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Instability Analysis of Uncertain Interval Continuous-time Systems

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Abstract: *This paper first proposes the concepts of $S(a,b)$ stability and $S(a,b)$ instability. Next, for a class of uncertain interval continuous-time systems, a simple criterion for guaranteeing $S(a,b)$ instability is offered. Furthermore, both parameters a and b can be quickly calculated and obtained from the upper and lower bounds of the unknown parameters of such uncertain interval continuous-time systems.Finally, several numerical simulation results are presented to demonstrate the validity and correctness of the main theorem*

Keywords: uncertain systems, interval systems, instability, continuous-time systems

I. INTRODUCTION

In recent years, various types of problems related to interval systems have been extensively and deeply explored and studied by scholars; see, for example, [1]-[8] and the references therein.Since interval systems usually have uncertain parameters, both their stability analysis and controller design become quite difficult.

When it comes to the analysis of uncertain interval systems, the sufficient conditions for ensuring their stability are as important as the sufficient conditions for ensuring their instability.However, as far as the authors know, no researchers have proposed criteria for determining whether an uncertain interval system is unstable.

For the reasons stated above, in this paper, we first propose the definitions of $S(a,b)$ stability and $S(a,b)$ instability. Then, applying algebraic theory, we derive a simple criterion for a class of uncertain interval continuous-time systems that guarantees the $S(a,b)$ instability of such systems. Furthermore, we present several numerical simulation results to illustrate the application and correctness of the main theorem.

II. DESCRIPTION AND MAIN RESULTS OF INTERVAL DYNAMIC SYSTEMS

Terminology and notation

$\Re^n$	the n-dimensional real space;
$[\underline{a}, \overline{b}]$	the set of $\{x \mid \underline{a} \le x \le \overline{b}\}$;
$C^{n\times n}$	the set of all complex m by n matrices;
A^*	the conjugate transport of the matrix A ;
$tr(A)$	the trace of the matrix A ;
$\mathrm{Re}(\lambda)$	the real part of a complex number λ .

This paper conducts instability analysis on the following uncertain interval continuous-time system. Its mathematical model is as follows:

$$\begin{aligned}\frac{dx(t)}{dt} &= \Delta A \cdot x(t) \\ &= [\Delta a_{i.j}]_{n\times n} \cdot x(t) \\ &= \begin{bmatrix} \Delta a_{1,1} & \Delta a_{1,2} & \cdots & \cdots & \Delta a_{1,n-1} & \Delta a_{1,n} \\ \Delta a_{2,1} & \Delta a_{2,2} & \cdots & \cdots & \Delta a_{2,n-1} & \Delta a_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta a_{n-1,1} & \Delta a_{n-1,2} & \cdots & \cdots & \Delta a_{n-1,n-1} & \Delta a_{n-1,n} \\ \Delta a_{n,1} & \Delta a_{n,2} & \cdots & \cdots & \Delta a_{n,n-1} & \Delta a_{n,n} \end{bmatrix}_{n\times n} x(t),\end{aligned} \tag{1}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad \cdots \quad x_n(t)]^T \in \Re^{n\times 1}$ is the state vector, $\Delta A := [\Delta a_{i.j}]_{n\times n}$ is unknown matrix, and $\Delta a_{i.j}$ are uncertain parameters of the systems (1).

The following assumption is made for the uncertain parameters of the above systems:

(A1) There exist finite real numbers $\underline{a_{i.j}}$ and $\overline{a_{i.j}}$ such that $\Delta a_{i.j} \in \left[\underline{a_{i,j}}, \overline{a_{i,j}}\right]$, $\forall\, i,j \in \{1,2,3,\cdots,n\}$.

The following two definitions are given for the uncertain interval continuous-time systems of (1).

Definition 1. $S(a,b) := \left\{p + q\times\sqrt{-1} \,\middle|\, a < p < b, q \in \Re\right\}$, where a and b are both real numbers with $a < b$.

Definition 2. Given $a < b$, if all the eigenvalues of the uncertain interval continuous-time system (1) are in the interval $S(a,b)$, then the uncertain interval continuous-time systems (1) are called $S(a,b)$ stable; otherwise, the uncertain interval continuous-time systems (1) are called $S(a,b)$ unstable.

From the above definitions and combined with linear system theory, we can easily obtain the following inference.

Remark 1. If the uncertain interval continuous-time systems of (1) are $S(-\infty,b)$ unstable system, with $b \geq 0$, then the systems (1) are unstable systems.

The main theorem of this paper is introduced as follows.

