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$B[r]$ Stability Analysis of Uncertain Interval Discrete-Time Systems

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Abstract: *This paper first defines the condition for achieving $B[r]$ stability for a class of uncertain interval discrete-time systems. Next, we apply algebraic methods to derive a criterion guaranteeing $B[r]$ stability for a class of uncertain interval discrete-time systems. Finally, we present several numerical simulation results to illustrate the application and correctness of the main theorem*

Keywords: interval systems, $B[r]$ stability, uncertain systems, discrete-time systems

I. INTRODUCTION

Both the stability analysis of interval systems and the design of various controllers for these systems have recently been the subject of extensive research and exploration by numerous scholars; see, for example, [1]-[9] and the references therein.Because uncertain interval systems contain uncertain parameters or factors, stability analysis and controller design for these systems are significantly more challenging.

This paper proposes a definition of $B[r]$ stability for uncertain interval discrete-time systems. Then, for a class of uncertain interval discrete-time systems, a sufficient condition for achieving $B[r]$ stability is rigorously derived. Besides, we propose several numerical simulation results to show the application and correctness of the main theorem.

II. DESCRIPTION AND MAIN RESULTS OF INTERVAL DISCRETE-TIME SYSTEMS

Terminology and notation

$\Re^n$ the n-dimensional real space;

$[\underline{a}, \bar{b}]$ the set of $\{x \mid \underline{a} \le x \le \bar{b}\}$;

$R^{m\times n}$ the set of all real m by n matrices;

A^T the transport of the matrix A.

This paper analyzes the relative stability of the following uncertain interval discrete-time systems. The mathematical model is as follows:

$$x(k+1) = \Delta A \cdot x(t) = [\Delta a_{i.j}]_{n\times n} x(k)$$

$$= \begin{bmatrix} \Delta a_{1,1} & \Delta a_{1,2} & \cdots & \cdots & \Delta a_{1,n-1} & \Delta a_{1,n} \\ \Delta a_{2.,1} & \Delta a_{2.,2} & \cdots & \cdots & \Delta a_{2.,n-1} & \Delta a_{2.,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta a_{n-1,1} & \Delta a_{n-1,2} & \cdots & \cdots & \Delta a_{n-1,n-1} & \Delta a_{n-1,n} \\ \Delta a_{n,1} & \Delta a_{n,2} & \cdots & \cdots & \Delta a_{n,n-1} & \Delta a_{n,n} \end{bmatrix}_{n\times n} x(k), \quad \forall\, k \in \{0,1,2,3,\cdots\} \tag{1}$$

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where $x(k) := [x_1(k) \quad x_2(k) \quad x_3(k) \quad \cdots \quad x_n(k)]^T \in \Re^{n\times 1}$ is the state vector, $\Delta A := [\Delta a_{i.j}]_{n\times n}$ is uncertain matrix, and $\Delta a_{i.j}$ are unknown parameters of the systems (1).

Below we make the following assumptions about the unknown parameters of the above uncertain interval systems of (1).

(A1) There exist finite real numbers $\underline{a_{i.j}}$ and $\overline{a_{i.j}}$ such that $\Delta a_{i.j} \in [\underline{a_{i,j}}, \overline{a_{i,j}}]$, $\forall\, i, j \in \{1,2,3,\cdots,n\}$.

For convenience, we define the following parameters:

$$m_{i.j} := \max \left\{ |r| \,\middle|\, r \in [\underline{a_{i,j}}, \overline{a_{i.j}}] \right\}, \quad \forall\, i, j \in \{1,2,3,\cdots,n\}; \tag{2a}$$

$$r_i := \sum_{j=1}^{n} m_{i,j}, \quad \forall\, i \in \{1,2,3,\cdots,n\}; \tag{2b}$$

$$\alpha := \max \{r_1, r_2, r_3, \cdots, r_n\}; \tag{2c}$$

$$c_j := \sum_{i=1}^{n} m_{i,j}, \forall\, j \in \{1,2,3,\cdots,n\}; \tag{2d}$$

$$\beta := \max \{c_1, c_2, c_3, \cdots, c_n\}. \tag{2e}$$

The following two definitions are made for the uncertain interval discrete-time systems of (1).

Definition 1. $B[r] := \left\{ p + q \times \sqrt{-1} \,\middle|\, p^2 + q^2 \le r^2, p, q \in \Re \right\}$, with $r \ge 0$.

Definition 2. If the eigenvalues of uncertain interval discrete-time systems of (1) all lie in the set $B[r]$, then the uncertain interval discrete-time systems (1) are called $B[r]$ stable.

