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The New Integral Transform Mu'azu-Bayawa Transform with Applications.

Sa'adu Bello Mu'azu[1] and Zayyanu Balarabe Bayawa[2]
Department of Mathematics, Abdullahi Fodio University of Science and Technology, Aliero, Nigeria[1]
Department of Mathematics, Federal College of Education Gidan Madi, Sokoto, Nigeria[2]
saadbm13.sbm@gmail.com and bayawabala3@ gmail.com

Abstract: *In this paper, we apply new integral transform namely Mu'azu -Bayawa transform to solve linear ordinary differential equations with constant coefficients. The fundamental properties, linearity property and convolution theorem for the Mu'azu-Bayawa transform has been proved. Some applications are given to demonstrate the effectiveness to propose the technique.*

Keywords: Mu'azu -Bayawa Transform, Integral Transform , Differential Equations, Linearity property and Convolution Theorem

I. INTRODUCTION

The integral transform methods are among the most utilized mathematical techniques to determine the analytical solutions of problem in engineering and sciences without large computational work. There are many integral transform to solve differential equations and integral equations, Laplace transform is the oldest integral transform and the most popular method used in differential equations in [1]. Other recently the new integral transformations develop, Sumudu Transform which was introduced by G.K .Watugula [2].The Elzaki Transform was presented by Tarig M.Elzaki [3].The Aboodh transform was introduced by Khalid S. Aboodh [4]. A. kamal and H.sedeeg proposed the Kamal transform [5].The Mohand transform was introduced by mohand M. Mahgoub [6].The Shehu transform was introduce by Shehu Maitama ,Weidong Z [7].The Sohan transform was presented by D.P.Patil, Savita Santu Khakale [8]. Finally, The Emad -Faith transform was introduced by Emad Kuffi, Sara faith [9].Iman Ahmed Almardy proposed by Iman Transform [10]. Zayyanu B. Bayawa and Aisha A. Haliru introduced by Bayawa transform [11] etc. All these transforms are convenient mathematical tools for solving differential equations. Mu'azu - Bayawa Integral Transform has been proposed to facilitate the process of solving Ordinary differential equations and partial differential equations in time domain, it has been derived from the classical Fourier integral. We now use Also Mu'azu -Bayawa Transform and some of its fundamental properties to solve differential equations. The aims of this work is to show that applicability of interesting new transform which is called Mu'azu -Bayawa Transform and its capability in solving the linear differential equations.

1.1 Mu'azu - Bayawa Integral Transform

The proposed Integral Transform is defined for an exponential order function:

$H = \{ f(t) : \exists M, r_1, r_2 > 0.\ |f(t)| < Me^{-\frac{|t|}{r_j}}$ if $t \in (-1)^j \times [0,\infty)\}$

For a given function in the set H the constant M must be finite number, r_1, r_2 may be finite or infinite.

Mu'azu- Bayawa Transform denoted by the operator M_B (.) defined by the integral equations.

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$M_B\{f(t)\} = U\ (v) = \int_0^{\infty} f(t)\, e^{-\frac{t}{v^2}}\, dt, t \geq 0, r_1 \leq v \leq r_2$

The variable v in the transform is used to factor the variable t in the argument of the function f this transform has deeper connection with the Laplace, Sumudu, Elzaki, Kamal transforms etc.

1.2 Mu'azu- Bayawa transform of some functions

In Mu'azu- Bayawa integral transform, the existence condition must be met to apply the transform in to any function. The existence condition to apply $M_B\{f(t)\}$to any function $f(t)$ is for $t \geq 0$ must be piecewise continuous and in exponential order.

Mu'azu- Bayawa integral transform can use in solving the following functions.

(i) let $f(t) = 1$, then, by definition, we have

$M_B\ \{1\} = U(v) = \int_0^{\infty} f(t)\, e^{-\frac{t}{v^2}} dt$

$= \int_0^{\infty} 1. e^{-\frac{t}{v^2}}\ dt$

$M_B\ \{1\} = v^2$

(ii) let $f(t) = t$, then

$M_B\ \{t\} = \int_0^{\infty} t. e^{-\frac{t}{v^2}}\ dt$

Applying integration by part, we have

$M_B\{t\} = v^4$

(iii) let $f(t) = t^2$, then

$M_B\{t^2\} = \int_0^{\infty} t^2. e^{-\frac{t}{v^2}}\ dt$

Applying integration by part, we have

$M_B\{t^2\} = 2!\, v^6$

(iv) let $f(t) = t^3$, then

$M_B\{t^3\} = \int_0^{\infty} t^3. e^{-\frac{t}{v^2}} dt$

Applying integration by part, we haves

(v) let $f(t) = t^n$, then

$M_B\{t^n\} = \int_0^{\infty} t^n. e^{-\frac{t}{v^2}}\ dt$

Let $x = \frac{t}{v^2}$ or $t = v^2 x$, $dt = dx v^2$

$= \int_0^{\infty} e^{-x} (v^2 x)^n . v\, dx$, Since gamma n +1 = $\int_0^{\infty} e^{-x} (x)^n = n!$

