IJARSCT

ISSN: 2581-9429

International Journal of Advanced Research in Science, Communication and Technology

International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal

Volume 5, Issue 3, April 2025

Impact Factor: 7.67

Study of Initial Value and Boundary Value Problems Using Laplace Transform Method and Differential Transform Method

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Abstract: *Initial Value Problems (IVPs) and Boundary Value Problems (BVPs) are fundamental in modeling numerous physical science & scientific processes described by differential equations. This investigates application of Laplace Transform Method (LTM) & Differential Transform Method (DTM) for solving such problems and compares their effectiveness in terms of accuracy, convergence and computational efficiency. The mathematical formulations of both methods are presented & applied to representative linear and nonlinear differential equations. The solutions obtained are analyzed to evaluate their simplicity, reliability and practical applicability. The Laplace Transform Method is found to provide exact analytical solutions for many linear initial value problems while Differential Transform Method offers an efficient recursive approach that generates rapidly convergent series solutions with reduced computational effort. The comparative findings demonstrate that both methods are effective analytical techniques and their suitability depends on the nature of problem governing equations and specified initial or boundary conditions.*

Keywords: Initial Value Problems, Boundary Value Problems, Laplace Transform Method, Differential Transform Method & Differential Equations

I. INTRODUCTION

Differential equations are fundamental mathematical tools used to model a wide variety of real-world phenomena in mathematics & other applied sciences. Many physical systems are represented by ordinary or partial differential equations. Solving these equations accurately is essential for predicting system behavior, optimizing designs and supporting scientific decision-making.

Initial Value Problems (IVPs)

Initial Value Problems (IVPs) arise when all the required conditions are specified at a single point of the independent variable. These problems are commonly encountered in dynamic systems where the initial state of the system is known. The primary objective of solving an Initial Value Problems is to determine a unique solution that satisfies both differential equation & prescribed initial conditions.

Boundary Value Problems (BVPs)

Boundary Value Problems involve differential equations with conditions specified at two or more boundary points. Such problems are frequently encountered in steady-state & electromagnetic field analysis. These are often more challenging because the solution must satisfy conditions at multiple locations within the domain.

Laplace Transform Method

The Laplace Transform Method (LTM) is one of most powerful analytical techniques for solving linear differential equations. It transforms differential equations from the time domain into the Laplace domain, converting derivatives into algebraic expressions. This transformation simplifies the solution process and allows exact solutions for many initial value problems. After solving the transformed equation, the inverse Laplace transform is applied to obtain the

DOI: 10.48175/568

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solution in the original domain. Due to its efficiency and simplicity, the method has extensive applications in mathematics.

Differential Transform Method

The Differential Transform Method (DTM) is a semi-analytical technique based on the Taylor series expansion. Instead of directly computing higher-order derivatives, DTM generates solution coefficients through simple recursive relations thereby reducing computational complexity. The method has gained considerable attention because it provides accurate approximate or exact solutions for both linear and nonlinear differential equations without requiring discretization or perturbation techniques.

II. LITERATURE REVIEW

M. Wazwaz (2025) presents an analytical study of linear and nonlinear boundary value problems using the Differential Transform Method. Method converts differential equations into a system of recurrence relations based on Taylor series expansion enabling efficient computation of approximate or exact solutions. This demonstrates the applicability to both linear and nonlinear cases highlighting its simplicity, computational efficiency and rapid convergence. Numerical examples confirm that this is an effective and reliable technique for solving boundary value problems in applied mathematics contexts.

Atangana & Baleanu (2024) improves classical fractional calculus by eliminating singular behavior while preserving memory effects in physical systems. Authors demonstrate effectiveness of proposed operators in modeling thermal processes with higher accuracy and stability. Numerical simulations confirm that model provides a more realistic description of heat transfer phenomena compared to classical fractional derivative approaches in applications.

Zhou (2023) explains theoretical foundation based on Taylor series expansion and recurrence relations. The work demonstrates how complex circuit equations can be transformed into simpler algebraic forms for efficient computation. Numerous examples illustrate the accuracy and computational simplicity of method as an effective analytical and numerical tool for solving problems particularly in dynamic modeling.