Theorem 1. Consider the uncertain interval continuous-time systems (1) with (A1), then

(i) the uncertain interval continuous-time systems (1) are $S(-\infty,b)$ unstable systems;

(ii) the uncertain interval continuous-time systems (1) are $S(a,\infty)$ unstable systems;

(iii) the uncertain interval continuous-time systems (1) are $S(a,b)$ unstable systems,

where

$$a = \frac{\sum_{i=1}^{n} \underline{a_{i,i}}}{n} \text{ and } b = \frac{\sum_{i=1}^{n} \overline{a_{i,i}}}{n}. \tag{2}$$

Proof. Let the eigenvalues of matrix ΔA be $\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n$, and make the following two definitions:

$\overline{R}(\Delta A) := Max\{\mathrm{Re}(\lambda_1), \mathrm{Re}(\lambda_2), \mathrm{Re}(\lambda_3), \cdots, \mathrm{Re}(\lambda_n)\}$;

$\underline{R}(\Delta A) := \min\{\mathrm{Re}(\lambda_1), \mathrm{Re}(\lambda_2), \mathrm{Re}(\lambda_3), \cdots, \mathrm{Re}(\lambda_n)\}$.

Using linear algebra theory, there exists a unitary matrix $P \in C^{n\times n}$ and a Jordan matrix or diagonal matrix $J \in C^{n\times n}$, such that $\Delta A = P^* JP$, where the main diagonal elements of matrix J are $\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n$ in order. Thus, it can be readily obtained that

$$\begin{aligned} tr(\Delta A) &= tr(P^{*}JP) \\ &= tr(PP^{*}J) \\ &= tr(J) \\ &= \lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n . \end{aligned}$$

It follows that

$$\begin{aligned} tr(\Delta A) &= \mathrm{Re}[tr(\Delta A)] \\ &= \mathrm{Re}(\lambda_1) + \mathrm{Re}(\lambda_2) + \mathrm{Re}(\lambda_3) + \cdots + \mathrm{Re}(\lambda_n). \end{aligned}$$

It can be further deduced that

$$n \times \underline{R}(\Delta A) \le tr(\Delta A) \le n \times \overline{R}(\Delta A)$$

$$\Rightarrow \quad \underline{R}(\Delta A) \le \frac{tr(\Delta A)}{n} \le \overline{R}(\Delta A).$$

This means

$$\begin{cases} b = \underset{\Delta A}{Max} \dfrac{tr(\Delta A)}{n} \le \underset{\Delta A}{Max}\, \overline{R}(\Delta A); \\ \underset{\Delta A}{\min}\, \underline{R}(\Delta A) \le \underset{\Delta A}{\min} \dfrac{tr(\Delta A)}{n} = a. \end{cases} \tag{3}$$

Therefore, the uncertain interval continuous-time systems of (1) with (A1) are not only $S(-\infty, b)$ unstable systems, but also are $S(a, \infty)$ unstable systems. The result of (3) also shows that the uncertain interval continuous-time systems (1) with (A1) are $S(a,b)$ unstable system. The proof is thus completed.□

III. NUMERICAL SIMULATION RESULTS

Below we provide a computer simulation example to illustrate the main theorem.

Example: Consider the uncertain interval continuous-time systems

$$\frac{dx(t)}{dt} = \begin{bmatrix} \Delta a_{1,1} & \Delta a_{1,2} & \Delta a_{1,3} & \Delta a_{1,4} & \Delta a_{1,5} \\ \Delta a_{2,1} & \Delta a_{2,2} & \Delta a_{2,3} & \Delta a_{2,4} & \Delta a_{2,5} \\ \Delta a_{3,1} & \Delta a_{3,2} & \Delta a_{3,3} & \Delta a_{3,4} & \Delta a_{3,5} \\ \Delta a_{4,1} & \Delta a_{4,2} & \Delta a_{4,3} & \Delta a_{4,4} & \Delta a_{4,4} \\ \Delta a_{5,1} & \Delta a_{5,2} & \Delta a_{5,3} & \Delta a_{5,4} & \Delta a_{5,5} \end{bmatrix} x(t), \tag{4a}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t)]^T \in \Re^{5\times 1}$ is the state vector, $\Delta A := [\Delta a_{i.j}]_{5\times 5}$ is unknown matrix, and $\Delta a_{i.j}$ are uncertain parameters of the systems with

$$\Delta a_{1,1} \in [-1,0] \text{ , } \Delta a_{1,2} \in [0,1] \text{ , } \Delta a_{1,3} \in [1,2] \text{ , } \Delta a_{1,4} \in [3,4] \text{ , } \Delta a_{1,5} \in [-2,-1] \text{ ,} \tag{4b}$$