$M_B\ \{t^n\} = v^{2n+2}. n!$ Where n is an integer number

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$(vi)\ \{e^{at}\} = \int_0^{\infty} e^{at}.e^{-\frac{t}{v^2}}\ dx$

$= \int_0^{\infty} e^{-\left(\frac{1}{v^2}-a\right)t}\ dt$

$= \frac{v^2}{1-av^2}$

$(vii)\quad M_B\{e^{-at}\} = \frac{v^2}{1+av^2}$

$(viii)\ M_B\ \{sinat\} = \int_0^{\infty} sinat.e^{-\frac{t}{v^2}}dt$

Since $sinat = \frac{e^{iat}-e^{-iat}}{2i}$

$= \frac{av^4}{1+a^2v^4}$

$(ix)\ M_B\ \{cosat\} = \int_0^{\infty} cosat.e^{-\frac{t}{v^2}}\ dt$

Since $cosat = \frac{e^{iat}-e^{-iat}}{2}$

$M_B\{cosat\} = \frac{v^2}{1+a^2v^4}$

$(x)\quad M_B\ \{sinhat\} = \frac{av^4}{1-a^2v^4}$

$(xi)\ M_B\ \{coshat\} = \frac{v^2}{1-a^2v^4}$

1.3 Theorem: let $U\ (v)$ be Mu'azu- Bayawa integral transform the function $f(t)$ denote by M_B, that is $M_B\{f(t)\} = U$ (v), then:

The integral transform Mu'azu -Bayawa of

$(i)\quad M_B\{f^{'}(t)\}\ = \frac{\mathbf{1}}{\boldsymbol{v^2}}U(v) - f(0)$

$(ii)\ M_B\{f^{''}(t)\} = \frac{\mathbf{1}}{\boldsymbol{v^4}}U(v) - \frac{\mathbf{1}}{\boldsymbol{v^2}}f(0) - f^{'}(0)$

$(iii)\ \ M_B\{f^{n}(t)\} = \frac{1}{v^{2n}}U(v) - \sum_{k=0}^{n-1} v^{-2n+2k+2}f^{(k)}(0)$

Proof:

$(i) M_B\{f^{'}(t)\} = \int_0^{\infty} f(t)\,e^{-\frac{t}{v^2}}dt$

Applying integration by part, we have

$M_B\{f^{'}(t)\} = \frac{\mathbf{1}}{\boldsymbol{v^2}}U(v) - f(0)$

(ii) Let $g(t) = f^{'}(t)$, then

$M_B[g^{'}(t)] = \frac{\mathbf{1}}{\boldsymbol{v^2}}A_B\{f'(t)\} - f(0)$

We find that by using (i)

$M_B\{f''(t)\} = \frac{1}{v^4}U(v) - \frac{1}{v^2}f(0) - f'(0)$

(iii) Can be proved by Mathematical Induction.

1.4 Linearity Property theorem for Mu'azu- Bayawa Transform. Let $f_1(t)$ and $f_2(t)$ are Mu'azu- Bayawa transform of functions $U_1(v)$ and $U_2(v)$, respectively, the Mu'azu- Bayawa transform of $\{cf_1(t) + df_2(t)\}$is given by $\{cU_1(v) + dU_2(v)\}$, where c and d are arbitrary constants.

Proof:

Using the definition, we obtain

$M_B\{f(t)\} = U\,(v) = \int_0^\infty f(t)\, e^{-\frac{t}{v^2}}\, dt$

$M_B\{cf_1(t) + df_2(t)\}$

$= \int_0^\infty [cf_1(t) + df_2(t)]\ e^{-\frac{t}{v^2}}\, dt$

$M_B\{cf_1(t) + df_2(t)\}$

$= c\left\{\int_0^\infty f_1(t)\ e^{-\frac{t}{v^2}}\right\} + d\left\{\int_0^\infty f_2(t)\ e^{-\frac{t}{v^2}}\right\}$

