8,9,11,12\},\ \overline{d_1} = \overline{d_3} = 6.75,\ \overline{d_{10}} = 2,\ \overline{K_j} = 2,\ \forall\, j \in \{1,2\}$, and $k_1 = k_3 = k_4 = 1$, and $k_2 = \sqrt{2}$, (A1) and (A2) are evidently satisfied. From (2a) and (2b), one has $\lambda_1 = 14.318,\ \lambda_2 = \lambda_4 = 2,\ \lambda_3 = 2.121$ and $\lambda_5 = 2.828$. Thus, from (4) and (5), we select $h_1 = 15$ and $h_2 = 20$, in view of $\frac{\lambda_1^2 \underline{b} + \lambda_3^2 \underline{a}}{4\underline{a}\underline{b}} \approx 13.94$, $\det(R) = -48.91$, and $\frac{\left(4\lambda_2^2 \underline{b} + 4\lambda_4^2 \underline{a}\right)h_1 - \det(R)}{16\underline{a}\underline{b}h_1 - 4\lambda_1^2 \underline{b} - 4\lambda_3^2 \underline{a}} \approx 18.38$. Therefore, by Theorem 1, we conclude that the uncertain systems (1) and (10), combined with control gains $K_1 = -18$ and $K_2 = -23$, are globally exponentially stable. Besides, according to (6), the guaranteed exponential convergence rate can be precisely calculated as

$$\alpha = \frac{1}{2} \times \lambda_{\min}\left(\begin{bmatrix} 2\underline{a} & 0 & -\lambda_1 & -\lambda_2 \\ 0 & 2\underline{b} & -\lambda_3 & -\lambda_4 \\ -\lambda_1 & -\lambda_3 & 2h_1 & -\lambda_5 \\ -\lambda_2 & -\lambda_4 & -\lambda_5 & 2h_2 \end{bmatrix}\right) = 0.0149.$$

The time-domain signal trajectory diagrams of the uncontrolled system and the feedback-controlled system are displayed in Figure 5 and Figure 6, respectively. In addition, the input signal trajectory diagram and the electronic circuit that implements this control law are shown in Figure 7 and Figure 8, respectively

IV. CONCLUSION

In this paper, the linear non-fragile controller design for a class of 4D nonlinear control systems with multiple uncertainties has been explored. Using differential-integral inequalities, a non-fragile linear controller has been proposed to achieve global exponential stability for a class of 4D nonlinear control systems with multiple uncertainties. At the same time, the exponential convergence rate of the system has also been precisely calculated. Finally, two numerical simulation examples have been provided, along with the electronic circuit implementation diagram of the proposed linear controller, to demonstrate the design flow of the main theorem and verify its correctness.

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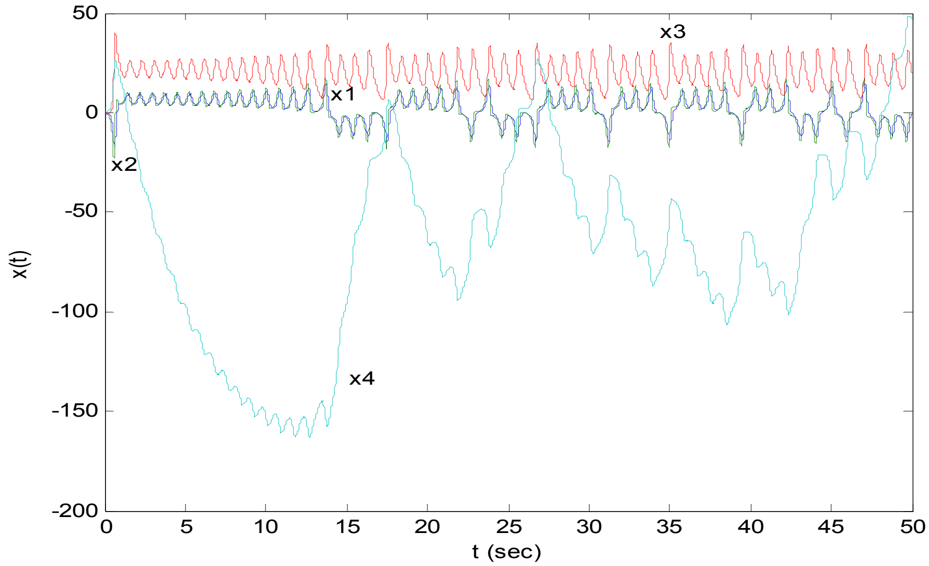

Figure 1: Typical state trajectories of the uncontrolled system of Example 1.