Through above definitions and combined with the relevant theorems of linear systems, we can easily obtain the following reasoning.

Remark 1. If the uncertain interval discrete-time systems of (1) are $B[r]$ stable system, with $0 \le r < 1$, then the systems (1) must be globally exponentially stable systems.

So far, the main theorem of this paper is introduced as follows.

Theorem 1. Consider the uncertain interval discrete-time systems (1) with (A1). If

$$\min \{\alpha, \beta\} < 1, \tag{3}$$

then such interval systems must be $B[r]$ stable systems, with

$$r = \min \{\alpha, \beta\}. \tag{4}$$

Proof. Assume that the eigenvalue and eigenvector of ΔA are λ and $z := [z_1 \quad z_2 \quad z_3 \quad \cdots \quad z_n]^T \in C^{n\times 1}$, respectively. Furthermore, we let $|z_k| = \max \{|z_1|, |z_2|, |z_3|, \cdots, |z_n|\}$, with $k \in \{1,2,3,\cdots,n\}$.

(i) According to $\lambda \cdot z = \Delta A \cdot z$, we can obtain

$$\lambda \cdot z_k = \Delta a_{k,1} z_1 + \Delta a_{k,2} z_2 + \Delta a_{k,3} z_3 + \cdots + \Delta a_{k,n} z_n.$$

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$$
\begin{aligned}
\Rightarrow\quad |\lambda\cdot z_k| &= |\lambda|\cdot|z_k| \\
&\le \sum_{i=1}^{n}|\Delta a_{k,i}|\cdot|z_i| \\
&\le \left(\sum_{i=1}^{n}|\Delta a_{k,i}|\right)\cdot|z_k| \\
&\le \left(\sum_{i=1}^{n}m_{k,i}\right)\cdot|z_k|.
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad |\lambda| &\le \sum_{i=1}^{n}m_{k,i} \\
&= r_k \\
&\le \alpha.
\end{aligned}
$$

This means

$$\lambda\in B[\alpha]. \tag{5}$$

(ii) $\lambda\cdot z=\Delta A\cdot z \quad\Rightarrow\quad \lambda\cdot z^T=z^T\cdot\Delta A^T$. Thus, one has

$$\lambda\cdot z_k=\Delta a_{1,k}z_1+\Delta a_{2,k}z_2+\Delta a_{3,k}z_3+\cdots+\Delta a_{n,k}z_n.$$

$$
\begin{aligned}
\Rightarrow\quad |\lambda\cdot z_k| &= |\lambda|\cdot|z_k| \\
&\le \sum_{i=1}^{n}|\Delta a_{i,k}|\cdot|z_i| \\
&\le \left(\sum_{i=1}^{n}|\Delta a_{i,k}|\right)\cdot|z_k| \\
&\le \left(\sum_{i=1}^{n}m_{i,k}\right)\cdot|z_k|.
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad |\lambda| &\le \sum_{i=1}^{n}m_{i,k} \\
&= c_k \\
&\le \beta.
\end{aligned}
$$

This shows

$$\lambda\in B[\beta]. \tag{6}$$

Due to (3)-(6), we can deduce that $\lambda\in B[\alpha]\cap B[\beta]=B[r]$, with $r=\min\{\alpha,\beta\}<1$. Therefore, we know that the uncertain interval discrete-time systems (1) must be $B[r]$ stable systems, and this is proved.□

III. COMPUTER SIMULATION RESULTS

Several computer simulation results are provided below to illustrate the applicability of Theorem 1.

Example: Consider the uncertain interval discrete-time systems

$$x(k+1)=\begin{bmatrix}\Delta a_{1,1} & \Delta a_{1,2} & \Delta a_{1,3} & \Delta a_{1,4}\\ \Delta a_{2,1} & \Delta a_{2,2} & \Delta a_{2,3} & \Delta a_{2,4}\\ \Delta a_{3,1} & \Delta a_{3,2} & \Delta a_{3,3} & \Delta a_{3,4}\\ \Delta a_{4,1} & \Delta a_{4,2} & \Delta a_{4,3} & \Delta a_{4,4}\end{bmatrix}x(k), \tag{7a}$$

where $x(k):=[x_1(k)\quad x_2(k)\quad x_3(k)\quad x_4(k)]^T\in\Re^{4\times1}$ is the state vector, $\Delta A:=[\Delta a_{i,j}]_{4\times4}$ is an uncertain matrix, and $\Delta a_{i,j}$ are unknown parameters of the systems with