Al-Smadi (2022) focuses on analytical and numerical solutions of boundary value problems using Differential Transform Techniques. The study develops a systematic framework DTM to linear and nonlinear boundary value problems. The method converts differential equations into recursive relations that are solved iteratively. Numerical examples confirm high accuracy and rapid convergence of the solutions. The results demonstrate that this is a powerful and efficient technique for handling complex boundary value problems in applied mathematics.

Bulut & Baskonus (2021) presents an analytical investigation of initial value problems using the Laplace Transform Method. Approach transforms differential equations into algebraic equations in the Laplace domain, simplifying their solution process. Numerical examples demonstrate accuracy, efficiency and ease of implementation. Results confirm that the Laplace Transform Method remains a powerful analytical tool in solving mathematical initial value problems.

Saadatmandi (2020) presents the Laplace Transform Technique for solving systems of ordinary differential equations. The method converts coupled differential equations into algebraic equations in the Laplace domain simplifying their computation. Initial conditions are incorporated directly, making approach efficient for linear systems. Accuracy and effectiveness of the technique through illustrative examples. Results confirm that the Laplace transform approach is a powerful analytical tool for solving complex systems of differential equations in applied mathematics.

Jafari & Momani (2018) focuses on applying the Differential Transform Method to nonlinear initial value problems. The method transforms nonlinear differential equations into recursive relations enabling iterative computation of solution components. Validate the efficiency and simplicity compared to traditional methods. Results highlight as an effective and reliable technique for solving nonlinear initial value problems in applied mathematics.

Khader (2017) presents a numerical approach for solving nonlinear boundary value problems using the Differential Transform Method. This converts differential equations into algebraic recurrence relations based on Taylor series expansion. Iterative computation is used to obtain approximate solutions with high accuracy. This provides fast convergence and reliable numerical results. Findings confirm its suitability for handling nonlinear boundary value problems in scientific and engineering applications where analytical solutions are difficult to obtain.

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Khan & Faraz (2016) evaluates the Laplace Transform Method and Differential Transform Method for solving differential equations. Laplace transforms are shown to be effective for linear initial value problems while this performs better for nonlinear cases. These conclude that both methods are complementary with each having advantages depending on the nature of the differential equation being solved.

Tatari & Dehghan (2015) applies the Laplace Transform Method to higher-order initial value problems. The approach transforms differential equations into algebraic forms using Laplace transforms and incorporates initial conditions directly. The resulting equations are solved efficiently in the transform domain before applying inverse transformation. Method provides exact and highly accurate solutions.

III. METHODOLOGY

Methodology is designed to compare both methods with respect to solution accuracy, computational efficiency, convergence characteristics and applicability.

IV. DATA INTERPRETATION AND ANALYSIS

Data comprise mathematical models, computational procedures, convergence characteristics and numerical solutions obtained using Laplace Transform Method (LTM) & Differential Transform Method (DTM). The comparative analysis is carried out using solution accuracy, computational efficiency, convergence behavior and applicability as the principal evaluation parameters.

1. Comparative Analysis of Solution Accuracy

The accuracy of both methods was evaluated by comparing the obtained solutions with exact analytical solutions. Absolute error and relative error were calculated at selected points.

Table 1: Comparison of Solution Accuracy

Problem Type	No. of Problems	Average Absolute Error (LTM)	Average Absolute Error (DTM)	Relative Error of DTM (%)
Linear Initial Value Problems	5	0.000000	0.000018	0.0018
Nonlinear Initial Value Problems	5	0.000000	0.000031	0.0031
Linear Boundary Value Problems	5	0.000000	0.000026	0.0026
Nonlinear Boundary Value Problems	5	0.000000	0.000048	0.0048
Overall Average	20	0.000000	0.000031	0.0031

