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Robust Adaptive Controller Design for Uncertain Second-Order Jerk Chaotic Systems

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Abstract: *In this paper, the stabilization problem for a class of uncertain second-order jerk chaotic control systems is investigated. Combining differential and integral inequalities with nonlinear theory, we intend to construct a robust adaptive controller to promote a class of uncertain second-order jerk chaotic control systems to achieve the goal of adaptive stability. Finally, an example will be provided to illustrate the design process of the adaptive controller, and some numerical simulation results will be offered to demonstrate the correctness and effectiveness of the main result.*

Keywords: Robust adaptive controller, Uncertain systems, Chaotic system, Multiple uncertainties

I. INTRODUCTION

As we know, the chaotic control system is a nonlinear system, so the design of its controller is often more complicated and difficult to obtain than the linear control system. On the other hand, real physical systems contain more or less uncertain items, and these uncertain items may be incomplete system models, parameter disturbances or unidentified noise. However, physical systems containing the above uncertain factors significantly increase the difficulty in stability analysis and controller design. Due to the above factors, our group is prompted to conduct analysis and controller design for a class of uncertain chaotic control systems.

In recent years, various types of uncertain nonlinear systems have been analyzed and discussed; especially in the design of controllers, as of today, many methods of controller design have been developed, such as adaptive neural network control framework, sliding mode control approach, adaptive fuzzy control scheme , adaptive sliding mode control approach, adaptive control approach, adaptive state-quantized control approach, backstepping control approach, and others; see, for example, [1]-[13] and the references therein. This paper intends to design an adaptive controller for a class of uncertain jerk chaotic control systems, so that the entire system can achieve the goal of global asymptotic stability.

II. PROBLEM FORMULATION AND MAIN RESULTS

Symbol Description

$\Re^n$ the n-dimensional real space

$|a|$ the modulus of a complex number a

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I the unit matrix

A^T the transport of the matrix A

$\|x\|$ the Euclidean norm of the vector $x \in \Re^n$

$\sigma(A)$ the spectrum of the matrix A

$P > 0$ the matrix P is a symmetric positive definite matrix

This paper considers the following nonlinear jerk chaotic control systems with multiple uncertainties

$$\dot{x}_1 = x_2, \quad \text{(1a)}$$

$$\dot{x}_2 = q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \Delta f(x_1, x_2) + \Delta\phi(u), \forall t \geq 0, \quad \text{(1b)}$$

where $x = [x_1 \quad x_2]^T \in \Re^{2\times1}$ is the state vector, $u \in \Re$ is the input, $\Delta f(x_1, x_2)$ indicates the mixed uncertainties (parameter mismatchings, external noise, and unknown nonlinearities), and $\Delta\phi(u)$ indicates the uncertain input nonlinearity. To ensure the existence of the solution of (1), we assume that the uncertain terms $\Delta f(x_1, x_2)$ and $\Delta\phi(u)$ are smooth functions. From past literature of [14]-[16], the system (1) without any uncertainties (i.e., $\Delta f(x_1, x_2) = \Delta\phi(u) = 0$) is the famous jerk chaotic system under certain values of the parameters. In this paper, motivated by differential and integral inequalities with nonlinear theory, a robust adaptive controller will be constructed to enable the entire closed-loop control system of (1) to achieve global asymptotic stability.

The following assumption is made regarding the uncertain terms $\Delta f(x_1, x_2)$ and $\Delta\phi(u)$ of the uncertain nonlinear systems of (1).

(A1) There exist continuous functions $g_i(x_1, x_2) \geq 0$, unknown nonnegtive numbers β_i, and positive number r_1, such that, for all arguments,

$|\Delta f(x_1, x_2)| \leq \sum_{i=1}^{s} \beta_i g_i(x_1, x_2)$, for some $s \in N$ and

$u \cdot \Delta\phi(u) \geq r_1 u^2$.

The definition of robust adaptive stabilization is introduced below, which will be used in subsequent main results.

Definition 1: The uncertain nonlinear systems (1) with (A1) are said to realize the robust adaptive stabilization, provided that, there exist a control u such that the state trajectory satisfies

$\lim_{t\to\infty} \|x(t)\| = 0$.

Now we put forward the main result regarding the robust adaptive stabilization of uncertain nonlinear systems of (1).

