

Evolution in the Methods of Measuring Price Elasticity: An Illustrative Framework

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Abstract: Price elasticity of demand (E_d) quantifies the responsiveness of quantity demanded to price changes, assuming other factors remain constant. Traditional methods of calculating E_d , such as the percentage method and point method, often produce mathematically inconsistent results in the computation of marginal revenue (MR) and total revenue (TR) changes when analysed bidirectionally along the demand curve. This paper critically reviews exiting methods of calculating price elasticity of demand such as percentage method and point method with the help of illustrative explanation and highlights inconsistencies in them and to overcome these inconsistencies, the midpoint (arc elasticity) method is introduced, which employs the arithmetic mean of price and quantity between two points on the demand curve. Using geometric and algebraic analysis, the paper shows that this method ensures mathematically consistent and direction-independent calculation of E_d , MR and TR changes. The findings provide a more reliable framework for both theoretical modelling and applied economic analysis, enhancing the accuracy of revenue projections and pricing strategies.

Keywords: Price elasticity of demand

I. INTRODUCTION

Price Elasticity of Demand (E_d) is a ratio of percentage change in demand for a commodity to a percentage change in the price of that commodity. It explains how much percentage change will there be in a demand for a particular commodity if the price of that commodity changes by 1 percent when other factors affecting on a demand for that commodity remain constant. When other factors affecting on a demand for that commodity remain constant, there is an inverse relationship between quantity demanded for a that commodity and price of that commodity, the value of price elasticity of demand will always be negative.

II. LITERATURE REVIEW

Following is a review of the available literature on this topic: -

A. P. Lerner (1933). This paper provides a graphical framework to illustrate the concept of price elasticity of demand. Published in The Review of Economic Studies, Lerner's work aimed to provide a visual understanding of how quantity demanded responds to changes in price. This diagrammatic approach has been influential in economic pedagogy, offering students and practitioners a clear method to visualize elasticity concepts. While the paper itself is concise, its impact on economic education has been enduring, as it laid the groundwork for subsequent discussions and analyses of demand elasticity in both theoretical and applied contexts.

Hyson and Hyson (1949). This paper provides an early geometric interpretation of point elasticity, introducing methods that use tangent intercepts on demand curves to measure responsiveness. Although brief, this classic note has been widely cited in modern pedagogical literature for its intuitive approach, linking algebraic elasticity formulas with visual, geometric representations. The study laid foundational insights for teaching and understanding price elasticity, particularly in illustrating how local slopes of demand curves correspond to elasticity values.

Donald Bumpass (1979). Price elasticity of demand plays a critical role in determining the impact of price changes on a firm's revenue. Bumpass examined how the response of total revenue to price variations can be used to derive the price



elasticity of demand. A mathematically constructive approach already demonstrates that for elastic products ($E_d > 1$), revenue decreases with price increases; for inelastic products ($E_d < 1$), revenue rises with price increases; and for unit-elastic products ($E_d = 1$), revenue remains unchanged. Collectively, these studies emphasize the necessity for firms to consider elasticity when setting prices to optimize total revenue.

Michael B. Vaughan (1988). This paper critically examines the traditional arc elasticity formula commonly taught in introductory economics courses, highlighting its tendency to approach unity as price changes become large, potentially misleading students about total revenue effects. He argues that this midpoint method may suggest no change in total revenue when, in fact, revenue is increasing or decreasing. Vaughan proposes point elasticity as a more accurate alternative for teaching elasticity concepts, emphasizing its suitability for analysing finite price changes and its alignment with graphical interpretations of demand responsiveness. This critique contributes to ongoing discussions about refining elasticity pedagogy in economics education.

Daskin (1992). This paper investigates the limitations of using point price elasticity for finite price changes, highlighting conditions under which it may misrepresent the actual responsiveness of quantity demanded. The study demonstrates that while point elasticity is theoretically precise for infinitesimal price changes, its application to larger, discrete changes can lead to inaccurate predictions of total revenue effects. By comparing point and arc elasticity approaches, Daskin shows that relying solely on point elasticity may overstate or understate revenue impacts, particularly along nonlinear demand curves. This analysis underscores the importance of selecting an appropriate elasticity measure when evaluating price changes of practical magnitude, providing a critical perspective for studies that aim to link elasticity estimates with total revenue outcomes.

William A. Phillips (1994). This paper revisits Allen's arc elasticity formula, proposing the use of the geometric mean instead of the traditional arithmetic mean to calculate average prices and quantities. He argues that the geometric mean offers pedagogical advantages by more accurately reflecting proportional changes in price and quantity, thereby providing a more precise measure of elasticity. This modification enhances the formula's theoretical consistency and aligns better with economic intuition, particularly when teaching the responsiveness of demand to price changes. Phillips approach has been cited in subsequent educational literature for its contribution to refining elasticity pedagogy.

Yang, Loviscek, Cheng, and Hung (2012). This paper examines Allen's arc elasticity using arithmetic, geometric, and harmonic means, highlighting their applicability under different market conditions. They found that for small price and quantity changes, arithmetic and geometric means yield similar elasticity estimates, while the harmonic mean is more accurate in volatile markets or reciprocal-based pricing, such as commodities and foreign exchange. The study also showed that the geometric mean is consistently bounded by the arithmetic and harmonic measures, making it a stable and practical choice for most applications, whereas the harmonic mean is preferred in cases of extreme price fluctuations. This research refines the methodology for estimating price elasticity and emphasizes the importance of selecting an appropriate mean based on market dynamics.