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$\Delta a_{1,1} \in [-0.1, 0.1]$, $\Delta a_{1,2} \in [-0.2, 0.2]$, $\Delta a_{1,3} \in [-0.1, 0.1]$, $\Delta a_{1,4} \in [-0.4, 0.3]$, (7b)

$\Delta a_{2,1} \in [-0.2, 0.1]$, $\Delta a_{2,2} \in [-0.2, 0]$, $\Delta a_{2,3} \in [-0.2, 0.3]$, $\Delta a_{2,4} \in [-0.1, 0.1]$, (7c)

$\Delta a_{3,1} \in [-0.1, 0.1]$, $\Delta a_{3,2} \in [-0.2, 0.2]$, $\Delta a_{3,3} \in [-0.2, -0.1]$, $\Delta a_{3,4} \in [-0.3, 0.3]$, (7d)

$\Delta a_{4,1} \in [-0.1, 0]$, $\Delta a_{4,2} \in [0, 0.1]$, $\Delta a_{4,3} \in [-0.2, 0.2]$, $\Delta a_{4,4} \in [-0.1, 0.2]$. (7e)

Comparing the interval of the above unknown parameters with (2), we have

$m_{1,1} = 0.1, m_{1,2} = 0.2, m_{1,3} = 0.1, m_{1,4} = 0.4, m_{2,1} = m_{2,2} = 0.2, m_{2,3} = 0.3, m_{2,4} = 0.1,$

$m_{3,1} = 0.1, m_{3,2} = m_{3,3} = 0.2, m_{3,4} = 0.3, m_{4,1} = m_{4,2} = 0.1, m_{4,3} = m_{4,4} = 0.2;$

$r_1 = 0.8, r_2 = 0.8, r_3 = 0.8, r_4 = 0.6$;

$\alpha = 0.8$;

$c_1 = 0.5, c_2 = 0.7, c_3 = 0.8, c_4 = 1$;

$\beta = 1$.

Since $r = \min\{\alpha, \beta\} = 0.8 < 1$, we can see from Theorem 1 that the uncertain interval discrete-time systems of (7) must be $B[0.8]$ stable system. Combining this result with Remark 1, it can be obtained that such uncertain systems of (7) are also globally exponentially stable. Meanwhile, the state variable trajectory diagrams of the uncertain interval discrete-time systems (7) are demonstrated in Figure 1 to Figure 4. It is obvious from these figures that the trajectories of the state variables of uncertain systems (7) will converge to zero

IV. CONCLUSION

In this paper, the notion of stability has been first presented. Based on algebraic methods, a sufficient condition has been proposed to guarantee stability for a class of uncertain interval discrete-time systems. Finally, several numerical simulation results have been offered to demonstrate the application and correctness of the main theorem.

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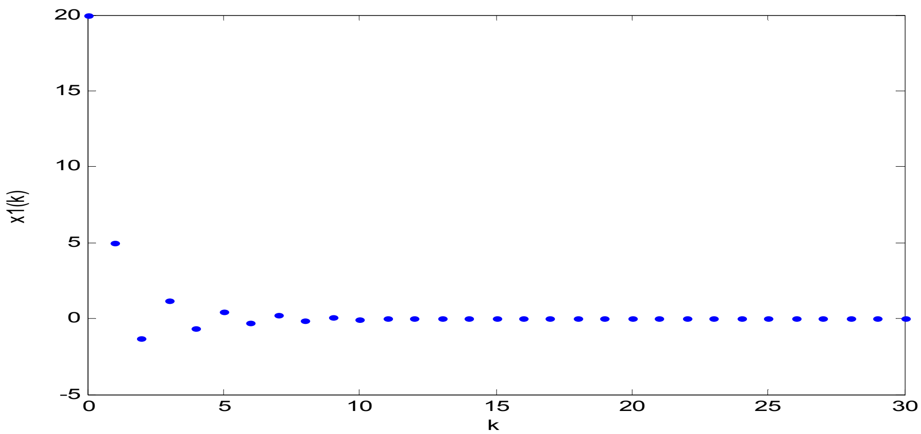

Figure 1: The trajectory of the state variable $x_1(k)$ of the uncertain interval discrete-time systemsof (7), with $x(0) = [20 \quad 10 \quad -10 \quad -20]^T$.

