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A Review of Graph Labeling Methods in Combinatorics

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Abstract: *We discuss a few items on various areas of graph labelling in this study, Labelled graphs serve as helpful mathematical models for a wide range of applications such as coding theory, including the design of good types codes, synch-set codes, missile guidance codes, and convolutional codes with optimal autocorrelation properties. Labelling a graph is the assignment of integers to the vertices or edges or both, subject to certain conditions that have been motivated by practical problems. In this study, we explore the neighbourhood prime labelling of the crown graph, the route union of a limited number of copies of the growth of the crown graph, and magic labelling of type (1, 1, 1) for wheels and subdivided wheels. Next, we demonstrate that a wheel allows for magic labelling of type (0, 1, 1).*

Keywords: Graph Labelling, Problems, Crown graph, Neighbourhood prime labelling, Magic labelling of type (a, b, c).

I. INTRODUCTION

In order to solve the Konigsberg bridge puzzle on the Russian river Pregel in 1735, Leonhard Euler developed the earliest version of graph theory. The proposed solution to the issue takes the shape of edges and points, often known as Eulerian graphs, which are connected graphs with even-numbered points as nodes. The study and development of various types of labelling are motivated by the graph theory's fascination with the topic of graph labelling [1]. A finite undirected graph without loops or multiple edges is referred to as a graph with the formula $G = (V, E)$. n and m, respectively, stand for the order $|V|$ and size of $|E|$. We cite Chartrand and Lesniak [2] when referring to graph theoretic terminology. We cite Gallian's dynamic survey [3] for basic descriptions of the various types of graph labelling.

One of the topics in a branch of mathematics known as discrete mathematics is graph theory. In discrete mathematics, the issues and approaches to solving them differ fundamentally from those in constant mathematics. While in constant mathematics we "measure" the sizes of the articles, in discrete mathematics we "check" their numbers. While continuous mathematics has long since dominated the history of mathematics, discrete mathematics was first developed as soon as man learned how to perform calculations. Beginning in the 20th century, this perception began to shift. The modification in how mathematics was created was the most notable advance. Its primary problem changed from the concept of a number to the concept of a set, which was progressively more consistent with discrete mathematics approaches than with those of consistent mathematics. The rising use of PCs in public life was the second spectacular point. Discrete mathematics concepts are heavily used in software engineering theory. There are a staggering number of applications for graph theory, a member of the discrete mathematics family, in software engineering, as well as in a wide range of other sciences designing, and commerce. Figure 1 displayed some of the important graph theory issues.

This study aims to give a few outcomes in a class of problems arranged as Graph labelling. Leave G alone an undirected graph without loops or twofold associations between vertices. In labelling (valuation or numbering) of a graph G, we partner unmistakable nonnegative whole numbers to the vertices of G as vertex labels (vertex esteems or vertex numbers) so that each edge gets a particular positive whole number as an edge name (edge worth or edge number) contingent upon the vertex labels of vertices which are occurring with this edge. Enthusiasm for graph labeling started in the mid-1960s with a guess by Kotzig-Ringel and a paper by Rosa[90].

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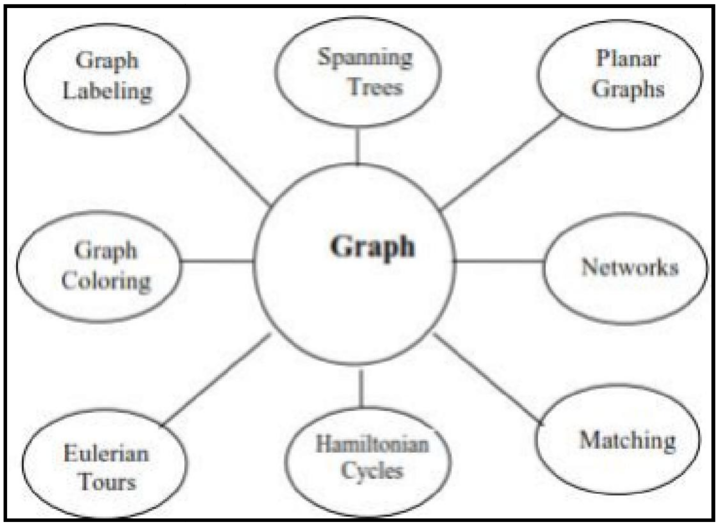

Fig: Some game theory

In 1967, Rosa distributed a spearheading paper on graph labeling problems. He called a function f a β-labelling of a graph G with n edges (Golomb [4] along these lines called such labeling graceful and this term is presently the well-known one) if f is an infusion from the vertices of G to the set {0, 1, …, n} to such an extent that, when each edge is labeled with the supreme estimation of the contrast between the labels of the two end vertices, the subsequent edge labels are particular. This labeling gives a successive labeling of the edges from 1 to the number of edges. Any graph that can be gracefully labeled is a graceful graph.