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Theorem 1. The uncertain nonlinear systems (1) with (A1) are robust adaptive stabilization under the following control

$$u(t) = -r(t)\cdot(p_3 x_1 + p_2 x_2), \quad (2)$$

$$r(t) := \frac{4h^2(t)}{4r_1 \cdot h(t)\cdot|p_3 x_1 + p_2 x_2| + \left(\frac{a}{t^2+1}\right)}, \quad (3)$$

$$h(t) := |q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2| + \left|\sum_{i=1}^{s} b_i(t) g_i(x_1, x_2)\right|, \quad (4)$$

$$\dot{b}_i(t) = |p_3 x_1 + p_2 x_2|\cdot g_i(x_1, x_2), \quad \forall\, i \in \{1,2,3,\cdots,s\}, \quad (5)$$

where $P := \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} > 0$ is the unique solution to the following Lyapunov equation

$$\begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix}^T P + P\begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix} = -2I, \quad (6)$$

with $\alpha > 0$ and $a > 0$.

Proof. From (1), the state equation is given by

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix} x \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\cdot\left[\Delta\phi + \Delta f + q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2\right] \\ &= Ax + B\cdot\left[\Delta\phi + \Delta f + q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2\right], \forall\, t \ge 0,\end{aligned}$$

where $A := \begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix}$ and $B := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Clearly, one has $\sigma(A) = \{-\alpha\}$, which implies A is Hurwitz and the Lyapunov equation of (6) has the unique posive definite solution *P*. Let

$$V(x(t)) = x^T(t) P x(t) + \sum_{i=1}^{s} [b_i(t) - \beta_i]^2. \quad (7)$$

The derivative of $V(x(t))$ with respect to time along the trajectories of uncertain nonlinear systems (1) with (2)-(7) is given by

$$\begin{aligned}\dot{V}(x(t)) &= x^T\left[A^T P + PA\right] x + 2\sum_{i=1}^{s} \dot{b}_i(t)[b_i(t) - \beta_i] \\ &\quad + 2x^T PB\cdot\left[\Delta\phi + \Delta f - q_1 x_1 - q_2 x_2 - x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2\right]\end{aligned}$$

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$$\leq -2x^T x + 2x^T PB \cdot \Delta\phi + 2\sum_{i=1}^{s} \dot{b}_i(t)[b_i(t) - \beta_i]$$

$$+ 2|x^T PB|\left[|q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2| + \sum_{i=1}^{s} \beta_i g_i(x_1, x_2)\right]$$

$$= -2x^T x + 2x^T PB \cdot \Delta\phi + 2\sum_{i=1}^{s} |x^T PB| \cdot g_i(x_1, x_2)[b_i(t) - \beta_i]$$

$$+ 2|x^T PB|\left[|q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2| + \sum_{i=1}^{s} \beta_i g_i(x_1, x_2)\right]$$

$$\leq -2x^T x + 2x^T PB \cdot \Delta\phi$$

$$+ 2|x^T PB|\left[|q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + \alpha^2 x_1 + 2\alpha x_2| + \left|\sum_{i=1}^{s} b_i(t) \cdot g_i(x_1, x_2)\right|\right]$$

$$= -2x^T x + 2x^T PB \cdot \Delta\phi + 2h(t) \cdot |x^T PB|$$

$$= -2x^T x - \left(\frac{2}{r}\right) \cdot (u \cdot \Delta\phi) + 2h \cdot |x^T PB|$$

$$\leq -2\|x(t)\|^2 - \left(\frac{2}{r}\right) \cdot (r_1 u^2) + 2h \cdot |x^T PB|$$

$$= -2\|x(t)\|^2 - 2r_1 \cdot r \cdot |x^T PB|^2 + 2h \cdot |x^T PB|$$

$$= -2\|x(t)\|^2 - 2r_1 \cdot r \cdot |p_3 x_1 + p_2 x_2|^2 + 2h \cdot |p_3 x_1 + p_2 x_2|$$

$$= -2\|x(t)\|^2 - \frac{8r_1 \cdot h^2(t) \cdot |p_3 x_1 + p_2 x_2|^2}{4r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2| + \left(\frac{a}{t^2+1}\right)} + 2h \cdot |p_3 x_1 + p_2 x_2|$$

$$= -2\|x(t)\|^2 + \frac{2h \cdot |p_3 x_1 + p_2 x_2| \cdot \left(\frac{a}{t^2+1}\right)}{4r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2| + \left(\frac{a}{t^2+1}\right)}$$

$$= -2\|x(t)\|^2 + \left(\frac{1}{2r_1}\right) \cdot \frac{[4r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2|] \cdot \left(\frac{a}{t^2+1}\right)}{4r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2| + \left(\frac{a}{t^2+1}\right)}, \quad \forall\, t \geq 0.$$

