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Optimal Solution of Practical Unbalanced Assignment Problems in Fuzzy Environment

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Abstract: *A Practical Unbalanced Assignment Problem (PUAP) arises when the number of jobs exceeds the number of machines, requiring a many-to-one assignment rather than introducing dummy machines to balance the model. In real-world scenarios, cost coefficients are often imprecise and can be represented using fuzzy numbers. This paper reformulates the PUAP in a fuzzy environment using triangular fuzzy numbers to model uncertain assignment costs. Multiple defuzzification techniques including robust ranking, Yager's ranking, centroid method, and mean of removal are employed to convert the fuzzy cost matrix into crisp equivalents. Classical Hungarian approach is then applied to obtain optimal assignments under each method. Comparative analysis demonstrates the effectiveness and consistency of robust ranking for PUAP. Numerical examples illustrate the proposed methodology.*

Keywords: Unbalanced Assignment Problem, Practical Assignment, Triangular Fuzzy Numbers, Robust Ranking, Yager's Ranking, Centroid Method, Defuzzification, Fuzzy Optimization, Hungarian Algorithm.

MSC 2010 Subject Classification: 90C08, 90C10, 90C05, 03E72.

I. INTRODUCTION

The classical assignment problem (AP) assumes a one-to-one matching between an equal number of jobs and machines with the objective of minimizing total cost or maximizing total profit [1]. However, practical situations often involve an unequal number of jobs and machines, leading to the unbalanced assignment problem (UAP). Conventional methods handle UAP by adding dummy rows or columns with zero cost to create a balanced square matrix, then applying the Hungarian algorithm [2]. This approach, while mathematically convenient, may not reflect real operational constraints where all jobs must be completed and all machines must be utilized at least once. To address this, Kumar [16] introduced the Practical Unbalanced Assignment Problem (PUAP), where m jobs are assigned to n machines with $m > n$, allowing multiple jobs per machine.

In many decision-making environments, the cost or time parameters are not precisely known due to uncertainty, vagueness, or incomplete information. Fuzzy set theory, introduced by Zadeh [3] , provides a powerful framework to model such uncertainty. This paper extends the PUAP to a fuzzy environment by representing cost coefficients as triangular fuzzy numbers (TFNs).

A critical step in solving fuzzy optimization problems is de-fuzzification — converting fuzzy numbers to crisp values. Various ranking methods exist, each with distinct properties. This paper compares robust ranking [14], Yager's ranking [4] , centroid method [8], and mean of removal [6] for solving fuzzy PUAP. The defuzzified problem is then solved using a replication-based Hungarianapproach.

In this research paper, we contribute the following:

Formulation of PUAP with triangular fuzzy costs,

Comparative analysis of four defuzzification techniques,

A step-by-step algorithm,

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Extensive literature review on unbalanced fuzzy assignment problems.

2. Literature Review

The assignment problem was first solved by Kuhn [2] using the Hungarian method. Unbalanced assignment problems have received attention due to their practical relevance. Kumar [16] formally defined the PUAP and proposed a direct method without dummy variables. Yadaiah et al [17]. developed a Lexi-search algorithm for PUAP, though it does not guarantee optimality. Textbook approaches [1, 11] typically convert UAP to balanced form by cloning machines. Fuzzy assignment problems (FAP) were introduced to handle imprecision in cost coefficients. Chen [5] and Lin & Wen [10] solved FAP using linguistic variables and fuzzy numbers. Mukherjee & Basu [13] used interval-valued fuzzy sets. Dinagar & Palanivel [12] investigated FAP with trapezoidal fuzzy numbers using ranking methods. Kumar et al. [15] applied a revised approach to solve FAP with generalized fuzzy numbers.

For defuzzification methods, Yager [4] proposed ranking functions based on the first and second moments. Cheng [8] introduced the centroid method for fuzzy numbers. Kaufmann and Gupta [6] developed the mean of removal method. Fortemps and Roubens [7] compared ranking methods for fuzzy quantities. Wang and Kerre [9] provided an axiomatic approach to fuzzy ranking.

For unbalanced fuzzy assignment problems (UFAP), Sakthi et al. [19] proposed a method using TFNs and the ones assignment method. Mohanaselvi& Ganesan [21] solved UFAP using robust ranking for trapezoidal fuzzy numbers. Thorani& Shankar [18] presented fuzzy Hungarian MODI method for UFAP. Khalifa et al. [23] addressed multi-objective UFAP in intuitionistic fuzzy environments. Senthil Kumar & Jahir Hussain [22] used centroid ranking to solve UFAP with heptagonal fuzzy numbers. Stephen Dinagar and Kamalanathan [20] compared Yager's ranking and robust ranking for fuzzy transportation. Despite these advances, systematic comparison of defuzzification methods for PUAP remains limited. This paper addresses that gap.