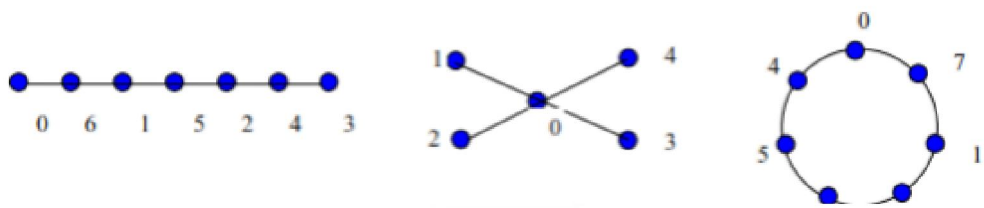

Fig 2: Examples of graceful labelling of graphs

Graph labelling is a value (with integers) at the point or side of the graph or both so that it meets certain conditions. The labels used are positive integers or natural numbers. Graph labelling was first introduced in the late 1960s by Rosa. Furthermore, labelling whose domain is a set of points, a set of sides, or both is usually referred to as point labelling, side labelling, and total labelling. Until now there are several types of graph labelling discussed, including graceful labelling, harmony labelling, irregular total labelling, magic labelling, radiolabelling, anti-magic labelling, and many other graph labelling's [5]. One type of graph labelling that is widely studied and developed is magic labelling.

The idea of labelling was first introduced by Sedlacek, then formulated by Kotzig and Rosa [6]. Wallis [7] defines magic labelling as the one-to-one mapping of graph elements, that is, a set of points and sets of sides to a set of positive integers, where there is a magic sum. Then Stewart explains that magic labelling is called super magic labelling if the smallest label is placed at the dot [8]. In the development of magic labelling, it is also known that the labelling of the total magic point, the total labelling of the super magic point, total labelling of the magic side, and total labelling of the super magic side. Total labelling of the magic side is a magic labelling that has been studied extensively, in more than 50 classes of graphs it has been proven to have total labelling of the magic side. From the development of total magic side labelling, it has been found a magic blanket labelling as follows. Let $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ be a finite simple graph. Gutierrez and Llado defined edge-covering of G as a family of different subgraphs$\boldsymbol{H}1, \ldots ,k$ such that any edge of E belongs to at least one subgraph$\boldsymbol{H}$i , $\boldsymbol{1 \le i \le k}$. Then, it is said that G admits an $(\boldsymbol{H}1, \ldots ,k) - (\boldsymbol{edge})$ covering. If every $\boldsymbol{H}$i is isomorphic to a given graph H, then we say that G admits an H-covering[9]. This paper will discuss the magic covering of a simple graphs and edge magic labeling on a simple graph. The purpose of this research is a secret sharing scheme

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with magic covering on domino graphs. The prime labelling concept was introduced by Roger Entringer and a detailed discussion is in the paper [4]. Let $G = ((G), (G))$ be a graph with p vertices. A bijection $f: V(G) \to \{1,2,3, \ldots \ldots \ldots .p\}$ is called prime labelling if for each edge $e = uv$, $gcd(f(u), f(v)) = 1$. A graph that admits prime labelling is called a prime graph. In [5] proved that the graph obtained by the path union of the finite number of copies of the crown graph * Cn is a prime graph for all n ≥ 3

CROWN GRAPHS: Definition: Let $G = ((G), (G))$ be a graph with p vertices. A bijection $f: V(G) \to \{1,2,3, \ldots \ldots \ldots .p\}$ is called neighbourhood prime labelling if for each vertex $v\epsilon V(G)$, with $deg(v) > 1$, $gcd(f(u), u\epsilon N(v)) = 1$.A graph that admits neighbourhood prime labelling is called a neighbourhood prime graph. The neighbourhood of v denoted by (v) is the set of all vertices in G which are adjacent to v. Let G be a graph in which every vertex is of degree at most 1, then G is a neighbourhood prime graph. If there exists a vertex $v0(G)$ of degree $n-1$, then G is a neighbourhood prime graph. Definition: The crown graph * Cn is obtained from a cycleby attaching a pendant edge at each vertex of the n cycle. Theorem: The crown graph * Cn is a neighbourhood prime labelling. Proof:The degree of the vertices og a crown graph is either 3 or 1. Consider vertices set n v ,v ,.....v 1 2 with degree 3 and ' ' 2 ' 1 , ,..... n v v v be the vertices with degree 1.