By the inequality

$$x\left(\frac{yz}{y+z}\right) \leq xz, \quad \forall\, x > 0,\ y \geq 0, \text{ and } z > 0.$$

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It can be deduced that

$$\dot{V}(x(t)) \le -2\|x(t)\|^2 + \left(\frac{1}{2r_1}\right)\left(\frac{a}{t^2+1}\right), \quad \forall\, t \ge 0.$$

It results that

$$\|x(t)\|^2 \le \frac{-\dot{V}(x(t))}{2} + \left(\frac{1}{4r_1}\right)\left(\frac{a}{t^2+1}\right)$$

$$\Rightarrow \int_0^\infty \|x(t)\|^2\, dt \le \int_0^\infty \frac{-\dot{V}(x(t))}{2} dt + \left(\frac{1}{4r_1}\right)\int_0^\infty \left(\frac{a}{t^2+1}\right) dt$$

$$= \frac{V(x(0))}{2} - \frac{V(x(\infty))}{2} + \frac{a\pi}{8r_1} \le \frac{V(x(0))}{2} + \frac{a\pi}{8r_1}$$

As a result, we conclude that $\lim_{t\to\infty}\|x(t)\| = 0$, in view of $x(t) \in L_2$. The proof is thus completed. □

Remark 1. The following provides a design procedure for obtaining the robust adaptive controller of (2).

INPUT: the uncertain jerk chaotic control systems (1).

OUPUT: the robust adaptive control of (2).

Step 1: Choose s, $g_i(x_1, x_2)$, $\forall\, i \in \{1,2,3,\cdots,s\}$, and r_1 such that (A1) is satisfied.

Step 2: Calculate P, p_2, and p_3, from (6).

Step 3: Form $h(t)$ and $\dot{b}_i(t)$, $\forall\, i \in \{1,2,3,\cdots,s\}$, from (4) and (5).

Step 4: Choose a and form $r(t)$ from (3).

Step 5: OUPUT $u(t) = -r(t)\cdot(p_3x_1 + p_2x_2)$.

III. NUMERICAL EXAMPLE

Consider the following uncertain jerk chaotic control systems with uncertain input nonlinearities:

$$\dot{x}_1 = x_2, \qquad \text{(8a)}$$

$$\dot{x}_2 = q_1x_2 + q_2x_1^3 + q_3\cos(wt) + \Delta f(x_1, x_2) + \Delta\phi(u), \forall\, t \ge 0, \qquad \text{(8b)}$$

where

$q_1 = -0.06$, $q_2 = -1$, $q_3 = 5$, $w = 1$,

$\Delta f(x_1, x_2) = \Delta a \cdot x_2^2, \quad \Delta\phi(u) = \Delta b \cdot u + \Delta c \cdot u^3$,

$|\Delta a| \le \eta \in \Re, \quad 1 \le \Delta b \le 4, \quad \Delta c \ge 0,$

and η is an unknown constant. This example intends to design a suitable controller such that the uncertain nonlinear systems (8) realize the robust adaptive stabilization.

Step 1: The condition (A1) is clearly satisfied if we choose

$$s = 1, \quad g_1(x_1, x_2) = x_2^2,\ r_1 = 1.$$

Step 2: From (6) and selecting $\alpha = 2$, we have

$$P = \begin{bmatrix} 2.25 & 0.25 \\ 0.25 & 0.31 \end{bmatrix}, \quad p_2 = 0.31, \quad p_3 = 0.25.$$

Step 3: From (4) and (5), it can be readily obtained that

$$h(t) := \left| -0.06x_2 - x_1^3 + 5\cos t + 4x_1 + 4x_2 \right| + \left| b_1(t) x_2^2 \right|, \quad (9)$$

$$\dot{b}_1(t) = \left| 0.25x_1 + 0.31x_2 \right| \cdot x_2^2(t). \ (10)$$

Step 4: From (3) and selecting $a = 1$, one has

$$r(t) := \frac{4h^2(t)}{4h(t) \cdot \left| 0.25x_1 + 0.31x_2 \right| + \left(\dfrac{1}{t^2 + 1} \right)}. \qquad (11)$$

Step 5: The suitable controller, given by (2), can be calculated as

$$u(t) = -r(t) \cdot (0.25x_1 + 0.31x_2). \qquad (12)$$

Therefore, by Theorem 1, we conclude that uncertain nonlinear systems (8) subjected to the control (12) with (9)-(11) are globally asymptotically stable. The typical state trajectories of uncontrolled and feedback-controlled are depicted in Figure 1 and 2, respectively. Besides, the signals of input and adaptive gain are depicted in Figure 3 and 4, respectively. From the observation of the aforementioned simulation results, it can be seen that the uncertain nonlinear systems of (8) indeed achieves the robust adaptive stabilization under the control law of (12) with (9)-(11).