Jong Shin, Wei (2013). Price elasticity of demand quantifies the responsiveness of quantity demanded to changes in price. Wei distinguishes between directional elasticity which measures the effect of discrete price changes and point elasticity, applicable to linear or differentiable demand curves. The study highlights flaw in traditional textbook treatments, showing that simple rules about the relationship between price changes and total revenue often fail in both linear and non-linear cases. Wei develops a unified framework to analyse changes in total revenue due to price adjustments, providing methods to calculate revenue-maximizing prices and illustrating practical applications for linear, iso-elastic, and general demand functions, with or without calculus.

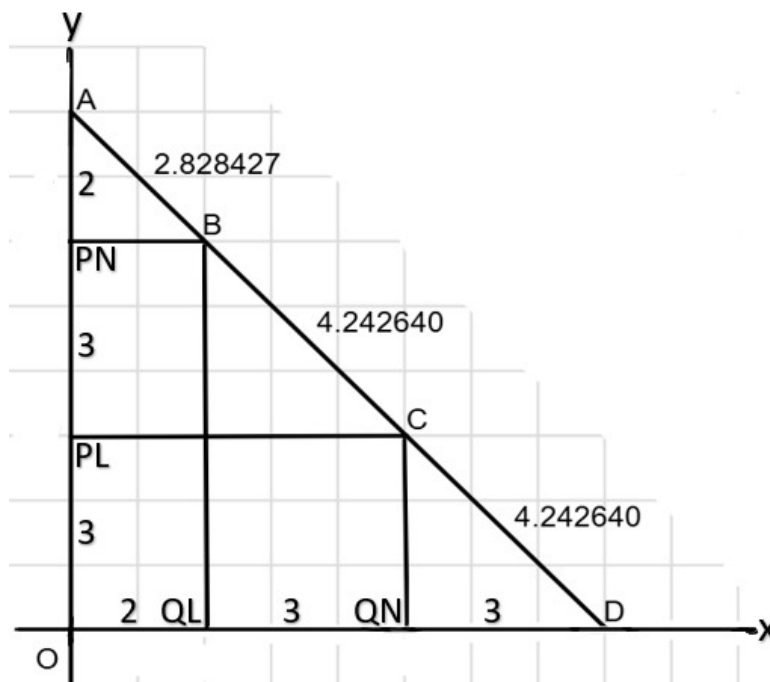
Samithamby Senthilnathan (2016). This paper emphasizes that price elasticity of demand plays a central role in determining a firm's revenue outcomes. The study develops a mathematically constructive framework to demonstrate how elasticity influences the relationship between price changes and total revenue. Specifically, elastic demand ($E_d > 1$) produces an inverse relationship between price and revenue, inelastic demand ($E_d < 1$) yields a positive relationship, and unit elasticity ($E_d = 1$) results in no change in revenue. By integrating theoretical and applied models, the paper highlights that understanding elasticity is essential for managerial pricing decisions, particularly when assessing how shifts in price affect both total revenue and marginal revenue.



Evolution in the Methods of Calculating a Value of Ed with Illustrations: -

Percentage Method of Calculating a Value of Ed with Illustration: -

Points	Price (P)	QD (Q) = $8 - (1 * P)$ ($y = a - bx$; $a = 8, b = 1, x = P, y = QD$)	TR = P * Q (Total Revenue)
A	8	0	$0 = 8 * 0$
B	6	2	$12 = 6 * 2$
C	3	5	$15 = 3 * 5$
D	0	8	$0 = 0 * 8$



on y axis, 1 cm = 1 unit, on x axis, 1 cm = 1 unit

$AB = 2.828427$, $BD = BC + CD = 4.242640 + 4.242640 = 8.48528$

$OPL = 3$, $OPN = OPL + PLPN = 3 + 3 = 6$

$OQL = 2$, $OQN = OQL + QLQN = 2 + 3 = 5$



In the above illustration,

Price (P) is taken on y axis while Quantity Demanded (QD) is taken on x axis. let's calculate price elasticity of demand using a percentage method: -

Under the percentage method of calculating Ed, $Ed = \% \Delta QD / \% \Delta P$

$$\% \Delta QD = (\text{new QD} - \text{original QD}) * 100 / \text{original QD}$$

$$\% \Delta P = (\text{new P} - \text{original P}) * 100 / \text{original P}$$

(While, new QD - original QD = a change in QD = $\Delta QD = Q_{LN}$, Similarly, new P - original P = a change in P = $\Delta P = P_{LPN}$)

Considering a movement along the demand curve from point B to point C, when price falls from OPN to OPL and quantity demanded rises from OQL to OQN,

$$\% \Delta QD = (OQN - OQL) * 100 / OQL$$

$$= (5 - 2) * 100 / 2 = 150 \%$$

$$\% \Delta P = (OPL - OPN) * 100 / OPN$$

$$= (3 - 6) * 100 / 6 = - 50 \%$$

$$Ed = 150 \% / - 50 \% = - 3 \text{ or } 3 \text{ (ignoring a negative sign)}$$

Now, considering a movement along the demand curve from point C to point B, when price rises from OPL to OPN and quantity demanded falls from OQN to OQL,

$$\% \Delta QD = (OQL - OQN) * 100 / OQN$$

$$= (2 - 5) * 100 / 5 = - 60 \%$$

$$\% \Delta P = (OPN - OPL) * 100 / OPL$$

$$= (6 - 3) * 100 / 3 = 100 \%$$

$$Ed = - 60 \% / 100 \% = - 0.6 \text{ or } 0.6 \text{ (ignoring a negative sign)}$$

For the same illustration, now, let's calculate price elasticity of demand using the point method.