$M_B\ \{cf_1(t) + df_2(t)\}$

$= c\ M_B\{f_1(t)\} + dA_B\{f_2(t)\}$

$M_B\{cf_1(t) + df_2(t)\}$

$= cU_1(v) + dU_2(v)$

1.5 Convolution of two Functions

The Convolution of two functions $f(t)$ and $u(t)$ is denoted by $f(t) * u(t)$ and it is defined by

$f(t) * u(t) = f * u = \int_0^t f(x)\, u(t-x)dx$

$= \int_0^t u(x)\, f(t-x)dx$

1.6 Convolution theorem for Mu'azu- Bayawa transform

Suppose $M_B\{f(t)\} = F(v)$ and $M_B\{u(t)\} = U(v)$ then $M_B\{f(t)\ * u(t)\} = M_B\ \{f(t)\}M_B\{u(t)\} = F(v)U(v)$

Proof:

The key step is to interchange two integrals. Let we start the product of the Mu'azu- Bayawa transforms, i.e $F(v)U(v)$

$F(v) = \int_0^\infty f(h)\, e^{-\frac{h}{v^2}}\, dh$ and $U(v) = \int_0^\infty u(k)\, e^{-\frac{k}{v^2}}\, dk$

$F(v)U(v) = \int_0^\infty f(h)\, e^{-\frac{h}{v^2}} \int_0^\infty u(k)\, e^{-\frac{k}{v^2}}\, dhdk$

$= \int_0^\infty f(h) \int_0^\infty e^{-\frac{(h+k)}{v^2}} u(k)\ dhdk$

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$= \int_0^\infty \int_0^\infty e^{-\frac{(h+k)}{v^2}} f(h)u(k)\, dhdk$

Let substituting $h + k = t,\ k = t - h,\ dk = dt$ and changing order integration

$= \int_0^\infty \int_0^t e^{-\frac{t}{v^2}} f(h)u(t-h)\, dhdt$

$= \left\{ \int_0^\infty e^{-\frac{t}{v^2}} \int_0^t f(h)u(t-h)\, dhdt \right\}$

$= \left\{ \int_0^\infty e^{-\frac{t}{v^2}} (f * u) dt \right\}$

$F(v)U(v) = M_B\{f(t) * u(t)\}$

Therefore $M_B\{f(t) * u(t)\} = F(v)\, U(v)$

1.7 Application Mu'azu -Bayawa transform in to ordinary differential equations

For linear systems that governed by differential equations, Mu'azu- Bayawa integral transform could be used as an efficient tool to analyze their basic characteristics in response into initial data.

The Efficiency Mu'azu- Bayawa can be expressed in solving some certain initial value problems described by ordinary differential equations.

Consider the linear first order ordinary differential equations:

$(i)\ \frac{dy}{dt} + py = f(t)\ , t > 0,\ y(0) = a$

Where a and p are constants and $f(t)$ is an external input function so that $M_B\{f(t)\}$ transform exists.

Now apply Mu'azu- Bayawa transform with initial condition.

$\frac{1}{v^2} U(v) - f(0) + pU(v) = \bar{f}(v)$

$\frac{1}{v^2} U(v) - a + pU(v) = \bar{f}(v)$

$U(v) \left\{ \frac{1}{v^2} + p \right\} = \bar{f}(v) + a$

$U(v) \left\{ \frac{1 + pv^2}{v^2} \right\} = \bar{f}(v) + a$

$U(v) = \frac{v^2(\bar{f}(v) + a)}{1 + pv^2}$

$U(v) = \frac{v^2 \bar{f}(v)}{1 + pv^2} + \frac{v^2 a}{1 + pv^2}$

The Inverse Mu'azu- Bayawa Transform give the solution

(ii) Consider the linear second order ordinary differential equations

For a second order ordinary differential equation that has the general form:

$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x),\ y(0) = a,\ y'(0) = b,\ a\ , b, p$ and q are constants

Apply Mu'azu- Bayawa transform with the initial value problem a, b, p and q would give :

