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Impact of Strong Domination on Connectivity in Fuzzy Graphs

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Abstract: *This study uses membership values of strong arcs in fuzzy graphs to establish the idea of strong dominance number. Bounds for the strong dominance number of fuzzy graphs are derived, and the strong domination number Ys of complete fuzzy graphs and complete bipartite fuzzy graphs are found. Additionally, the correlation between a fuzzy graph's strong dominance number and its complement is examined.*

Keywords: Fuzzy graph, Strong arcs, Weight of arcs, Strong domination

I. INTRODUCTION

Rosenfeld presented fuzzy graphs by describing and establishing some of the features of the fuzzy analogue of key graph theoretic notions, including routes, cycles, trees, and connectedness. The notion of strong arcs has been presented by Bhutani and Rosenfeld.

The following people have also completed many works on fuzzy graphs: Akram, Samanta, Dudek, Davvaz, Sunitha, Pal, and Pramanik. In the 1850s, the study of dominant sets in graphs began with a simple chess issue.

The topic of finding the bare minimum of queens to put on a chessboard to ensure that every square is either occupied or assaulted by a queen was discussed by European chess aficionados. Ore and Berge presented the idea of dominance in graphs in 1962, and Cockayne and Hedetniemi investigated it further. By employing effective edges, Somasundaram & Somasundaram established dominance in fuzzy graphs. Nagoor Gani and Chandrasekaran addressed dominance in fuzzy graphs with strong arcs. For a fuzzy graph G, they define a dominant set as a set D of its nodes such that each node in V - D has at least one strong neighbor in D. Additionally, they provided two definitions for the dominance number of G: the smallest scalar cardinality of any D and the minimum number of nodes in any D. In order to further decrease this parameter, we define the dominance number of the fuzzy graph in this study utilizing the weights of strong arcs.

The structure of this document is as follows. Preliminaries are presented in Section 2, and a traditional definition of a fuzzy graph's strong dominance number is provided in Section 3. The weight of the weakest arc in the fuzzy graph represents the strong dominance number of the whole fuzzy graph. It is possible to derive a necessary and sufficient condition that guarantees the magnitude of a fuzzy graph G's strong dominance number. A maximum value is determined by adding the strong dominance number of a fuzzy graph to its complement. The strong dominance number of fuzzy networks without isolated nodes is shown to have an upper limit. Additionally, an upper limit is given for the product of a fuzzy graph's strong dominance number and its complement. Finally, utilizing minimum arc strength, order, and maximum strong neighborhood degree, a lower limit and an upper bound for the strong dominance number of fuzzy graphs are derived.

Preliminaries

Graphs are recognized to be nothing more than models of relations. An easy approach to display information about the relationships between items is via a graph. Vertices and edges indicate the objects and relations, respectively. We naturally need to create a "fuzzy graph model" when there is ambiguity in the item description, its connections, or both. A concise summary of the fundamental definitions of fuzzy graphs is provided in.

A fuzzy graph is denoted by G : (V,cr,µ) where Vis a node set, er andµ are mappings defined as er : V ➔ [0, 1] andµ : V x V ➔ [0, 1], where er andµ represent the membership values of a vertex and an edge respectively. For any fuzzy graph µ(x, y) ;:; min{cr(x), cr(y)}. We consider fuzzy graph G with no loops and assume that Vis finite and non-empty,µ is reflexive (i.e., µ(x, x) = cr(x) for all x) and symmetric (i.e., µ(x,y) = µ(y, x) for all (x,y)). In all the examples, er is chosen suitably. Also, we denote the underlying crisp graph by G*: (er*,µ') where er*= {u EV I cr(u) > 0} andµ* = {(u, v) E V x V I µ(u, v) > O}. Throughout we assume that er* = V. The fuzzy graph H : (T, v) is said to be a partial fuzzy subgraph of G : (er,µ) if v <;;, µ and T <;;, er. In particular, we call H : (T, v) a fuzzy subgraph of G : (er,µ) if T(u) = cr(u) for all u ET* and v(u, v) = µ(u, v) for all (u, v) Ev*. A fuzzy graph G : (V,cr,µ) is Fuzzy Inf. Eng. (2015) 7: 369-377 371 called trivial if lcr*I = 1. Two nodes u and v in a fuzzy graph Gare said to be adjacent if µ(u, v) > 0.