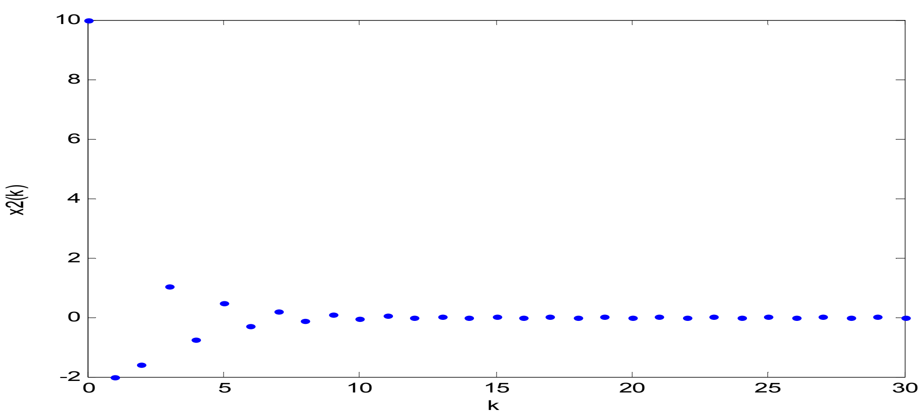

Figure 2: The trajectory of the state variable $x_2(k)$ of the uncertain interval discrete-time systemsof (7), with $x(0) = [20 \quad 10 \quad -10 \quad -20]^T$.

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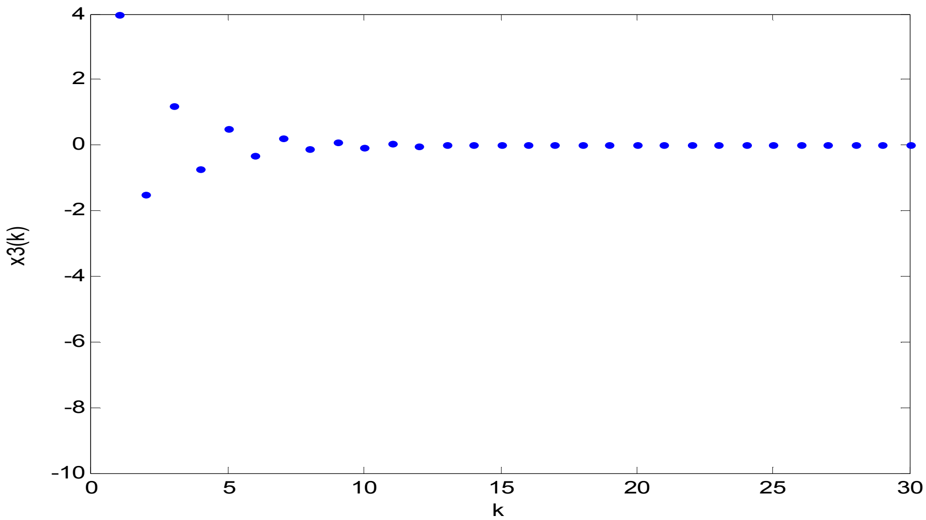

Figure 3: The trajectory of the state variable $x_3(k)$ of the uncertain interval discrete-time systemsof (7), with $x(0) = [20 \quad 10 \quad -10 \quad -20]^T$.

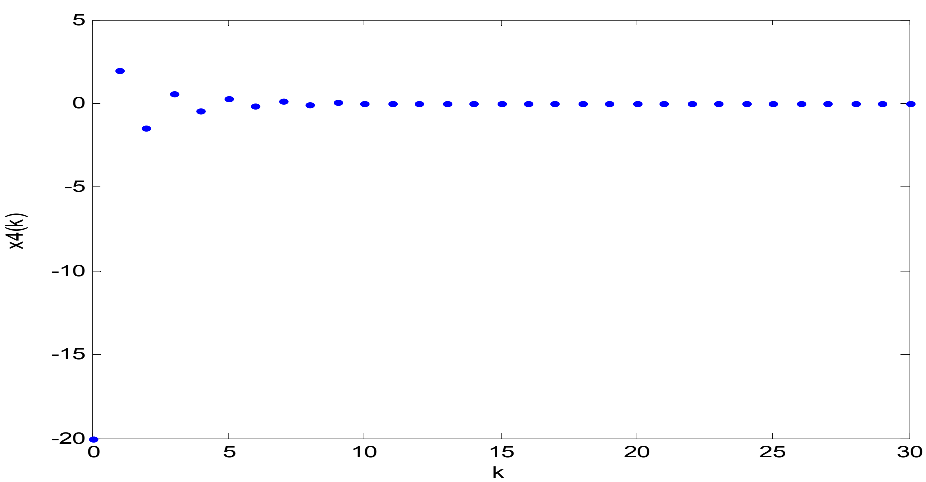

Figure 4: The trajectory of the state variable $x_4(k)$ of the uncertain interval discrete-time systemsof (7), with $x(0) = [20 \quad 10 \quad -10 \quad -20]^T$.

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