$\frac{1}{v^4} U(v) - \frac{1}{v^2} f(0) - f'(0) + 2p(\frac{1}{v^2} U(v) - f(0)) + qU(v) = \bar{f}(v)$

$$\frac{1}{v^4}U(v) - \frac{a}{v^2} - b + 2p(\frac{1}{v^2}U(v) - a) + qU(v) = \bar{f}(v)$$

$$\frac{1}{v^4}U(v) + \frac{2p}{v^2}U(v) + qU(v) = \frac{a}{v^2} + b + 2ap + \bar{f}(v)$$

$$U(v)\left\{\frac{1}{v^4} + \frac{2p}{v^2} + \mathrm{q}\right\} = \frac{a}{v^2} + b + 2ap + \bar{f}(v)$$

$$U(v)\left\{\frac{1 + 2pv^2 + qv^4}{v^4}\right\} = \frac{a + bv^2 + 2av^2p + v^2f(v)}{v^2}$$

$$U(v) = \frac{v^2(a + bv^2 + 2av^2p + v^2\bar{f}(v)}{1 + 2v^2p + qv^4}$$

$$U(v) = \frac{v^4\bar{f}(v) + v^4b + av^2(1 + 2v^2p)}{qv^4 + 2v^2p + 1}$$

$$U(v) = \frac{v^4\bar{f}(v)}{qv^4 + 2v^2p + 1} + \frac{v^4b}{qv^4 + 2v^2p + 1} + \frac{av^2(1 + 2v^2p)}{qv^4 + 2v^2p + 1}$$

The inverse Mu'azu -Bayawa transform gives the solution

Example 1

Consider the first order ordinary differential equations

$$\frac{dy}{dx} + y = 0, y(0) = 1$$

Apply Muazu -Bayawa transform with initial condition

$$\frac{1}{v^2}U(v) - f(0) + U(v) = 0$$

Where $U(v)$ is the Mu'azu- Bayawa transform of the function $y(x)$

$$\frac{1}{v^2}U(v) - (1) + U(v) = 0$$

$$U(v)\left\{\frac{1}{v^2} + 1\right\} = 1$$

$$U(v)\left\{\frac{1 + v^2}{v^2}\right\} = 1$$

$$U(v) = \frac{v^2}{1 + v^2}$$

Take inverse Mu'azu - Bayawa transform, we have

$$y(x) = e^{-x}$$

Example 2

Consider the first order ordinary differential equation

$$\frac{dy}{dx} + 2y = 0, y(0) = 1$$

Apply Mu'azu- Bayawa transform with initial condition

$$\frac{1}{v^2}U(v) - f(0) + 2U(v) = v^4$$

Where $U(v)$ is the Mu'azu -Bayawa transform of the function $y(x)$

$$\frac{1}{v^2}U(v) - 1 + 2U(v) = v^4$$

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$U(v)\left\{\frac{1}{v^2} + 2\right\} = v^4 + 1$

$U(v) = \frac{v^2(v^4+1)}{1+2v^2}$

Take synthetic division method and inverse Mu'azu - Bayawa Transform, we have

$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$

Example 3

Consider the first order ordinary differential equations

$\frac{dy}{dx} + y = 3, y(0) = 1$

Apply Mu'azu -Bayawa transform with initial condition

$\frac{1}{v^2}U(v) - f(0) + U(v) = 0$

Where $U(v)$ is the Mu'azu - Bayawa transform of the function $y(x)$

$\frac{1}{v^2}U(v) - (1) + U(v) = 3v^2$

$\frac{1}{v^2}U(v) + U(v) = 3v^2 + 1$

$U(v)\left\{\frac{1}{v^2} + 1\right\} = 3v^2 + 1$

$U(v) = \frac{v^2(3v^2+1)}{1+v^2}$

Take synthetic division method and inverse Mu'azu- Bayawa Transform, we have

$y(x) = -2e^{-x} + 3$

Example 4

Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + y = 0\ y(0) = 1, y'(0) = 1$

Apply Mu'azu - Bayawa transform with initial condition

$\frac{1}{v^4}U(v) - \frac{1}{v^2}f(0) - f'(0) + U(v) = 0$

Where $U(v)$ is the Mu'azu -Bayawa transform of the function $y(x)$

$\frac{1}{v^4}U(v) - \frac{1}{v^2}(1) - (1) + U(v) = 0$

$U(v)\left\{\frac{1}{v^4} + 1\right\} = \frac{1}{v^2} + 1$

$U(v)\left\{\frac{1+v^4}{v^4}\right\} = \frac{1+v^2}{v^2}$

$U(v) = \frac{v^4(1+v^2)}{v^2(1+v^4)} = \frac{v^4+v^6}{v^2(1+v^4)}$

$U(v) = \frac{v^4}{v^2(1+v^4)} + \frac{v^6}{v^2(1+v^4)}$

$U(v) = \frac{v^4}{1+v^4} + \frac{v^2}{1+v^4}$

Take inverse Mu'azu - Bayawa transform, we have

$y(x) = sinx + cosx$

Example 5

Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0 \; y(0) = 1, y^{'}(0) = 4$