Point Method of Calculating a Value of Ed with Illustration: -

Under the point method of calculating Ed, at a particular point on the demand curve, the demand curve is divided into two segments; lower segment (the segment of the demand curve below that particular point) and upper segment (the segment of the demand curve above that particular point)

$Ed = \text{lower segment of the demand curve} / \text{upper segment of the demand curve}$

Considering a movement along the demand curve from point B to point C, when price falls from OPN to OPL and quantity demanded rises from OQL to OQN, let's calculate Ed using a point method at the initial point that is point B;

At initial point B; lower segment of the demand curve is BD and upper segment of the demand curve is AB.

$$Ed = BD / AB = 8.48528 / 2.828427 = 3$$

Now, considering a movement along the demand curve from point C to point B, when price rises from OPL to OPN and quantity demanded falls from OQN to OQL, let's calculate Ed using a point method at the initial point that is point C;

At initial point C; lower segment of the demand curve is CD and upper segment of the demand curve is AC.

$$\text{While } CD = 4.24264 \text{ and } AC = AB + BC = 2.828427 + 4.24264 = 7.071067$$

(Values of AB, BC, CD are verified using Pythagoras theorem and hence, values of AC, AD and BD are also automatically verified using Pythagoras theorem)

$$Ed = CD / AC = 4.24264 / 7.071067 = 0.6$$

(In both of the cases mentioned above, the demand curve is AD)

Consistency in Percentage and Point Method: -

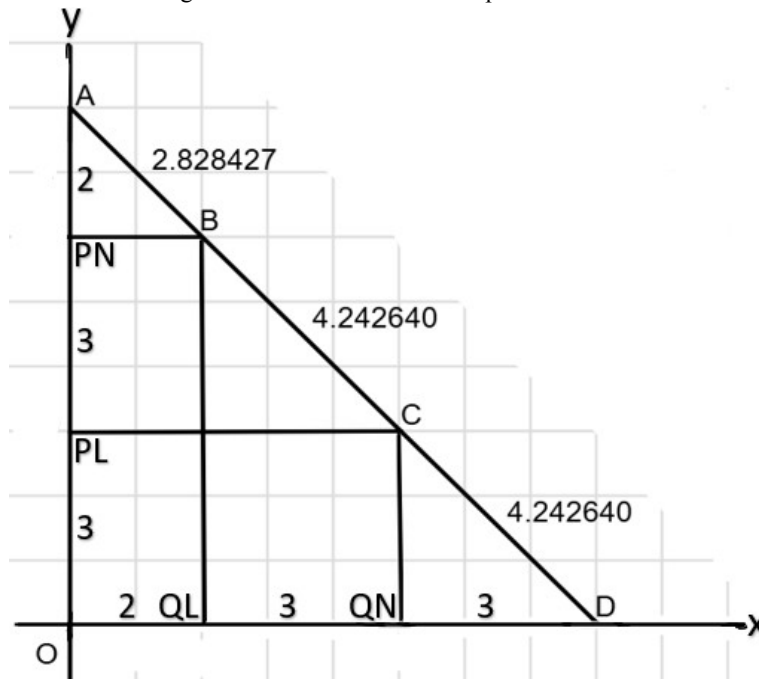
It can be observed that, in case of a movement along the demand curve from point B to point C where an initial point is point B, the x and y coordinates at point B which are OQL and OPN respectively, are also the base values which are used in the calculation of % change in QD and % change in P respectively under a percentage method of calculating Ed. Similarly, in case of a movement along the demand curve from point C to point B where an initial point is point C, the x and y coordinates at point C which are OQN and OPL respectively, are also the base values which are



used in the calculation of % change in QD and % change in P respectively under a percentage method of calculating Ed. It is worth observing that in case of a movement along the demand curve from point B to point C, a value of Ed calculated using a percentage method and in the same case, a value of Ed calculated at the initial point B using a point method is same that is 3. Similarly, in case of a movement along the demand curve from point C to point B, a value of Ed calculated using a percentage method and in the same case, a value of Ed calculated at the initial point C using a point method is same that is 0.6. The technical explanation to this is given in the later part of this paper.

Geometric Reasoning behind Consistency in the Percentage and Point Method: -

It is already noted that, in case of a movement along the demand curve from one point to another point, the values of Ed calculated using a percentage method and in the same case, the values of Ed calculated at the initial point (of those two points) using a point method are same. So, both of the methods of calculating Ed bring same results. Now, let's consider the explanation in the form of geometric reasons behind this pattern.