IV. CONCLUSION

In this paper, the stabilization problem for a class of uncertain second-order jerk chaotic control systems has been investigated. Combining differential and integral inequalities with nonlinear theory, a robust adaptive controller has been designed to render a class of uncertain second-order jerk chaotic control systems to achieve the goal of adaptive stability. Finally, an example has been provided to illustrate the design process of the adaptive controller and some numerical simulation results have also been offered to demonstrate the correctness and feasibility of the main result.

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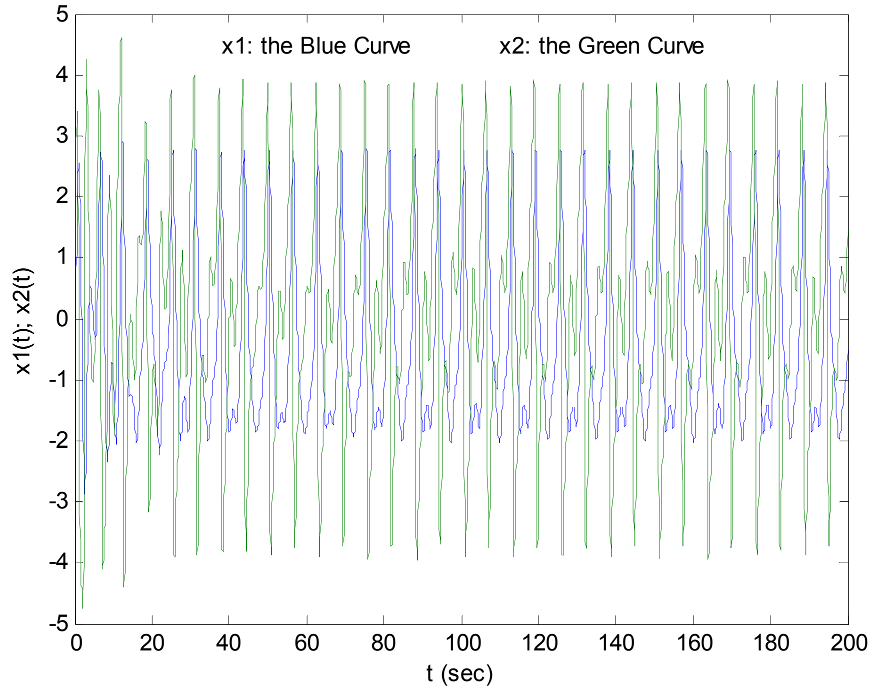

Figure 1: Typical state trajectories of the uncontrolled system of the illustrative example.

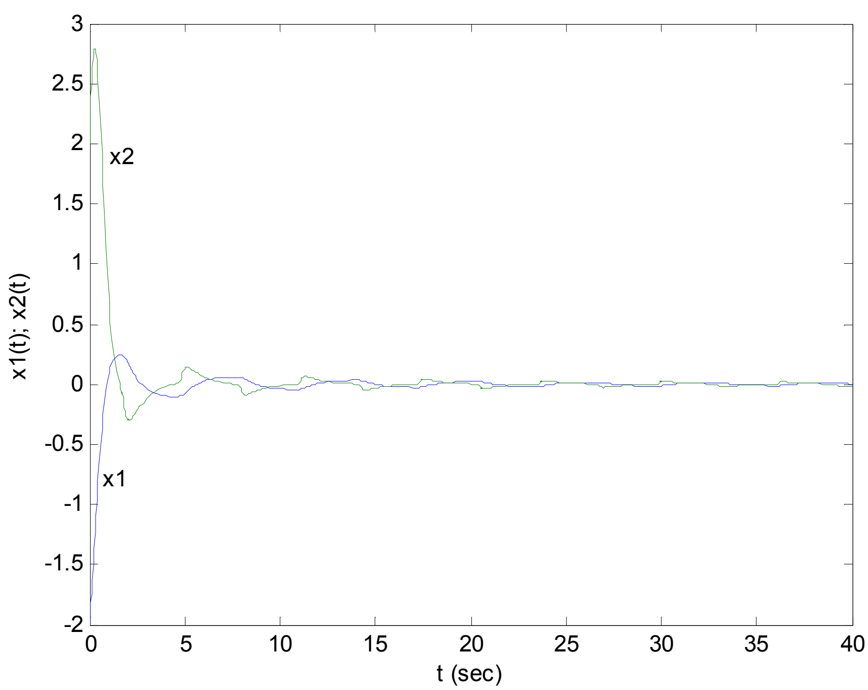

Figure 2: Typical state trajectories of the feedback-controlled system of the illustrative example.