The edge of the crown graph is $\{v_x v_x^{'} / 1 \le x \le n\} \bigcup \{v_x v_{x+1} / 1 \le x \le n-1\} \bigcup \{v_1 v_n\}$.

Here $|V(G)| \& |E(G)| = 2n$,here n is a Z^+ Consider v be any arbitrary vertex of G, then the degree of v is either 1 or 3. Illustration 1: When v of degree 1

Let $v = v_i^{'}$ for some $i \in \{1,2,....,n\}$, Consider the labelling $f: V(G) \to \{1,2,....,2n\}$ as follows

$$f(v) = f(v_i^{'}) = 1$$
$$f(v_x^{'}) = \{$$
$$2(n+x-i+1), x = 1,2,....,i-1.$$
$$2(x-i+1), x = i+1, i+2,.....n$$
$$f(v_i) = 2$$
$$f(v_x) = \{$$
$$2(n+x-i)+, x = 1,2,....i-1.$$
$$2(x-i)+1, x = i+1, i+2,.....i+n.$$

Clearly f is injective mapping In the crown graph ' i v are vertices of degree 1.So, to check the neighborhood prime labeling on vi only. Consider the arbitrary vertex vj of degree 3 for $j \neq i-1$. There exists at least two vertices in the neighborhood of vj have consecutive integers are labeling and they are relatively prime. Also for the vertex 1−i v the neighborhood contains the vertex i v and v . The labeling on i v is 2 and the labeling on is an odd integer. Then, the god of the labeling on the neighborhoods vertices of v is one. So f is a neighborhood prime labeling. Illustration 2: when v is of degree 3.

Let $v = v_i$ for some $i \in \{1,2,....,n\}$ consider the labelling $f: V(G) \to \{1,2,.....,2n\}$ as follows (v) =

$$f(v) = f(v_i) = 1$$
$$f(v_x) = \{$$
$$2(n+x-i)+1, x = 1,2,.....,i-1.$$
$$2(x-i)+1, x = i+1, i+2,.......,n$$
$$f(v_i^{'}) = 2$$
$$f(v_x^{'}) = \{$$
$$2(n+x-i+10, x = 1,2,.....i-1.$$
$$2(x-i+1), x = i+1, i = 2,......n.$$

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Clearly f is an injective map. Consider the arbitrary vertex j v of degree 3 for $j \neq i-1$. There exists at least two vertices in the neighbourhood of j v have consecutive integers are labelling and they are relatively prime. Also for the vertex 1–i v the neighbourhood contains the vertex vi and the labelling on i v is 1. Then, the gcd of the labelling on the neighbourhood vertices of v is one. So f is a neighbourhood prime labelling. Theorem: Let G be the graph obtained by the path union of finite number of copies of the crown graph * Cn .Then G is a neighbourhood prime graph for $n \geq 3$. Proof: Let G G Gk , ,..... 1 2 be k copies of the crown graph * Cn and G be the path union of the k copies of * Cn . Let us denote the vertices of the cycle on Gi as i i in x , x ,...., x 1 2 for $1 \leq i \leq k$ and the pendant vertices attached to the above vertices in Gi as ' ' 2 ' 1 , ,.... i i in x x x respectively. The edge joining Gi and 1+Gi are denoted as 1)1+i1 (i x x for $i = 1,2, \ldots \ldots \ldots \ldots k-1$. The total number of vertices in G is $2nk$. Consider the labelling {1,2,,.....2nk}→f :V(G) as follows

$$f(x_{ij}) = 2j-1, 1 \leq j \leq n$$

$$f(x_{ij}) = 2n(i-1)+2j-1, 2 \leq i \leq k, 1 \leq j \leq n$$

$$f(x_{ij}^{'}) = f(x_{ij})+1$$

Clearly f is an injective map.

In the graph$G$, the vertices ' ij x have degree 1. So, to check the neighbourhood prime labelling condition on ij x only. The neighbourhood vertices for any vertex ij x contains the vertices ' ij x and 1)+i(j x . The labelling on these vertices are consecutive integers and they are relatively prime.The gcd of labelling on the neighbourhood of all the vertices in G is one. So, G is a neighbourhood prime graph. GRAPH DECOMPOSITION: Definition 1: A decomposition of a graph G is a family H = (H1, H2, … ,Hn) of sub graphs of G such that each edge of G is contained in exactly one member of H. In fact, G is the edge disjoint union of its subgraphs Hi

$$E(H_i) \cap E(H_j) = 0 \quad for\, i \neq j \; E(G)$$

$$= \cup E(H_i) \; i = 1,2,\ldots\ldots, n; \; V(G)$$

$$= \cup V(H_i) \; i = 1,2,\ldots\ldots, n.$$

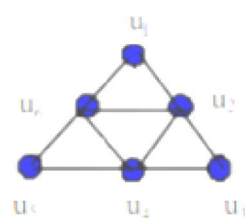

Fig 3: Decomposition of a graph.