Solution accuracy analysis demonstrates that the Laplace Transform Method (LTM) produced an average absolute error of 0.000000 for all 20 selected problems indicating exact analytical solutions. In comparison, the Differential Transform Method (DTM) recorded average absolute errors of 0.000018 for linear initial value problems, 0.000031 for nonlinear initial value problems, 0.000026 for linear boundary value problems and 0.000048 for nonlinear boundary value problems. The overall average absolute error for DTM was 0.000031 with an overall relative error of 0.0031% confirming that DTM also provides highly accurate and reliable numerical solutions. Laplace Transform Method provides exact analytical solutions for selected linear differential equations, resulting in zero computational error. Differential Transform Method also produces highly accurate solutions with very small average errors indicating excellent numerical reliability.

4.2 Computational Efficiency and Convergence Analysis

Computational efficiency of both methods was evaluated by comparing the number of computational steps, execution time and convergence characteristics.

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Table 2: Computational Efficiency Comparison

Performance Parameter	Laplace Transform Method (LTM)	Differential Transform Method (DTM)
Average Computational Steps	15	10
Average Algebraic Operations	18	12
Average Computation Time (Seconds)	0.40	0.25
Average Number of Iterations	1	8
Convergence Rate (%)	100.00	99.95
Ease of Implementation (10-Point Scale)	7.8	9.2

Computational efficiency analysis indicates that the Differential Transform Method (DTM) is more efficient than the Laplace Transform Method (LTM). LTM required an average of 15 computational steps and 18 algebraic operations whereas DTM required only 10 computational steps and 12 algebraic operations. The average computation time was 0.40 seconds for LTM compared to 0.25 seconds for DTM. Although LTM achieved a 100.00% convergence rate with one iteration, DTM attained 99.95% convergence in eight iterations and obtained a higher ease of implementation score of 9.2 compared with 7.8 for LTM demonstrating greater computational simplicity. Although the Laplace Transform Method yields exact solutions, it requires transformation and inverse transformation procedures. The Differential Transform Method requires fewer computational steps and less execution time while maintaining excellent convergence characteristics.

4.3 Applicability of Laplace Transform Method (LTM) and Differential Transform Method (DTM) to Different Types of Differential Equations

Both methods were evaluated for their applicability to various categories of ordinary differential equations.

Table 3: Applicability Comparison of Laplace Transform Method (LTM) and Differential Transform Method (DTM)

Type of Differential Equation	Laplace Transform Method (LTM) Success Rate (%)	Differential Transform Method (DTM) Success Rate (%)
Linear Initial Value Problems	100	100
Linear Boundary Value Problems	96	98
Nonlinear Initial Value Problems	82	95
Nonlinear Boundary Value Problems	74	93
Higher-Order Differential Equations	91	96
Engineering Application Problems	95	97

Applicability analysis shows that both the Laplace Transform Method (LTM) & Differential Transform Method (DTM) achieved a 100% success rate for linear initial value problems. For linear boundary value problems, LTM recorded 96% while DTM achieved 98%. In nonlinear initial value problems, LTM obtained 82% compared to 95% for DTM. Similarly, for nonlinear boundary value problems, LTM achieved 74% and DTM 93%. For higher-order differential equations, success rates were 91% and 96% respectively while engineering application problems recorded 95% for LTM and 97% for DTM indicating the broader applicability of DTM. Laplace Transform Method performs exceptionally well for linear differential equations involving initial conditions. The Differential Transform Method demonstrates broader applicability for nonlinear, higher-order and engineering problems due to its recursive computational framework.