(On y axis, 1 cm = 1 unit of P, On x axis, 1 cm = 1 unit of QD)

In case of a movement along the demand curve from point B to point C, the value of Ed calculated using a percentage method is stated as: -

$$Ed = \% \Delta QD / \% \Delta P$$

$$= ((OQN - OQL) * 100 / OQL) / ((OPL - OPN) * 100 / OPN) \text{ Since, } OQN - OQL = QLQN \text{ and } OPL - OPN = PLPN$$

$$= (QLQN / PLPN) * (OPN / OQL)$$

$$\text{Since, } QLQN = WC, PLPN = WB \text{ and } OPN = QLB$$

$$= (WC / WB) * (QLB / OQL)$$

$$\text{Since, } \Delta BWC \text{ and } \Delta BQLD \text{ are similar } \Delta\text{'s } WC / WB = QLD / QLB$$

$$= (QLD / QLB) * (QLB / OQL)$$

$$= QLD / OQL$$

$$\text{And, since, } \Delta BQLD \text{ and } \Delta AOD \text{ are similar } \Delta\text{'s } QLD / OQL = OPN / APN = BD / AB$$

$$Ed = BD / AB$$

Also, in the same case, at the initial point that is point B, the value of Ed calculated using a point method is stated as: -

$$Ed = \text{lower segment of the demand curve} / \text{upper segment of the demand curve} = BD / AB$$



Similarly, in case of a movement along the demand curve from point C to point B, the value of E_d calculated using a percentage method and in the same case, at the initial point C, the value of E_d calculated using point method is going to be same which can be mathematically proven in a similar way as explained above. (Both of the methods finally give $E_d = CD / AC$)

(Note that, In case of a movement along the demand curve from point C to point B, Under a percentage method of calculating E_d , $E_d = (QLQN / PLPN) * (OPL / OQN)$)

And, In the same case,

At the initial point that is point C,

When a point method is applied for calculating the value of E_d ,

$E_d = \text{lower segment of the demand curve} / \text{upper segment of the demand curve} = CD / AC$)

(In both of the cases mentioned above, the demand curve is AD)

Inconsistencies in Change in TR and MR Calculations with Percentage or Point Method: -

In the above illustration, let's consider the TR at a point B which is 12 and TR at a point C which is 15. In case of a movement along the demand curve from point B to point C, TR increases by 3 (TR becomes 15 from 12) while in case of a movement along the demand curve from point C to point B, naturally TR decreases by 3 (TR becomes 12 from 15). Please ignore the negative sign attached to a decrease in the value of TR for a time being. So, in both of the cases, a value of change in TR remains same that is 3. Again, in the above illustration, let's consider the QD at a point B which is 2 and QD at a point C which is 5. In case of a movement along the demand curve from point B to point C, QD increases by 3 (QD becomes 5 from 2) while in case of a movement along the demand curve from point C to point B, naturally QD decreases by 3 (QD becomes 2 from 5). Please ignore the negative sign attached with a decrease in the value of QD for a time being. So, in both of the cases, a value of change in QD remains same that is 3.

It is already known that, Marginal Revenue (MR) is a change in TR divided by a change in QD.

$MR = \Delta TR / \Delta QD$

Since, in the both of the cases mentioned above, a value of change in TR and change in QD remains same that is 3 and 3 respectively;

$MR = 3 / 3 = 1$.

In both of the cases mentioned above, a value of MR also remains the same that is 1.

This is a manual calculation of a value of change in TR and MR. It is already known that,

$MR = P * (1 - (1 / E_d))$

(Multiplying both sides by ΔQD ;

$MR * \Delta QD = P * (1 - (1 / E_d)) * \Delta QD$

Since, $MR = \Delta TR / \Delta QD$ $\Delta TR = MR * \Delta QD$

Hence, $\Delta TR = \Delta QD * P * (1 - (1 / E_d))$

Here, P is a value of P which is taken at the base (as a base value) for the calculation of % change in P which is used in calculating E_d under a particular method of calculating E_d while E_d is nothing but a value of E_d which is calculated under that particular method of calculating E_d .

Now, using the above formulae of calculating a value of change in TR and MR, let's calculate a value of change in TR and MR in both of the cases mentioned above.

Referring to the percentage method of calculating E_d ;

P is original P while E_d is a value of E_d which is calculated using a percentage method.

In case of a movement along the demand curve from point B to point C, $P = OPN = 6$ and $E_d = 3$.

$MR = 6 * (1 - (1 / 3)) = 4$. (While, $\Delta QD = 3$, $MR = 4$, $\Delta TR = 3 * 4 = 12$)

In case of a movement along the demand curve from point C to point B,

$P = OPL = 3$ and $E_d = 0.6$

$MR = 3 * (1 - (1 / 0.6)) = -2$ or 2 (Please ignore a negative sign for a time being). (While, $\Delta QD = -3$, $MR = 2$,

$\Delta TR = -3 * 2 = -6$ or 6

(Please ignore a negative sign for a time being))

Limitation of Percentage or Point Method: -

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In case of a movement along the demand curve from point B to point C, a value of MR is 4 (while a value of change in TR is 12) when calculated using the above formulae and in case of a movement along the demand curve from point C to point B, a value of MR is 2 (while a value of change in TR is 6) when calculated using the above formulae but in both of the cases mentioned above, a value of MR remains the same that is 1 (while a value of change in TR also remains the same that is 3) when calculated manually.