$$\Delta a_{2,1} \in [1,2] \text{ , } \Delta a_{2,2} \in [-2,-1] \text{ , } \Delta a_{2,3} \in [3,4] \text{ , } \Delta a_{2,4} \in [-2,-1] \text{ , } \Delta a_{2,5} \in [0,1] \text{ ,} \tag{4c}$$

$$\Delta a_{3,1} \in [-2,-1] \text{ , } \Delta a_{3,2} \in [3,4] \text{ , } \Delta a_{3,3} \in [1,2] \text{ , } \Delta a_{3,4} \in [-1,0] \text{ , } \Delta a_{3,5} \in [2,3] \tag{4d}$$

$$\Delta a_{4,1} \in [-1,0] \text{ , } \Delta a_{4,2} \in [3,4] \text{ , } \Delta a_{4,3} \in [-2,-1] \text{ , } \Delta a_{4,4} \in [2,3] \text{ , } \Delta a_{4,5} \in [1,2] \text{ ,} \tag{4e}$$

$$\Delta a_{5,1} \in [-2,-1] \text{ , } \Delta a_{5,2} \in [1,2] \text{ , } \Delta a_{5,3} \in [3,4] \text{ , } \Delta a_{5,4} \in [-1,0] \text{ , } \Delta a_{5,5} \in [2,3] \text{ .} \tag{4f}$$

From the ranges of the above uncertain parameters and comparing with (A1), we can deduce that

$$\underline{a_{1,1}} = -1,\ \overline{a_{1,1}} = 0,\ \underline{a_{2,2}} = -2,\ \overline{a_{2,2}} = -1,$$

$$\underline{a_{3,3}} = 1,\ \overline{a_{3,3}} = 2,\ \underline{a_{4,4}} = 2,\ \overline{a_{4,4}} = 3,\ \underline{a_{5,5}} = 2,\ \overline{a_{5,5}} = 3.$$

Thus, one has

$$a = \frac{-1-2+1+2+2}{5} = \frac{2}{5},$$
$$b = \frac{0-1+2+3+3}{5} = \frac{7}{5},$$

in view of (2). Therefore, by Theorem 1, we conclude that the uncertain interval continuous-time systems (4) are simultaneously

(i) $S\left(-\infty, \frac{7}{5}\right)$ unstable systems;

(ii) $S\left(\frac{2}{5}, \infty\right)$ unstable systems;

(iii) $S\left(\frac{2}{5}, \frac{7}{5}\right)$ unstable systems.

From (i) above and in conjunction with Remark 1, we can also see that the uncertain interval continuous-time systems of (4) are unstable systems. Besides, the state variable trajectory diagrams of the uncertain interval continuous-time systems (4) are shown in Figures 1 to Figure 3. As shown in Figure 1 to Figure 3, the state variables of the uncertain interval continuous-time system (4) all exhibit divergence.

IV. CONCLUSION

In this paper, the concepts of stability and instability have been first proposed. A simple criterion has been offered to guarantee instability for a class of uncertain interval continuous-time systems. Besides, both parameters and can be quickly calculated and obtained from the upper and lower bounds of the unknown parameters of such uncertain interval continuous-time systems. At last, several numerical simulation results have beenpresented to illustratethe validity and correctness of the main theorem.

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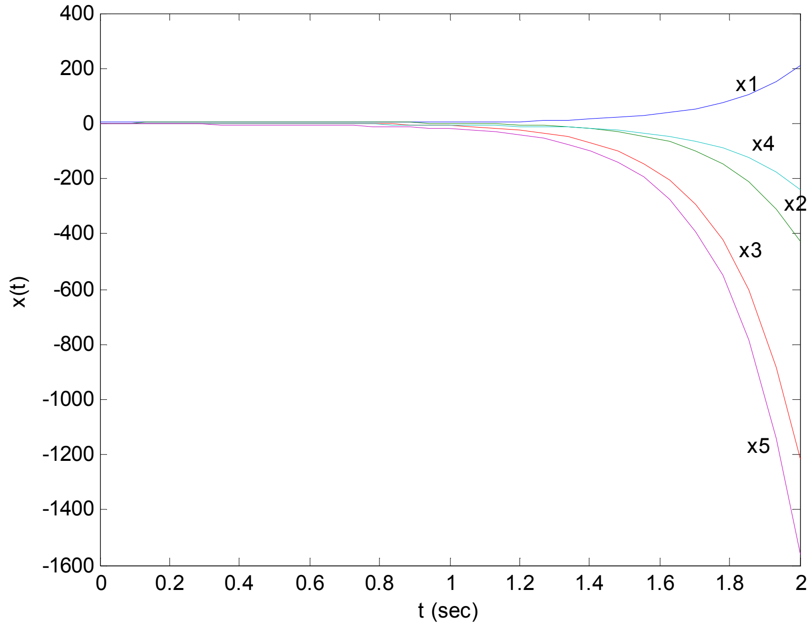

Figure 1: State variable trajectory diagram of the uncertain interval continuous systems (4)with $x(0)=[3 \quad 2 \quad 1 \quad -1 \quad -2]^T$.