A path P of length n is a sequence of distinct nodes uo, u1, Un such that µ(u;-1, u;) > 0, i = 1, 2, n and the degree of membership of a weakest arc is defined as its strength. If uo = Un and n c'. 3, then P is called a cycle and P is called a fuzzy cycle if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and is denoted by CONNG(x,y).

A fuzzy graph G: (V, er,µ) is connected if for every x,y in er*, CONNG(x,y) > 0. An arc (u, v) of a fuzzy graph is called an effective arc (M-strong arc) if µ(u, v) = cr(u) II cr(v). Then u and v are called effective neighbors. The set of all effective neighbors of u is called effective neighborhood of u and is denoted by EN(u). A fuzzy graph G is said to be complete if µ(u, v) = cr(u) II cr(v) for all u, v E er*. The order p and size q of a fuzzy graph G: (er,µ) are defined to be p = Lxcv cr(x) and q = L(x,y)cvxv µ(x,y).

Let G : (V, er,µ) be a fuzzy graph and S \: V. Then the scalar cardinality of S is defined to be Lves cr(v) and it is denoted by IS 1,- Let p denotes the scalar cardinality of V, also called the order of G. The complement of a fuzzy graph G, denoted by G is defined to be G = (V, er,µ) where µ(x,y) = cr(x) II cr(y) - µ(x,y) for all x,y EV.

An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. Depending on CONNG(x,y) of an arc (x,y) in a fuzzy graph G, Mathew and Sunitha defined three different types of arcs. Note that CONNG-(x.y)(x,y) is the strength of con nectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x,y). An arc (x,y) in G is a-strong if µ(x,y) > CONNG-(x,yi(x,y). An arc (x,y) in G is ,(:l-strong if µ(x,y) = CONNG-(x,y)(x,y). An arc (x,y) in G is 8-arc if µ(x,y) < CONNG-(x,y)(x,y).

Thus an arc (x,y) is a strong arc if it is either a-strong or f:l-strong. A path Pis called strong path if P contains only strong arcs. If µ(u, v) > 0, then u and v are called neighbors. The set of all neighbors of u is denoted by N(u). Also vis called strong neighbor of u if arc (u,v) is strong. The set of all strong neighbors of u is called the strong neighborhood of u and is denoted by N,(u). The closed strong neighborhood N, is defined as N, = Ns(u) U {u}.

The strong degree of a node v E V is defined as the sum of membership values of all strong arcs incident at v. It is denoted by d,(v). That is d,(v) = LucN,(v) µ(u, v). The minimum strong degree of G is 8,(G) = /\ {d,(v) Iv E V} and maximum strong degree of G is li,(G) = V {d,(v) I v E V}.

The strong neighborhood degree of a node vis defined as dsN(v) = LucN,(v) cr(u). The minimum strong neighborhood degree of G is 8SN(G) = /\ {dSN(v) Iv EV} and the maximum strong neighborhood degree of G is liSN(G) = V {dsN(v) I v E V}.

A fuzzy graph G is said to be bipartite if the vertex set V can be partitioned into two non empty sets Vi and V2 such that µ(vi, v2) = 0 if vi, v2 E Vi or vi, v2 E V2. Further if µ(u, v) = cr(u) cr(v) for all u E Vi and v E V2, then G is called a complete bipartite graph and is denoted by K<ri,<r,, where cr1 and cr2 are respectively the restrictions of cr to V1 and V2.

A node u is said to be isolated if µ(u, v) = 0 for all v * u.