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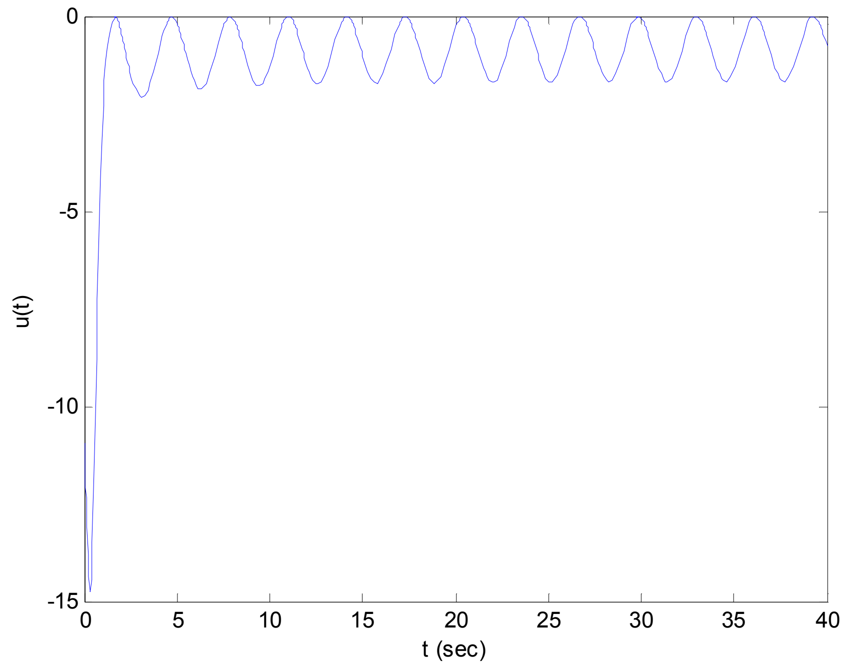

Figure 3: Control signal of the illustrative example.

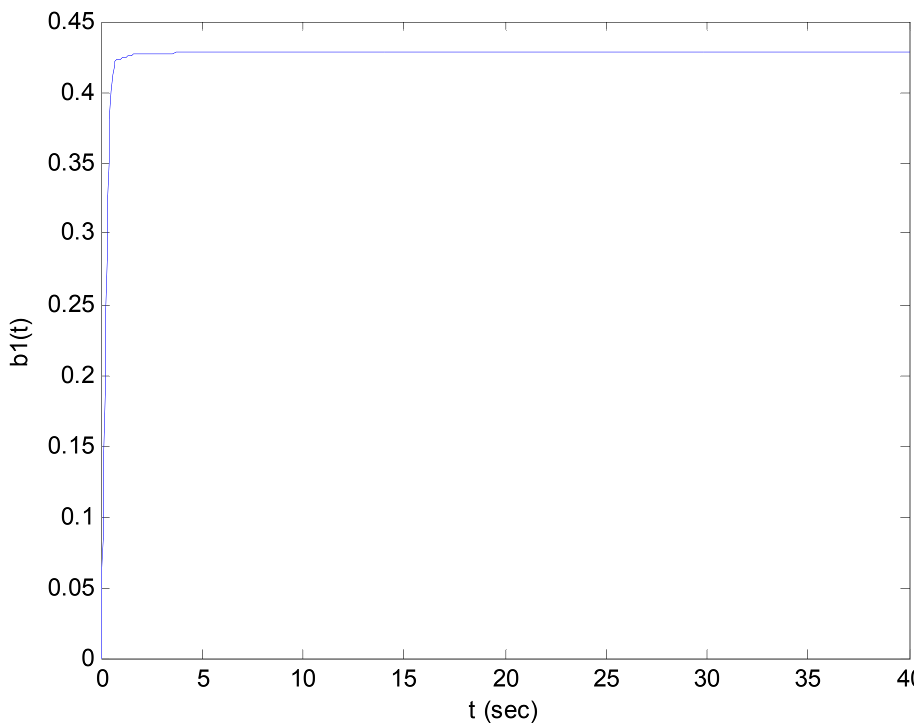

Figure 4: The signal of adaptive gain.