So, in both of the cases mentioned above, when the calculation of Ed is done by using a percentage method or a point method, there is a lack of mathematical consistency or mathematical equality in the computation of MR (and the computation of a change in TR) which is done by using the above formulae and the computation of MR (and the computation of a change in TR) which is done by using a manual calculation. The reason behind this can be simply understood through the fact that in case of a movement along demand curve from one point to another point and again returning back from that point to the starting point, (in our illustration, a movement along the demand curve from point B to point C and again returning back from point C to the point B); first of all, a value of change in QD and change in P remains the same because, a movement is between the two points (point B and point C) which are same. So, a distance (a value of change in QD and change in P) between same two points (point B and point C) will be same only. (Although, a movement is in two opposite directions, (in which one direction is going from one point (point B) to another point (point C) while another direction is returning back from that point (point C) to a starting point (point B))). Secondly, under a percentage method or a point method, we use two different bases of QD and P in the calculation of Ed at two different points on a demand curve (point B and point C) for the movement along the demand curve between those two points itself (point B and point C). So, considering a formula of calculating Ed under percentage method or point method, we get two different values of Ed at two different points on a demand curve (point B and point C) and therefore, when calculated using the above formulae, we obtain two different values of a change in TR and MR at two different points on a demand curve (point B and point C). Now, considering a manual calculation of a value of change in TR and MR, in case of a movement along demand curve from one point to another point and again returning back from that point to the starting point, (in our illustration, a movement along the demand curve from point B to point C and again returning back from point C to the point B); a value of change in TR, change in QD and MR (because, $MR = \text{change in TR} / \text{change in QD}$ when calculated manually) remains the same (when a negative sign attached to a value of change in TR, change in QD and MR is ignored for a time being) because, a movement is between the two points (point B and point C) which are same. So, a distance (a value of change in TR and change in QD) between same two points (point B and point C) will be same only. (Although, a movement is in two opposite directions, (in which one direction is going from one point (point B) to another point (point C) while another direction is returning back from that point (point C) to a starting point (point B))). This is a major limitation of a percentage method or a point method of calculating Ed.

Need for an Introduction of Arc Elasticity Method (or Midpoint Method) of Calculating a Value of Ed: -

It is already known that in both of the cases mentioned above, when the calculation of Ed is done by using a percentage method or a point method, there is a lack of mathematical consistency or mathematical equality in the computation of a value of MR (and computation of a value of change in TR) by using the formulae and computation of a value of MR (and computation of a value of change in TR) by using a manual calculation, this is a major limitation of a percentage method or a point method and to overcome this limitation, a new method of calculating Ed has been introduced. It is called arc elasticity or midpoint method of calculating Ed.

Midpoint Method of Calculating a Value of Ed with Illustration: -

Under the arc elasticity or midpoint method of calculating Ed, $Ed = \% \Delta QD / \% \Delta P$

$\% \Delta QD = (\text{new QD} - \text{original QD}) * 100 / ((\text{new QD} + \text{original QD}) / 2)$

$\% \Delta P = (\text{new P} - \text{original P}) * 100 / ((\text{new P} + \text{original P}) / 2)$

(While, $\text{new QD} - \text{original QD} = \text{a change in QD} = \Delta QD = QLQN$, Similarly, $\text{new P} - \text{original P} = \text{a change in P} = \Delta P = PLPN$)

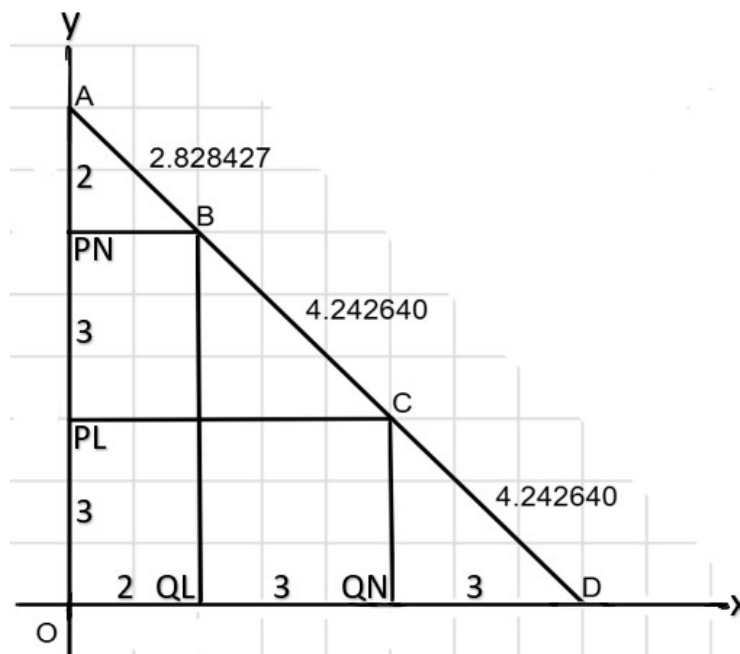
So, under a midpoint method, a midpoint or a simple arithmetic mean of new QD and original QD is taken (instead of just an original QD which is used under a percentage method) as a base value in the calculation of % change in QD



similarly, a midpoint or a simple arithmetic mean of new P and original P is taken (instead of just an original P which is used under a percentage method) as a base value in the calculation of % change in P for the computation of Ed.