Strong Domination in Fuzzy Graphs

The concept of domination in graphs was introduced by Ore and Berge in 1962 and further studied by Cockayne and Hedetniemi.

We refer to for the terminology of domination in crisp graphs .

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For a vertex v of a graph G : (V, E), recall that a neighbor of v is a vertex adjacent to v in G. Also the neighborhood N(v) of vis the set of neighbors of v. The closed neighborhood N is defined as N[v] = N(v) U {v}. A vertex v in a graph G is said to dominate itself and each of its neighbors, that is, v dominates the vertices in N[v]. A set S of vertices of G is a dominating set of G if every vertex of V(G) - S is adjacent to some vertex in S. A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the domination number of G and is denoted by y(G).

These ideas are extended to fuzzy graphs using strong arcs as follows.

Nagoor gani and Chandrasekaran introduced the concept of domination using strong arcs. These concepts have motivated researchers to reformulate some of the concepts in domination more effectively. The studies in is our main motivation and we have modified the definition of domination number of a fuzzy graph. This modification is required due to the fact that the parameter 'domination number' de fined by Nagoor gani and Chandrasekharan is inclined more towards graphs than to fuzzy graphs. Using the new definition of domination number we have reduced the value of old domination number and extracted classic results in a fuzzy graph. According to Nagoor gani a node v in a fuzzy graph G is said to strongly dominate itself and each of its strong neighbors, that is, v strongly dominates the nodes in N, [v]. A set D of nodes of G is a strong dominating set of G if every node of V(G) - Dis a strong neighbor of some node in D. They defined a minimum strong dominating set in a fuzzy graph G as a strong dominating set with minimum number of nodes. Also in Nagoor gani defined a minimum strong dominating set as a strong dominating set of minimum scalar cardinality. The scalar cardinality of a minimum strong dominating set is called the strong domination number of G. The concept of strong domination in fuzzy graphs has applications to several fields. Strong domination arises in fuzzy location problems in networks. In such applica tions, the membership values of strong arcs in fuzzy graph give more optimum re sults for strong domination number than using membership values of nodes. Hence we have modified the definition of strong domination number using membership val ues of strong arcs and extracted some interesting results using the new definition.

Definition 1. The weight of a strong dominating set D is defined as W(D) = L,ueDμ(u, v), where μ(u, v) is the minimum of the membership values (weights) of the strong arcs incident on u. The strong domination number of a fa:a,y graph G is defined as the minimum weight of strong dominating sets of G and it is denoted by y,(G) or simply y,. A minimum strong dominating set in a fuzzy graph G is a strong dominating set of minimum weight.

Let y s(G) or y, denote the strong domination number of the complement of a fuzzy graph G.

Remark 1. Note that for any undirected fuzzy graph for any x,y E V, if (x,y) is a strong arc, then (y, x) is also a strong arc. That is if x strongly dominates y, then y strongly dominates x and hence strong domination is a symmetric relation on V.

Remark 2. If all the nodes are isolated, then V is the only strong dominating set of G of order p and 'Ys = 0. That is Ns(u) = <fJ for each u EV.

Example 1 Consider the fuzzy graph given in Fig. 1.

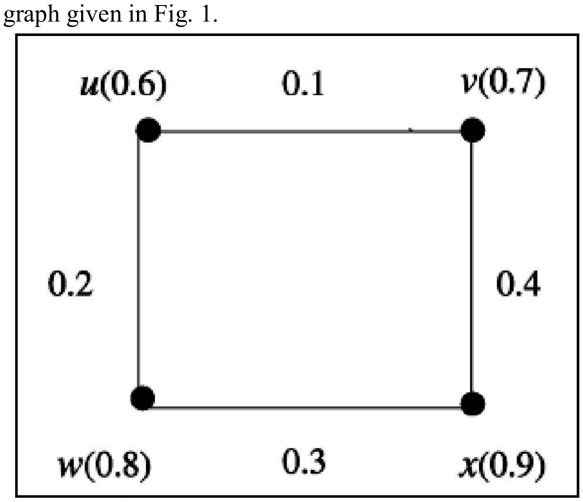

Fig. 1. Illustration of strong domination in fuzzy graphs

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In this fuzzy graph, strong arcs are (u, w), (w, x) and (x, v). The minimum strong dominating sets are D1 = {u, x) and D2 = {w, x), where
W(D1) = 0.2 + 0.3 = 0.5 and W(D2) = 0.2 + 0.3 = 0.5.
Hence