Now apply Mu'azu - Bayawa transform with initial condition

$\frac{1}{v^4}U(v) - \frac{1}{v^2}f(0) - f^{'}(0) - \frac{3}{v^2}U(v) + f(0)) + 2U(v) = 0$

Where $U(v)$ is theMuazu -Bayawa transform of the function $y(x)$

$\frac{1}{v^4}U(v) - \frac{1}{v^2}(1) - 4 - \frac{3}{v^2}U(v) + 3 + 2U(v) = 0$

$U(v)\,[\frac{1}{v^4} - \frac{3}{v^2} + 2] = \frac{1}{v^2} + 1$

$U(v) = v^2\left\{\frac{1+v^2}{1-3v+2v^4}\right\}$

Apply partial fraction, we find

Now, $\frac{1+v^2}{1-3v+2v^4} = \frac{1+v^2}{(1-2v^2)(1-v^2)} = \frac{A}{1-2v^2} + \frac{B}{1-v^2}$

Therefore $A = 3, B = -2$

$U(v) = v^2\left\{\frac{3}{1-2v^2} - \frac{2}{1-v^2}\right\}$

Take inverse Mu'azu- Bayawa transform, we have

$y(x) = 3e^{2x} - 2e^x$

Example 6 Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + 4y = 12t, \; y(0) = 0, y^{'}(0) = 7$

Now apply Mu'azu- Bayawa transform with initial condition

$\frac{1}{v^4}U(v) - \frac{1}{v^2}f(0) - f^{'}(0) + 4U(v) = 12v^4$

Where $U(v)$ is the Muazu -Bayawa transform of the function $y(x)$

$\frac{1}{v^4}U(v) - \frac{1}{v^2}(0) - 7 + 4U(v) = 12v^4$

$U(v)\;\left\{\frac{1}{v^2} + 4\right\} = 12v^4 + 7$

$U(v) = \frac{12v^8 + 7v^4}{4v^4 + 1}$

Take synthetic division method and inverse Mu'azu - Bayawa

$U(v) = 3v^4 + \frac{4v^4}{4v^4+1}$

$y(x) = 3x + 2sin2x$

II. CONCLUSION

In this paper, we introduced a new integral transform namely Mu'azu - Bayawa Transform for solving ordinary differential equations we demonstrated the applicability and have shown new efficient the method is in solving ordinary differential equations. Mu'azu - Bayawa transform is a simple powerful tool for solving ordinary differential equations without a need too much time consuming and without computational work.

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REFERENCES

[1] L. Debnath, D.Bhatta, Integral Transform and its applications, Third Ediotion, CRC press Tailor and Francis Group, Boca Raton London New York. (2015)

[2] G.K watugala, sumudu transform a new integral transform to solve differential equation and control engineering problem.Math.Engrginduct.6(1998),no 4,319-329

[3] TarigM.Elzaki, The New Integral Transform "Alzaki Transform" Global journal of pure and applied Mathematics ISSN 0973-1768, (2011),N0.1,pp.57-64

[4] K.S Aboodh, The New Integral Transform "Aboodh Transform" Global Journal of Pure and Applied mathematics,9(1),35-43(2013)

[5] AbdelilahKamal.H.Sedeeg The New Integral Transform " Kamal Transform", Advance in Theoretical and Applied Mathematics,vol.11,no.2(2016),pp.451-458

[6] Mohand. M.AbderahimMahgoub .The New Integral Transform "Mahgoub Transform" Advance in Theoretical and Applied Mathematics ISSN:0973-4554 vol.11,No.4(2016).pp.391-398

[7] Shehu M, Weidng Z, New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations,International Journal of Analysis and Application.Vol.17,no.2(2019),167-190.

[8] D.P.Patil, SavitaSantuKhakale, The New Integral Transform "Sohan Transform". International Journal of Advance in Engineering and Management (IJAEM),vol.3,Issue 10,10(2021),pp:126-132,ISSN:2395-5252

[9] Emadkuffi, Sara faith Maktoof, "Emad – Faith transform" a new integral transform. Journal of Interdisciplinary Mathematics ISSN: 0972-05029(print),ISSN:2169-12x(online),vol.24(2021),No.6 pp.2381-2390.

[10] Iman Ahmed Almardy, The New Integral Transform "Iman Transform" International journal of Advanced Research in Science, Communication and Technology(IJARSCT)vol.3,1ssue1,3(2023)

[11] Zayyanu B. Bayawa, Aisha A.Haliru, The New Integral Transform "BayawaTransform" International journal of Advanced Research in Science, Communication and Technology(IJARSCT)vol.4,Issue2,5(2024)