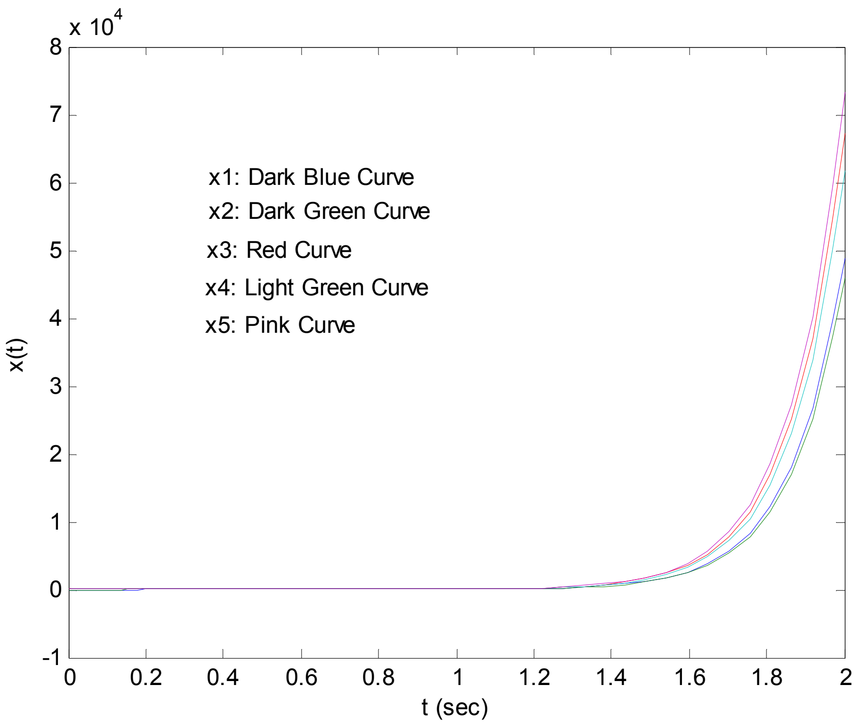

Figure 2: State variable trajectory diagram of the uncertain interval continuous systems (4)with $x(0)=[-0.1 \quad 0 \quad 0 \quad 0.1 \quad 0.1]^T$.

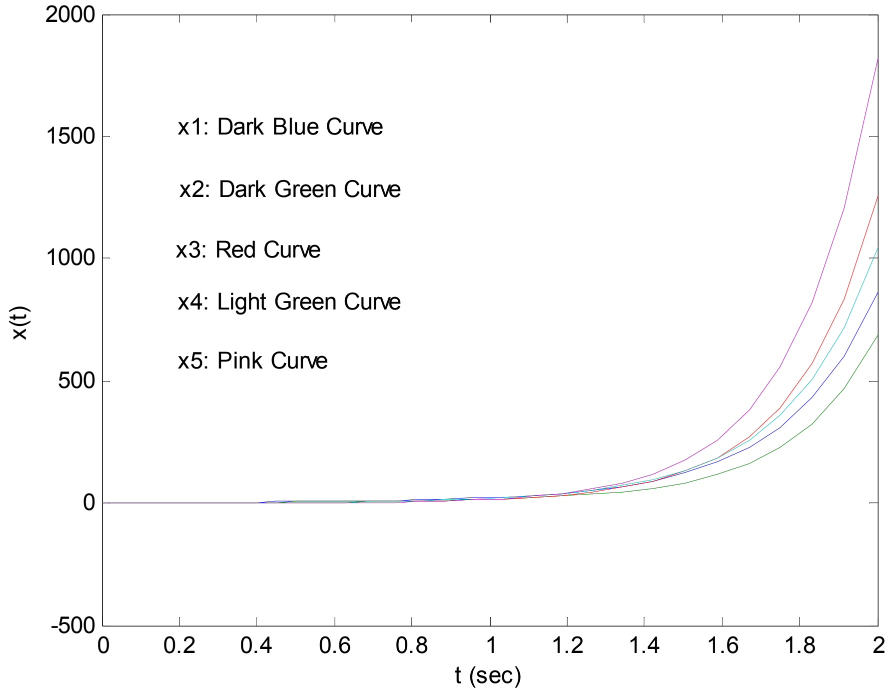

Figure 3: State variable trajectory diagram of the uncertain interval continuous systems (4)with $x(0)=[3 \quad 2 \quad 1 \quad -1 \quad -2]^T$.