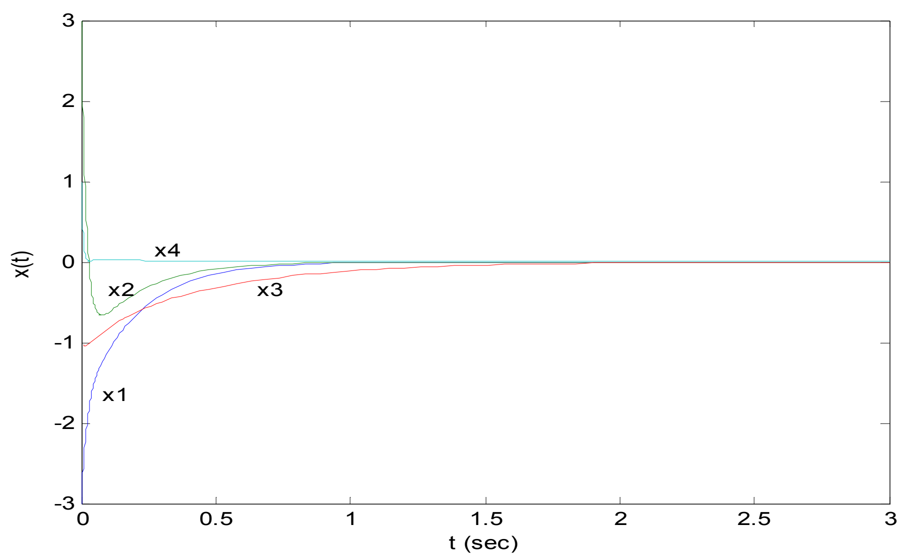

Figure 2: Typical state trajectories of the feedback-controlled system of Example 1.

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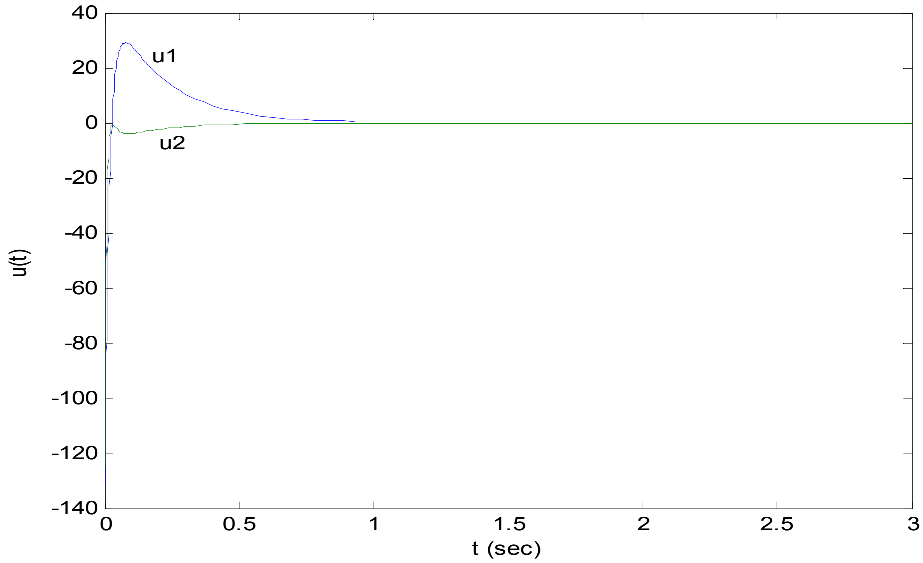

Figure 3: Control signal of Example 1.