Now, let's calculate Ed using a midpoint method for the following illustration: -

Points	Price (P)	QD (Q) = $8 - (1 * P)$ ($y = a - bx$; $a = 8$, $b = 1$, $x = P$, $y = QD$)	TR = $P * Q$ (Total Revenue)
A	8	0	$0 = 8 * 0$
B	6	2	$12 = 6 * 2$
C	3	5	$15 = 3 * 5$
D	0	8	$0 = 0 * 8$



(On y axis, 1 cm = 1 unit of P, On x axis, 1 cm = 1 unit of QD)

In the above illustration, considering a movement along the demand curve from point B to point C, when price falls from OPN to OPL and quantity demanded rises from OQL to OQN, (here, a midpoint would be $(B + C) / 2$)

$$\% \Delta QD = (OQN - OQL) * 100 / ((OQN + OQL) / 2)$$

$$= (5 - 2) * 100 / ((5 + 2) / 2) = 85.71428 \%$$

$$\% \Delta P = (OPL - OPN) * 100 / ((OPL + OPN) / 2)$$

$$= (3 - 6) * 100 / ((3 + 6) / 2) = - 66.6666 \%$$

$$Ed = 85.71428 \% / - 66.6666 \% = - 1.28571 \text{ or } 1.28571 \text{ (ignoring a negative sign)}$$

Now, considering a movement along the demand curve from point C to point B, when price rises from OPL to OPN and quantity demanded falls from OQN to OQL, (here, a midpoint would be $(C + B) / 2$)



$$\% \Delta QD = (OQL - OQN) * 100 / ((OQL + OQN) / 2)$$

$$= (2 - 5) * 100 / ((2 + 5) / 2) = - 85.71428 \%$$

$$\% \Delta P = (OPN - OPL) * 100 / ((OPN + OPL) / 2)$$

$$= (6 - 3) * 100 / ((6 + 3) / 2) = 66.6666 \%$$

$$Ed = - 85.71428 \% / 66.6666 \% = - 1.28571 \text{ or } 1.28571 \text{ (ignoring a negative sign)}$$

It is worth noting that first of all, $((B + C) / 2) = ((C + B) / 2)$ and secondly, ΔQD is same and ΔP is also same in both of the cases mentioned above and because of this mathematical equality, values of Ed (1.28571) calculated using a midpoint method yields same results in both of the cases mentioned above.

Consistencies in Change in TR and MR Calculations with Midpoint Method: -

Using the formulae of calculating a value of change in TR and MR (mentioned earlier), let's calculate a value of change in TR and MR in both of the cases mentioned above.

Now, referring to the midpoint method of calculating Ed ;

P is $(\text{new } P + \text{original } P) / 2$ and Ed is a value of Ed which is calculated using a midpoint method.

In case of a movement along the demand curve from point B to point C, $P = (OPL + OPN) / 2 = (3 + 6) / 2 = 4.5$ and $Ed = 1.28571$

$$MR = 4.5 * (1 - (1 / 1.28571)) = 1. \text{ (While, } \Delta QD = 3, MR = 1,$$

$$\Delta TR = 3 * 1 = 3)$$

In case of a movement along the demand curve from point C to point B, $P = (OPN + OPL) / 2 = (6 + 3) / 2 = 4.5$ and $Ed = 1.28571$

$$MR = 4.5 * (1 - (1 / 1.28571)) = 1 \text{ (While, } \Delta QD = - 3, MR = 1,$$

$$\Delta TR = - 3 * 1 = - 3 \text{ or } 3,$$

(Please ignore a negative sign attached to a value of change in TR for a time being))

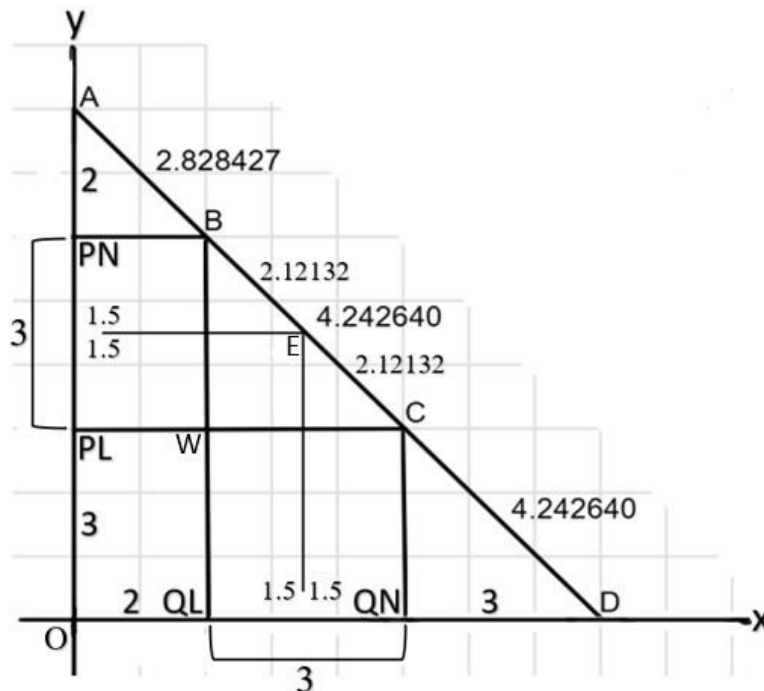
Importance of Midpoint Method: -

In case of a movement along the demand curve from point B to point C, a value of MR is 1 (while a value of change in TR is 3) when calculated using the formulae and in case of a movement along the demand curve from point C to point B, a value of MR is also 1 (while a value of change in TR is also 3) when calculated using the formulae. So, in both of the cases mentioned above, a value of MR remains the same that is 1 (while a value of change in TR also remains the same that is 3) when calculated using the formulae. These results are mathematically consistent with the fact that, in both of the cases mentioned above, a value of MR remains the same that is 1 (while a value of change in TR also remains the same that is 3) when calculated manually. Hence, it establishes a mathematical equality. So, in both of the cases mentioned above, the formulae-based calculation of a value of change in TR and MR exactly match with manually calculated value of change in TR and MR. So, a major limitation of a percentage method or a point method is overcome when a midpoint method is used for the calculation of Ed .