Definition 2: Let two graphs G and G' be given. A G-decomposition of a graph G' is a decomposition of G into subgraphs isomorphic to G. In other words, each member Hi in definition 2. must be isomorphic to G. We writewhenever a G-decomposition of G′ exists. The decomposition of graph G in Figure 3 is a K3-decomposition, i.e., K_3/G .

Definition 3: A decomposition H of a graph G into subgraphs $H_1, H_2, ...H_n$ is said to be cyclic if there exists an isomorphism f of G which induces a cyclic permutation fv of the set V(G) and satisfies the following implication: if $H_i \in H$ thenfor i = 1,2, … ,n. Here $f(H_i)$ is the subgraph of G with vertex set $\{f(u); u \in V(H_i)\}$ and edgeset $\{(f(u), f(v); e = (u,v) \in E(H_i)\}$.

Magic Covering:

Let T be the sum of labels of all vertices, the sum of labels of all edges is s, and the magic sum of star magic covering is m(f) . In Figure 4, the vertices are labelled by $a, b, ..., f$ and the edges are labeled by $g, h, ..., m$ The labels of vertices and edges are a positive integer between 1 to $|V(G) + E(G)|$. In this case, we have a domino graph. The basic idea is to determine the label of each vertex, and then the labels on each edge are interchanged to find cycle magic covering. The label of the edges is obtained from the remainder of the vertices labels. To get cycle magic covering, first, we search all possible T to label the vertices. There is no guarantee every combination of T has cycle magic covering.

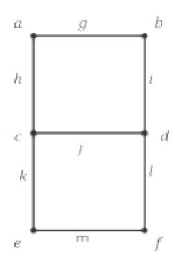

Fig 4: Domino Graph

G is a domino graph if every vertex of G is contained in at most two maximal cliques of G [10]. Here is an illustration to find T, s, and m(f) . There are 2 C_4-cycle magic covering on a Domino graph in Figure 5 The constant $m(f)$ can be obtained by adding all labels on star magic covering as follows.

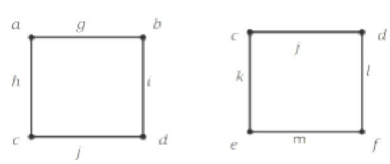

Fig 5: C4- Cycle Magic Covering

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$$2m(f) = (a + b + ... + m) + (c + d + j)$$

$$m(f) = \frac{a + b + ... + m}{2} + \frac{T}{2}$$

$$m(f) = \frac{91}{2} + \frac{T}{2}$$

From Equation, T is a vertex label sum mod 2. To obtain a (f), first, we combined all of possible T. However, not all combination labels on T has cycle magic covering. To find all possible labelling, we also use a duality [11]. Given cycle magic covering is λ, its dual labellingλ' is defined by $\lambda' = v + e + 1 - \lambda(x)$ for every vertex x and $\lambda'(x, y) = v + e + 1 - \lambda(x, y)$ for every edge $\{x, y\}$. Proposition: If there is no cycle magic covering on domino graphs, then the sum of labels of vertices T is odd. critical set of a graph G with a labellingλ is a set $Q\lambda(G) = \{(a, b)|a, b\epsilon\{1,2, \dots, |V(G) \cup E(G)|\}\}$ with ordered pair (a, b) represents label b in position a, which satisfy: λ is the only labelling of G which has label b in position a.¬ No proper subset of $Q\lambda(G)$ satisfies (1).¬ If a critical set has c members, thus it has size c In this case, we search a $C4$-cycle magic covering on domino graph with $T = 21$, $(f) = 56$, and $s = 70$, and construct its dual labelling Figure. Here are the steps to get a secret sharing. First, we determine the label of each vertex of the combination of label T, for example, the set of the label of the

Vertices obtained is $\{1,2,3,4,5,6\}$. Second, we label each edge with labels not used by T, for example, the label of an edge 7, 8, 9, 10, 11, 12, and 13

Third, we interchange each label of edge, until we get cycle magic covering. In the process of interchanging the labels of edges, we can obtain more than one label. Next, we will search for dual labelling.