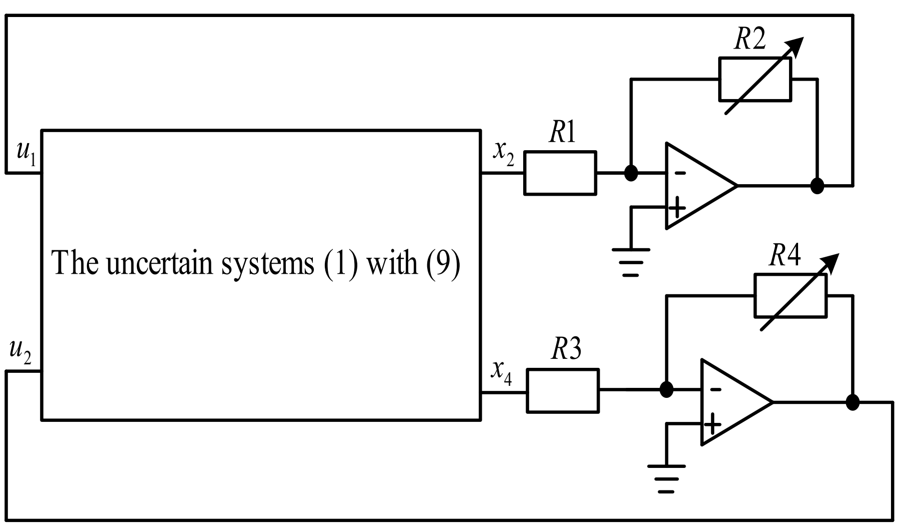

Figure 4: The diagram of implementation of Example 1, where $R1 = 1k\Omega$, $R2 = 44k\Omega$, $R3 = 1k\Omega$, , and $R4 = 125k\Omega$.

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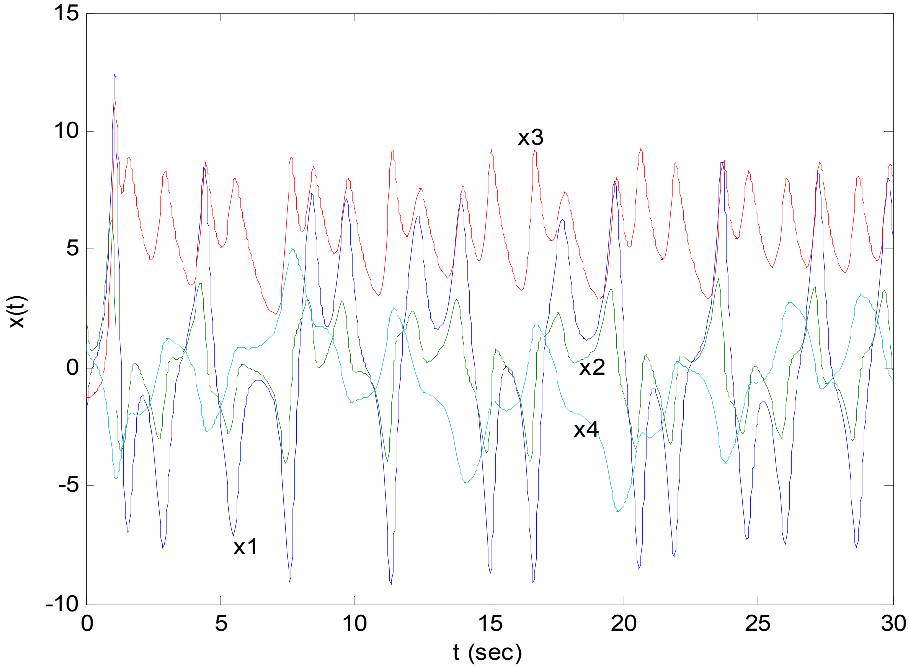

Figure 5: Typical state trajectories of the uncontrolled system of Example 2.