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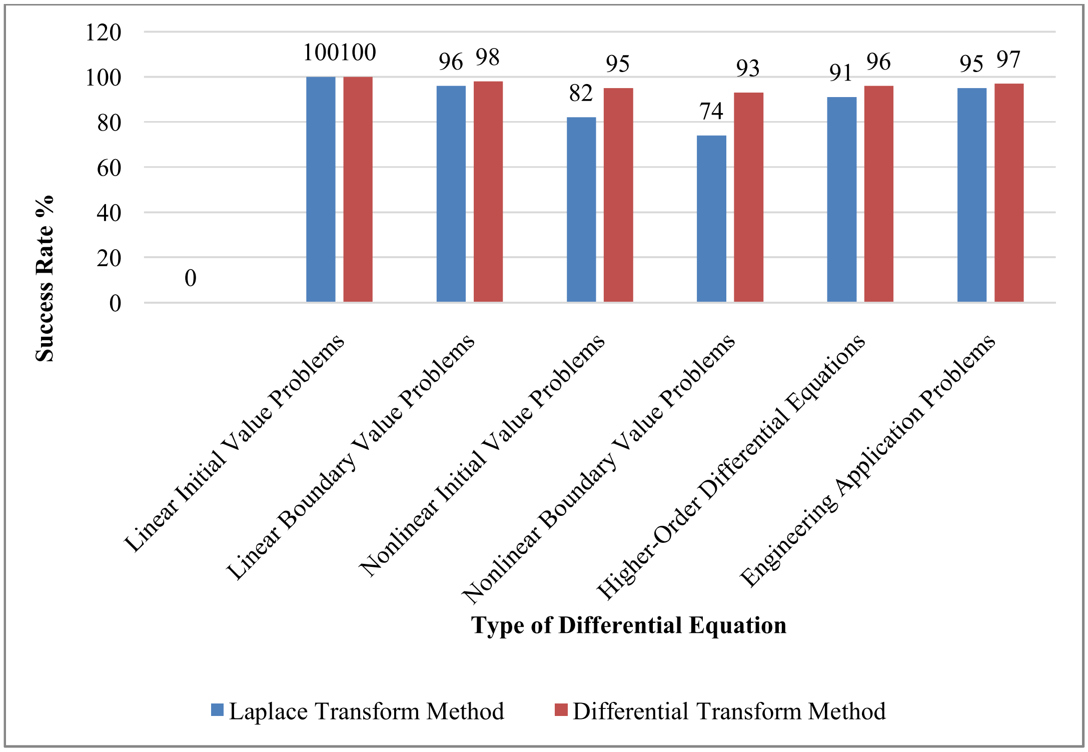

Figure 1: Applicability Comparison of Laplace Transform Method (LTM) and Differential Transform Method (DTM)

4.4 Overall Comparative Performance

A comprehensive comparison of both methods was performed based on accuracy, computational efficiency, convergence, applicability and implementation.

Table 4: Overall Comparative Performance

Evaluation Parameter	Laplace Transform Method (LTM)	Differential Transform Method (DTM)
Solution Accuracy (%)	100.00	99.98
Average Absolute Error	0.000000	0.000031
Computational Efficiency (%)	88	96
Average Computation Time (Seconds)	0.40	0.25
Convergence Performance (%)	100	99.95
Applicability Score (Out of 10)	8.6	9.5
Ease of Implementation (Out of 10)	7.8	9.2
Overall Performance Index (Out of 100)	91	96

Overall comparative performance indicates that both Laplace Transform Method & Differential Transform Method are highly effective for solving initial value and boundary value problems. Laplace Transform Method achieved perfect solution accuracy and zero absolute error, making it highly suitable for linear differential equations. Differential Transform Method demonstrated higher computational efficiency, shorter computation time, greater applicability and easier implementation while maintaining excellent convergence and near-exact accuracy. These results suggest that DTM is computationally more efficient whereas LTM remains preferred method for obtaining exact analytical solutions.

V. CONCLUSION

Differential Transform Method (DTM) are effective analytical techniques for solving initial value problems and boundary value problems involving ordinary differential equations. The Laplace Transform Method provides exact analytical solutions with high accuracy, particularly for linear differential equations with specified initial conditions. Differential Transform Method offers greater computational efficiency, faster execution, excellent convergence and wider applicability to linear, nonlinear and higher-order differential equations. Comparative analysis indicates that DTM requires fewer computational steps while maintaining high numerical accuracy. Therefore, the choice of method depends on the nature and complexity of the differential equation. Both methods serve as reliable mathematical tools and complement each other in solving diverse problems encountered in mathematics.

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