3. Mathematical Preliminaries

3.1 Hungarian Algorithm for Solving Assignment Problem

The Hungarian method is a combinatorial optimization algorithm that solves the assignment problem in polynomial time $O(n^3)$. Given an $n \times n$ cost matrix $C = (c_{ij})$, where c_{ij} is the cost of assigning worker i to job j, the goal is to find a one-to-one assignment that minimizes total cost:

$$min\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}x_{ij}$$

subject to $\sum_{j} x_{ij} = 1$ for all i, $\sum_{i} x_{ij} = 1$ for all j, and $x_{ij} \in \{0,1\}$.

Algorithm Steps

Row reduction. For each row i, let $u_i = min_j c_{ij}$. Replace $c_{ij} \leftarrow c_{ij} - u_i$.

Column reduction. For each column j, let $v_j = min_i c_{ij}$. Replace $c_{ij} \leftarrow c_{ij} - v_j$.

Cover zeros. Cover all zeros in the current matrix with the minimum number of horizontal and vertical lines. Let this number be L.

Optimality test. If $L = n$, an optimal assignment exists among the zeros. Go to Step 6. If $L < n$, continue to Step 5.

Create additional zeros.

Find the smallest uncovered element, say $k > 0$.

Subtract k from all uncovered elements.

Add k to all elements covered by two lines.

Return to Step 3.

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Make assignment. Choose a set of n independent zeros, i.e., no two in the same row or column. Start with rows or columns containing only one zero.

Two special cases in Assignment Problems are

Maximization Problem: To maximize total profit, convert the matrix using $c'_{ij} = M - c_{ij}$ where $M = max_{i,j} c_{ij}$. Then minimize C'.

Unbalanced Assignment Problem: If there are m workers and n jobs with $m \neq n$, add dummy rows or columns with zero cost to make the matrix square.

3.2. Triangular Fuzzy Numbers

Definition 1. *A triangular fuzzy number (TFN)* $\widetilde{A} = (a,b,c)$ *is a fuzzy number with membership function* $\mu_{\widetilde{A}}(x)$ *defined by:*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases}$$

Here $a \le b \le c$, *where* b *is the core, and* $[a,c]$ *is the support of* $\widetilde{A}$.

3.3. Defuzzification Methods

Definition 2 (Robust Ranking). *For a TFN* $\widetilde{A} = (a,b,c)$, *the robust ranking index is:*

$$R(A) = \int_0^1 0.5\left[a_\alpha^L + a_\alpha^U\right] d\alpha = \frac{a+2b+c}{4},$$

where $a_\alpha^L = a + \alpha(b - a)$ *and* $a_\alpha^U = c - \alpha(c - b)$.

Definition 3 (Yager's Ranking). *For a TFN* $\widetilde{A} = (a,b,c)$, *Yager's first index is:*

$$Y_1(A) = \frac{a+2b+c}{4}.$$

Note: For TFNs, Yager's first index coincides with robust ranking. The second index $Y_2(\widetilde{A}) = b$ *and third index* $Y_3(A) = \dfrac{a+b+c}{3}$ *are also used.*

Definition 4 (Centroid Method). *The centroid of a TFN* $\widetilde{A} = (a,b,c)$ *is:* $C(A) = \dfrac{a+b+c}{3}$.

Definition 5 (Mean of Removal). *For a TFN* $\widetilde{A} = (a,b,c)$, *the mean of removal is:* $M(A) = \dfrac{a+2b+c}{4}$.

Remark 6. *For triangular fuzzy numbers, robust ranking, Yager's first index, and mean of removal yield identical values. However, they differ for trapezoidal and other fuzzy numbers. Centroid method differs even for TFNs.*

3.4. Practical Unbalanced Assignment Problem in Fuzzy Environment

Consider m jobs $J_1, \ldots, J_m$ and n machines $M_1, \ldots, M_n$ with $m > n$. Let $\widetilde{C}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ be the TFN representing the cost of assigning job i to machine j. The fuzzy PUAP is:

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Minimize $$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}$$

Subject to $$\sum_{j=1}^{n} x_{ij} = 1, \forall i = 1,\ldots,m$$

$$x_{ij} \in \{0,1\}.$$

Proposed Algorithm to Solve PUAP using Triangular Fuzzy Number

Step 1: Represent the cost matrix of FUAP using triangular fuzzy numbers $\widetilde{C}_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

Step 2: Select a defuzzification method D from {Robust Ranking, Yager's Y_3, Centroid, Mean of Removal}. Compute crisp values: $C_{ij} = D(\widetilde{C}_{ij})$.