Consistency in the Midpoint and Point Method: -

Let's consider the following illustration: -





(On y axis, 1 cm = 1 unit of P, On x axis, 1 cm = 1 unit of QD) Here,

It is already known that, in case of a movement along the demand curve from point B to point C, where the initial point is point B, the value of E_d calculated using a percentage method and a point method yields same results. Similarly, in case of a movement along the demand curve from point C to point B where the initial point is point C, the value of E_d calculated using a percentage method and a point method also yields same results. Midpoint method of calculating E_d is going a step further and takes into consideration a midpoint or simple arithmetic mean of these two points $((B + C) / 2)$ which divides the segment BC into two equal parts of equal length namely, BE and EC through point E. So, point E is a midpoint of the segment BC and $BC / 2 = BE = EC$. (Keeping in mind that $((B + C) / 2) = ((C + B) / 2)$ and $BC = CB$).

This is because,

$$\Delta QD = (OQN - OQL) = QLQN = WC, \Delta P = (OPN - OPL) = PLPN = WB,$$

Midpoint method takes into consideration a midpoint or simple arithmetic mean of OQL and OQN (that is, $(OQL + OQN) / 2$), as a base value for the calculation of % change in QD. Similarly, it takes into consideration a midpoint or simple arithmetic mean of OPL and OPN (that is, $(OPL + OPN) / 2$), as a base value for the calculation of % change in P. (This % change in QD and % change in P is used in the calculation of E_d under a midpoint method).

$$\begin{aligned} ((OQL + OQN) / 2) &= (OQL + ((OQN - OQL) / 2)) = (OQN - ((OQN - OQL) / 2)) \\ &= (OQL + (WC / 2)) = (OQN - (WC / 2)), \end{aligned}$$

Similarly,

$$\begin{aligned} ((OPL + OPN) / 2) &= (OPL + ((OPN - OPL) / 2)) = (OPN - ((OPN - OPL) / 2)) \\ &= (OPL + (WB / 2)) = (OPN - (WB / 2)), \end{aligned}$$

So, a midpoint method divides WB and WC into two equal parts of equal length.

ΔAOD and ΔBWC are similar Δ 's. ΔAOD is a right angle Δ located at the origin of two axis namely, x axis and y axis. OA and OD are the two sides whereas AD is a hypotenuse of ΔAOD . So, ΔBWC is also a right angle Δ . WC and WB are the two sides whereas BC is a hypotenuse of ΔBWC . So, when WC and WB are divided into two equal parts of equal length, simultaneously, the segment BC also gets divided into two equal parts of equal length namely, BE and EC through point E. So, point E is a midpoint of the segment BC and $BC / 2 = BE = EC$. (Reference: - rules of proportions in mathematics and Pythagoras theorem in geometry)



In case of a movement along the demand curve between point B and point C (that is, from point B to point C or vice versa), the value of E_d calculated using a midpoint method and the value of E_d calculated at the midpoint (that is, point E) of those two points (that is, point B and point C) lying on a demand curve (that is, AD) using a point method yields same results. Technical explanation to this is given in the later part of this paper.

In the above illustration, it is already clear that, when a midpoint method of calculating E_d is used, a value of E_d is same that is 1.28571 in both of the cases mentioned above.

Since, the segment BC is divided into two equal parts of equal length namely BE and EC through point E,
 $BC = 4.24264$

$$BC / 2 = 4.24264 / 2 = 2.12132 = BE = EC$$

Since, the segment BC is divided into two equal parts of equal length namely BE and EC through point E,

Let's calculate E_d again using a point method at point E. Under a point method, it is already known that,

$$E_d = \text{Lower Segment of the demand curve} / \text{Upper Segment of the demand curve At point E,}$$

$$\text{Lower Segment of the demand curve} = ED = EC + CD = 2.12132 + 4.24264 = 6.36396$$

$$\text{Upper Segment of the demand curve} = AE = BE + AB = 2.12132 + 2.828427 = 4.949747$$