$$\gamma_s = 0.5.$$

Proposition 1. If G : (V, er,μ) is a complete fuzzy graph, then
Ys(G) = I\ {μ(u, v) I u, v E er*}.
Proof Since G is a complete fuzzy graph, all arcs are strong [29] and each node is adjacent to all other nodes. Hence D = {u) is a strong dominating set for each u E er*. Hence the result follows.
Proposition 2. Let G: (V,er,μ) be a non trivial fuzzy graph of size q. Then Ys(G) = q if and only if all arcs are strong and each node is either an isolated node or has a unique strong neighbor.
Proof If all arcs are strong and each node is either an isolated node or has a unique strong neighbor, then the minimum strong dominating set of G is a set D containing nodes each of which is either an isolated node or an end node of each unique strong arc. Hence weight of D is exactly

$$W(D) = \sum_{u \in D} \mu(u, v) = q.$$

Conversely, suppose that Ys = q. To prove that all arcs are strong and each node is either an isolated node or has a unique strong neighbor. If possible let (u, v) be an arc of G which is not strong. Then the weight of this arc is not counted for getting y,. Hence Ys < q, a contradiction. Hence all arcs are strong.
Let x be any node of G. If xis an isolated node, then clearly xis contained in the minimum strong dominating set. If possible suppose x has two strong neighbors say v and w. Then exactly one of the weights of the arcs (x, v) and (x, w) contribute to the weight of the minimum strong dominating set. Hence Ys < q, a contradiction. Hence each node has a unique strong neighbor.
In particular, Ys(G) = 0 when G is a complete fuzzy graph.
Remark 3. In Proposition 3.2, G is exactly S UN where S is a set of isolated nodes, may be empty and N is a union of K2' s.
Proposition 3. ysCKcr,.cr,) = μ(u, v) or 2μ(u, v), where μ(u, v) is the weight of a weakest arc in Kcr1ocr,
Proof In Kcr1ocr,, all arcs are strong. Also each node in Vi is adjacent with all nodes in V2. Hence in Kcr,.cr,, the strong dominating sets are V1, V2 and any set containing 2 nodes, one in V1 and other in V2. If Vi or V2 contains only one element say u, then D = {u} is the minimum strong dominating set in G. Hence y,(Kcr,,cr,) = μ(u, v) where μ(u, v) is the minimum weight of the arcs adjacent on u. If each Vi and V2 contains more than one element, then the end nodes say {u, v} of any weakest arc (u, v) in Kcr,,cr, form the minimum strong dominating set.
Hence Ys(Kcr,.cr,) = μ(u, v) + μ(u, v) = 2μ(u, v). So the proposition is proved.
Remark 4. In any fuzzy graph G : (V, er,μ), Ys < p always holds, since μ(x, y) ::,; er(x) (\ er(y) for all x, y E er*. [pis the scalar cardinality of G, which is got by using the node weights and Ys is the weight of the minimum strong dominating set, which is got by using the arc weights.
For the strong domination number Ys, the following theorem gives a Nordhaus Gaddum type result.
Theorem 1. For any fuzzy graph G: (V, er,μ), Ys + Ys < 2p.
Proof Since Ys < p and Ys < p by Remark 3.4, we have Ys + Ys < p + p = 2p.
Definition 2. A strong dominating set Dis called a minimal strong dominating set if no proper subset of D is a dominating set.
Example 2. In Fig.1 of Example 3.1, D = {u, v} is a minimal strong dominating set.
Theorem 2. Every non trivial connectedfuuy graph G has a strong dominating set D whose complement V - D is also a strong dominating set.