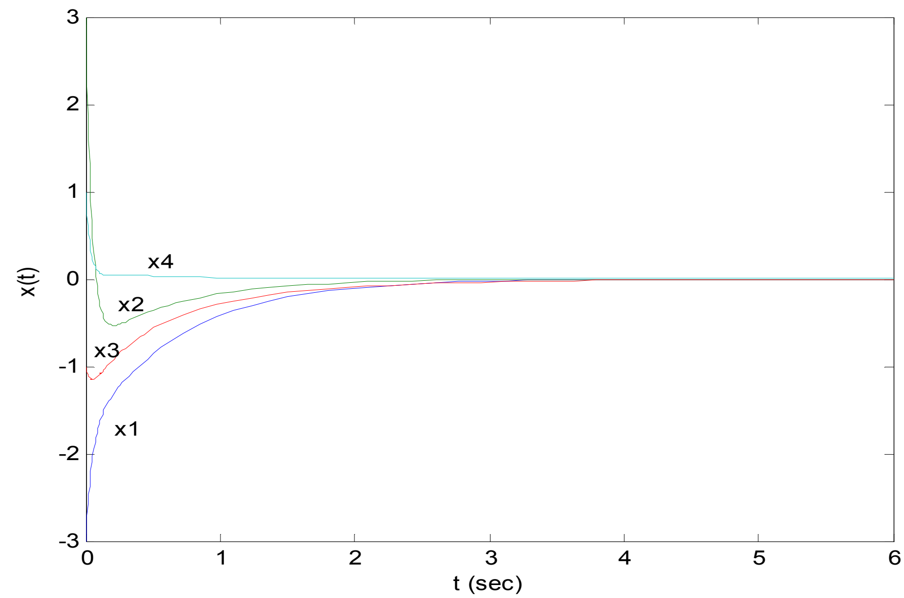

Figure 6: Typical state trajectories of the feedback-controlled system of Example 2.

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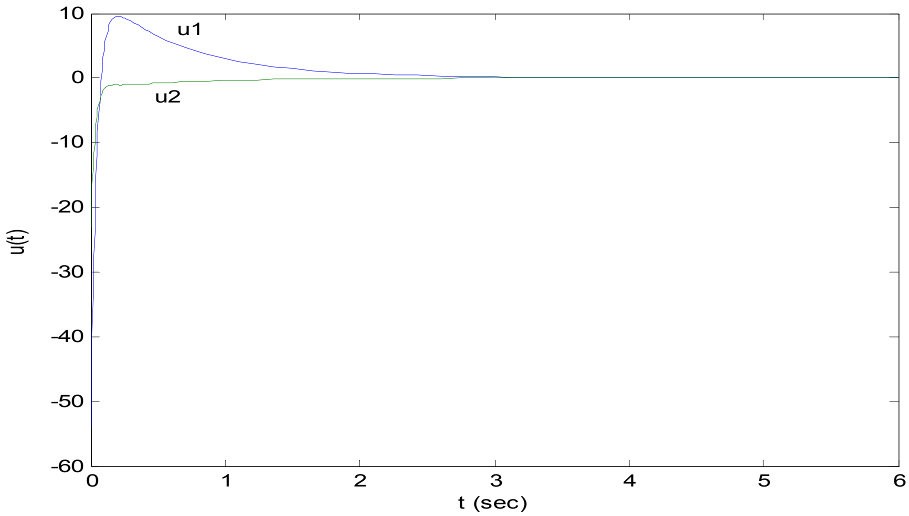

Figure 7: Control signal of Example 2.

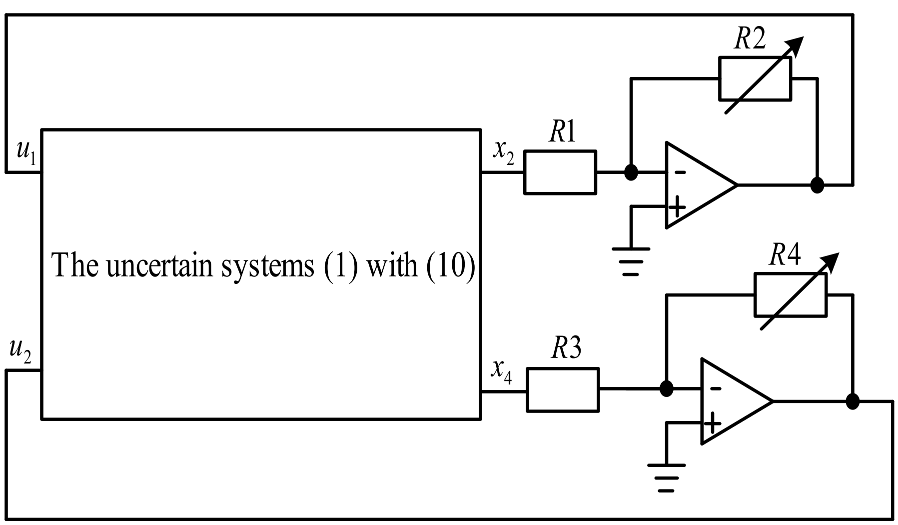

Figure 8: The diagram of implementation of Example 2, where $R1 = 1k\Omega$, $R2 = 18k\Omega$, $R3 = 1k\Omega$, , and $R4 = 23k\Omega$.