Once a critical set is found, it will be used to devise a secret sharing scheme [12]

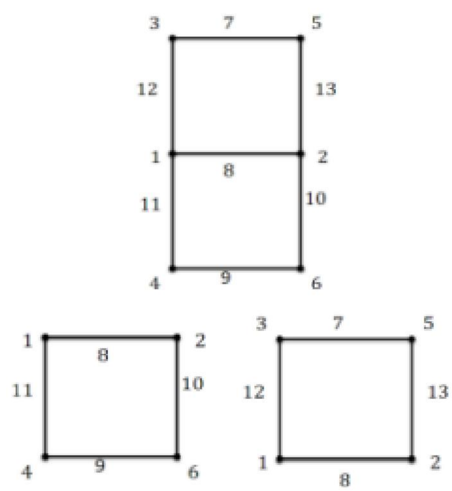

Fig 6:C4- cycle magic covering on domino graph with T=21, m (f)=56, and s=70.

A method of distributing secret data among a set of participants are a secret sharing scheme. So that only specified qualified subsets of participants can recover the secret [13]. Also, if the unqualified subsets collectively yield no extra information, i.e., the scheme called perfect if the joint shares are statistically independent of the secret. The access structure is the subset of qualified subsets among all possible subsets of participants. In this paper, we propose a secret sharing scheme with magic covering on domino graphs.

Magic labeling of type (a,b,c):

A labelling of a graph is a map that carries graph elements to numbers. Assume that a, b, c ∈ {0, 1}. A magic labelling of type (a, b, c) of a planar graph G with face set F is bijective function from $V \cup E \cup F$ onto the integers $\{1,2,\ldots..av+be+cf\}$ with the property that for every integer s, all s-sided faces have the same weight. The weight of a face is the sum of the labels (if present) of the face, the vertices and the edges surrounding that face. A magic type method for labelling the vertices, edges and faces of a planar graph was introduced by Lih [14]. Labelling's of type (1, 0, 0), (0, 1, 0) and (0, 0, 1) are also called vertex, edge and face labelling's respectively.

For n ≥ 3, $W_n \cong C_n + \{a\}$ denotes the wheel with a rim of order n.

$$\begin{aligned} v &= |V(W_n)| \\ \text{Let } e &= |E(W_n)| \, and \\ f &= |F(W_n)| \end{aligned}$$

then v = n +1, e = 2n and f = n +1.

We denote the vertex, edge and interior face-sets of $G \cong W_n$ as follows:

$$V(G) = \{a\} \cup \{v_i : 1 \le i \le n\}$$
$$E(G) = (av_i : 1 \le i \le n\} \cup \{1 \le i \le n\}$$
$$F(G) = \{f_i = av_i v_{i+1} a : 1 \le i \le n\} \cup f_{ext}.$$

where all indices are taken modulo n. Graph Wn consist of n 3-sided faces and one external n-sided face. Theorem: For 3≥n ,Wheel Wn admits a magic labelling of type (0,1,1)

Proof: To prove the result, let us define a labelling $\lambda : E(W_n) \cup F(W_n) \to (1,2,\ldots.3n+1\}$ as follows:

$$\lambda(v_i v_{i+1}) = 2n - i + 1$$
$$\lambda(av_i) = i$$
$$\lambda(f_i) = \begin{Bmatrix} 3n, if i = n \\ 3n - i, if 1 \le i \le n-1. \end{Bmatrix}$$
$$\lambda(f_{ext}) = 3n + 1.$$

It is clear that for every interior 3-sided face;

$f_i : 1 \le i \le n$, the sum of all edge and face labels is 5n+2.

Hence for $n \ge 3$, wheel admits a magic labelling of type (0,1,1).

Theorem:

For any odd , $n \ge 3$ Wheel W_n admits a mafic labelling of type (0,1,0).

Proof: In order to prove our result, let us define a labelling $\lambda : E(W_n) \to \{1,2,\ldots.2n\}$ as follows:

$$\lambda(v_i v_{i+1}) = 2n - i + 1$$
$$\lambda(av_i) = i$$
$$\lambda(f_i) = \begin{Bmatrix} 3n, if i = n \\ 3n - i, if 1 \le i \le n-1. \end{Bmatrix}$$
$$\lambda(f_{ext}) = 3n + 1.$$

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It is clear that for every interior 3-sided face: n,≤ i ≤1 the sum of all edge labels is $\frac{7n+3}{2}$. Hence for odd 3≥n ,wheel Wn admits a magic labeling of type (0,1,0). Like-wise we are proving different kind of theorems.

II. CONCLUSION

In this paper we presented some label ling's of graphs and path continuation and finally explains about labeling of the type (a,b,c) for wheels.

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