Step 3: There after we adopt the approach suggested in both Hillier and Lieberman , Winston as $m>n$, replicate each machine column $k = \lfloor m/n \rfloor$ times and introducing dummy rows if any necessary to balance the given assignment problems,

Step 4: Apply the Hungarian algorithm to the resulting $m \times m$ balanced matrix.

Step 5: Extract the assignment for the original Jobs by mapping replicated machines back to original machines.

Step 6: Compute total fuzzy cost $\widetilde{Z} = \sum \widetilde{C}_{ij} x_{ij}$ and total crisp cost is $Z = \sum C_{ij} x_{ij}$.

§4. Numerical Example and Comparative Analysis

Example 6.1: Consider 4 jobs and 3 machines with TFN costs:

Fuzzy Cost Matrix with TFNs

Jobs/Machines	M1	M2	M3
J1	(2,4,6)	(3,5,7)	(8,10,12)
J2	(3,5,7)	(1,3,5)	(5,7,9)
J3	(5,7,9)	(7,9,11)	(6,8,10)
J4	(7,9,11)	(6,8,10)	(5,7,9)

4.1. De-fuzzification Results:

First, fuzzy numbers given in the Table 1 are converted to corresponding crisp values using Robust ranking technique, followed by cloning each machines once and introducing two more dummy rows to balance the resultant Assignment problem in crisp values as suggest in the textbooks by Hillier and Liberman and Winston . Thus, we obtain

Balancing the PUAP with Crisp Values obtained by Robust Ranking Technique

Jobs/Machines	M1	M2	M3	M1	M2	M3
J1	4	5	10	4	5	10
J2	5	3	7	5	3	7
J3	7	9	8	7	9	8
J4	9	8	7	9	8	7
DUM1	0	0	0	0	0	0
DUM2	0	0	0	0	0	0

Upon applying Hungarian algorithm of solving assignment problem with cost values as given in Table 2, yields the optimal solution J1 to M1, J2 to M2, J3 to M1, J4 to M3 with total crisp cost 21 and total fuzzy cost (13, 21, 29).

4.2. Comparison of Optimal Assignment in Four De-fuzzification Methods

The crisp values obtained by four defuzzification methods under study are given in the next table.

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Table1. Complete Crisp Cost Matrices

	Robust/Yager Y_1/Mean			Yager Y_3			Centroid		
	M1	M2	M3	M1	M2	M3	M1	M2	M3
J1	4	5	10	4	5	10	4.0	5.0	10.0
J2	5	3	7	5	3	7	5.0	3.0	7.0
J3	7	9	8	7	9	8	7.0	9.0	8.0
J4	9	8	7	9	8	7	9.0	8.0	7.0

After cloning the machines as mentioned earlier followed by applying Hungarian algorithm, we obtain optimal assignments as J1 to M1, J2 to M2, J3 to M1, J4 to M3.

Table2. Optimal Assignments Using Different Defuzzification Methods

Method	J1-M1	J2-M2	J3-M1	J4-M3
Robust Ranking	4.0	3.0	7.0	7.0
Yager's Y_3	4.0	3.0	7.0	7.0
Centroid	4.0	3.0	7.0	7.0
Mean of Removal	4.0	3.0	7.0	7.0

4.3. Discussion

For triangular fuzzy numbers, robust ranking, Yager's first index, and mean of removal produce identical defuzzified values and hence identical assignments. The centroid method and Yager's third index $Y_3 = (a+b+c)/3$ yield slightly different values but result in the same optimal assignment for this example due to the structure of the cost matrix. However, for asymmetric fuzzy numbers or trapezoidal fuzzy numbers, these methods diverge significantly . Robust ranking is preferred for UFAP as it incorporates the α-cut approach and provides a more balanced representation of uncertainty .

Table3. Comparison of Total Costs

Method	Total Crisp Cost	Total Fuzzy Cost
Robust Ranking	21	(13, 21, 29)
Yager's Y_1	21	(13, 21, 29)
Yager's Y_3	21	(13, 21, 29)
Centroid	21.0	(13, 21, 29)
Mean of Removal	21	(13, 21, 29)

II. CONCLUSION

This paper presented a comparative study of defuzzification techniques for solving practical unbalanced assignment problems in fuzzy environment using triangular fuzzy numbers. Four methods — robust ranking, Yager's ranking, centroid, and mean of removal — were applied. For TFNs, robust ranking and Yager's first index coincide, while centroid differs slightly. All methods yielded the same optimal assignment for the given example, validating the stability of the proposed algorithm. The literature review indicates that robust ranking provides a reliable and computationally efficient defuzzification method for PUAP. Future work may extend this comparison to intuitionistic, Pythagorean, and neutrosophic fuzzy environments, and to cases where methods yield different assignments.

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