Theorem 3. Let G be a fuzzy graph without isolated nodes. If Dis a minimal strong dominating set, then V - D is a strong dominating set.

Theorem 4. For any fuzzy graph G : (V, er,µ) without isolated nodes Ys ::,; p/2.

Fuzzy Inf. Eng. (2015) 7: 369-377 375 Proof Let D be a minimal strong dominating set of G. Then by Theorem 3.3, V - D is a strong dominating set of G. Then 'Ys :'> W(D) and 'Ys :'> W(V - D). Therefore 2ys ='> W(D) + W(V - D) ='> p which implies 'Ys ='> p/2. Hence the proof.

Corollary 1. Let G be a fuzzy graph such that both G and G have no isolated nodes. Then 'Ys + Ys :'> p. Further equality holds if and only if'Ys = Ys = p/2.

Proof By Theorem 3.4, 'Ys ='> p/2, Ys ='> p/2

$$\Rightarrow \gamma_s + \overline{\gamma_s} \leq p/2 + p/2 = p,$$

$$\text{that is } \gamma_s + \overline{\gamma_s} \leq p.$$

If 'Ys = y, = p/2, then obviously 'Ys + y, = p. Conversely, suppose 'Ys + y, = p. Then by Theorem 3.4, we have 'Ys :'> p/2, y, :'> p/2. If either 'Ys < p/2 or y, < p/2, then 'Ys +'Ys < p, which is a contradiction. Hence the only possibility is that 'Ys = y, = p/2.

Remark 5 Note that Theorem 3.4 and Corollary 3.1 are true when we use the definition of domination number in.

Theorem 5. In any fuzzy graph G : (V, er,µ), 'Ys = p/2 if and only if the following conditions hold.

1) All nodes have the same weight.

2) All arcs are M-strong arcs.

3) For every minimum strong dominating set D of G, IDI = n/2, where n is the number of nodes of G and n is even.

Proof If all the conditions 1), 2), 3) hold, then obviously 'Ys = p/2. Conversely, suppose 'Ys = p/2. If some nodes say u and v have different weights, then the arc weight corresponding to these nodes is µ(u, v) ='> cr(u) /\ cr(v) . If µ(u, v) < cr(u) /\ cr(v), then obviously 'Ys < p/2, a contradiction. If µ(u, v) = cr(u) /\ cr(v), then (u, v) becomes an M-strong arc. If IDI < n/2, then clearly 'Ys < p/2, a contradiction.

Hence all the conditions are sufficient.

The following theorem gives a lower bound and an upper bound for the strong domination number of a connected fuzzy graph.

Theorem 6. For any connected fuzzy graph G: (V, er,µ) of order p,

$$\min_{u,v \in V} \mu(u,v) \leq \gamma_s \leq p - \Delta_{SN}(G).$$

Proof The first part is trivial. For the second part, let u be a node of G such that dsN(u) = l'lsN(G). Then V - Ns(u) is a strong dominating set. Therefore 'Ys ='> W(V - Ns(u)) ='> p - l'lsN(G). Therefore Ys ='> p - l'lSN(G).

Remark 6. The above inequality cannot be improved further. For example, for the complete fuzzy graph G, min µ(u, v) = 'Ys = p - l'lsN(G).

Remark 7. Clearly, ll,(G) :'> llsN(G) (since µ(x,y) :'> cr(x) I\ cr(y)) and hence Ys :'> p - ll,(G).

II. CONCLUSION

Theoretical advancements and practical applications abound in the idea of dominance in graphs. Numerous writers have examined over thirty domination parameters; in this research, the notion of strong domination number for fuzzy graphs is adapted by using the membership values of strong arcs.

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