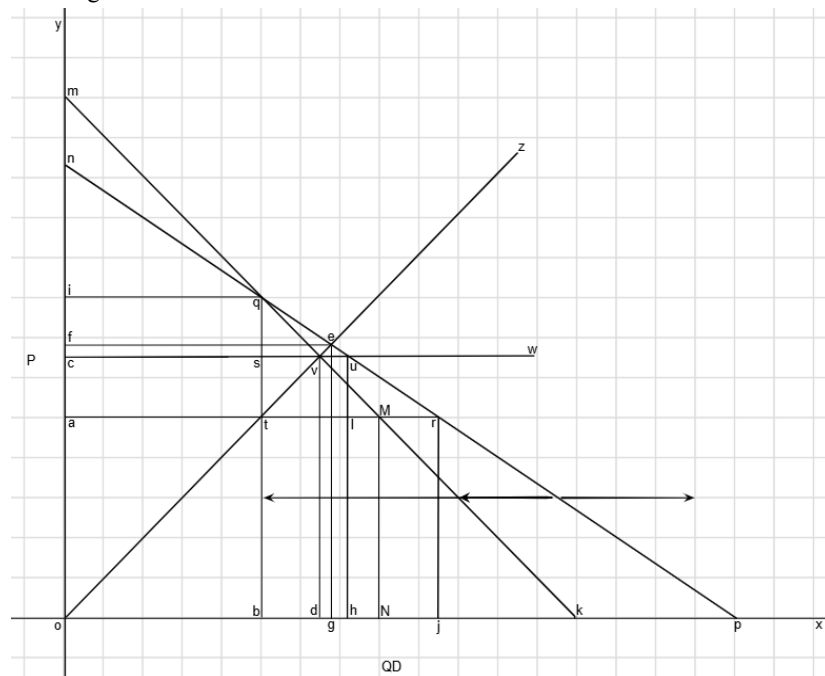
$$E_d = 6.36396 / 4.949747 = 1.28571$$

(The demand curve is AD)

So, a value of E_d calculated by using a midpoint method (that is, $E_d = 1.28571$) remains same irrespective of the direction of movement along the demand curve between two points (between point B and point C as in both of the cases mentioned above) and it is exactly matching with a value of E_d calculated by using a point method (that is, $E_d = 1.28571$) at a midpoint (point E) of those two points (point B and point C) or a midpoint (point E) of the segment of the demand curve (the segment BC of the demand curve AD) between those two points (point B and point C). The technical explanation to this is given below.

Geometric Reasoning behind Consistency in the Midpoint and Point Method: -

Let's consider the following illustration: -



Here, $oa = ob$

$oa = bt$ and $ob = at$, $bt = at$;

So, $oa = bt = ob = at$



That means obta is a square.

The line oz divides a 90° angle (a right-angle triangle) at point o (a point of origin which is also a lower left corner of the square obta) into two angles with an equal degree that is $45^\circ - 45^\circ$ each. It also divides a square obta into two equal parts of equal areas. The line oz passes through point t. (Obviously, it will also divide a 90° angle (a right-angle triangle) at point t (which is an upper right corner of the square obta) into two angles with an equal degree that is $45^\circ - 45^\circ$ each).

The line oz intersects the demand curve np at point e which divides that demand curve (that is np) into two segments namely, segment en (or upper segment) and segment pe (or lower segment). Thereafter ultimately it reaches point z.

When all of the values are in cm on a graph, $a = PL$, $b = QL$, $j = QN$ and $i = PN$

$OPL = oa$, $OQL = ob$, (Since $oa = ob$, $OPL = OQL$),

$OPN = oi$, $OQN = oj$, $\Delta QD = QLQN = OQN - OQL = oj - ob = bj$ and $\Delta P = PLPN$

$= OPN - OPL = oi - oa = ai$ Here,

In case of a movement along the demand curve np between point q and point r, (from point q to point r or vice versa), an attempt has been made here to explain a mathematical equality between the value of Ed which is calculated by using a midpoint method (that is, $(q + r) / 2$) and the value of Ed which is calculated at a midpoint (that is, point u) of those two points (that is, point q and point r) on the demand curve (that is, np) by using a point method.

Under a midpoint method of calculating Ed,

$$Ed = (QLQN / PLPN) * (((OPL + OPN) / 2) / ((OQL + OQN) / 2))$$

Since,

$$((OPL + OPN) / 2) = ((oa + oi) / 2) = oc \quad ((OQL + OQN) / 2) = ((ob + oj) / 2) = oh$$

$$Ed = (bj / ai) * (oc / oh) \text{ Since,}$$

$$oc = hu, bj = tr \text{ and } ai = tq, Ed = (tr / tq) * (hu / oh) \text{ Since,}$$

Δqtr , Δqbp and Δuhp are similar Δ ,

$$tr / tq = bp / bq = hp / hu \text{ So,}$$

$$Ed = (hp / hu) * (hu / oh) Ed = hp / oh$$

Δuhp and Δnop are similar Δ , $hp / oh = oc / cn = pu / un$

$$Ed = pu / un \text{ And,}$$

In the same case,

When a point method is applied for calculating the value of Ed at a midpoint (that is point u) of those two points (that is point q and point r) on the demand curve (that is np),

Under a point method of calculating Ed,

$$Ed = \text{lower segment of the demand curve np} / \text{upper segment of the demand curve np}$$

Since,

A lower segment is pu while an upper segment is un at point u on the demand curve np,

$$Ed = pu / un$$

This way, the value of Ed calculated by using a midpoint method and the value of Ed calculated by using a point method yields the same results.

II. CONCLUSIONS

A value of Ed which is calculated by using a percentage method and a point method exactly match.

Similarly,

There is an exact match between a value of Ed which is calculated by using a midpoint method and a value of Ed which is calculated by using a point method at the midpoint of those two points lying on the demand curve which are used in the calculation of a value of Ed under a midpoint method formula.

When a value of Ed is calculated by using a percentage method or point method, there is a lack of mathematical consistency or mathematical equality in a value of MR and change in TR calculated by using a formula and manual calculation.



But when a value of Ed is calculated by using a midpoint method, a value of MR and change in TR calculated by using a formula and manual calculation exactly match.

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