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Boundary Layer Flow Over a Stationary Wedge

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Abstract: *The primary aim of this research is to scrutinize the Falkner-Skan boundary layer flow past a wedge, taking into account the velocity slip condition. The governing partial differential equations describing the physical system are transformed into ordinary differential equations through similarity transformations. The ensuing ordinary differential equation is subsequently addressed utilizing the differential transform method (DTM) augmented by Pade approximations. The obtained velocity profiles are presented, and an in-depth analysis of the impact of the slip parameter on the flow is provided. The credibility of our solutions is affirmed through comparison with previously published results.*

Keywords: Velocity slip, Differential Transform Method, Pade Approximation, Similarity transformation, Stationary wedge.

I. INTRODUCTION

The initial examination of steady laminar flow around a stationary wedge was conducted by Falkner and Skan [1] as a demonstration of Prandtl's boundary layer theory application. They introduced a similarity transformation that effectively reduces the non-linear boundary layer equations to a well-known non-linear third-order ordinary differential equation termed the Falkner-Skan equation. This equation accounts for non-uniform flow, specifically outer flows expressed as ax^m when evaluated at the wall, where x represents the coordinate along the wedge wall and $a\ (> 0)$ and m are constants. Extensive literature on this subject has been explored by various researchers, including Schilichting and Gersten [2], Leal [3], and Ishak et al. [4].

Non-linear differential equations frequently emerge from the mathematical modeling of diverse physical systems. In many instances, scientific and engineering problems inherently exhibit non-linearity, often resisting analytical solutions. Consequently, numerical methods or analytic perturbation methods (as seen in Nayfeh [5] and Rand and Armbruster [6]) are employed. However, numerical approaches may yield discontinuous points, making it time-consuming to obtain comprehensive result curves. Issues related to stability and convergence must be carefully addressed to avoid divergence or inaccurate results. Furthermore, full comprehension of non-linear problems based solely on numerical outcomes can be challenging. Numerical complications also arise when dealing with singularities or multiple solutions in non-linear problems.

The advent of symbolic software, such as MATHEMATICA, MAPLE, MATLAB, and others, has directed significant attention toward the development of new methods for constructing approximate analytic solutions to non-linear equations, circumventing the limitations mentioned earlier. Methods like the homotopy perturbation method [7], homotopy analysis method (HAM) [9], variational iteration method [15], and the differential transform method (DTM) [17] have gained prominence. DTM, a semi-exact method, is notable for its application without the requirement for small parameters in the equation.

The inception of the Differential Transform Method (DTM) can be attributed to Zhou [17], who applied it to address electric circuit analysis problems featuring both linear and non-linear differential equations. DTM offers an iterative procedure for obtaining analytic solutions to ordinary and partial differential equations through a polynomial approach. Unlike the widely used Taylor series method, which involves extensive mathematical computation, DTM stands out by minimizing the computational burden. Notably, DTM is capable of handling non-linear differential equations without relying on discretization, linearization, or perturbation. Consequently, it remains unaffected by errors associated with discretization. Chen and Ho [18] expanded the application of DTM to solve partial differential equations, while Ayaz

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[19] employed it in investigating systems of differential equations. Arikoglu and Ozkol [20] extended its utility to difference equations, and Darania and Ebadian [21] successfully applied DTM to integro-differential equations.In recent years, DTM has demonstrated its efficacy in diverse problem domains, including linear partial differential equations of fractional order [22], non-oscillatory systems [23], multi-order fractional differential equations [24], hyperchaotic Rossler systems [25], fourth-order boundary value problems [26], Volterra integral equations with separable kernels [27], calculation of two-dimensional differential transforms of non-linear functions [28], linear and non-linear Schrödinger equations [29], free vibration analysis of circular plates [30], and solving fuzzy differential equations [31]. These successful applications affirm the validity, effectiveness, and flexibility of DTM.While DTM exhibits validity in solving differential equations on bounded domains, its applicability on unbounded domains is constrained due to the divergent nature of the closed series solution when the independent variable tends to infinity. Researchers, including Boyd [32], have established the limitations of power series in unbounded domains, particularly in handling boundary value problems. To overcome this challenge, the DTM-Pade technique has been recently adopted by Xiao-hong and Lian-cun [33] to provide approximate solutions across a broader range.
Motivated by these considerations, this paper aims to extend the application of DTM with Pade approximant to solvethe Falkner-Skan boundary layer flow over a stationary wedge. The investigation focuses on studying the influence of slip conditions, and comparisons are drawn between the results obtained using DTM-Pade and those presented by Bashir et al. [34].

Problem discussion and mathematical formulation

Consider a steady two-dimensional Falkner-Skan boundary layer flow around a wedge. The physical model and corresponding geometric coordinates are depicted in Fig. 1, where $\beta\pi$ represents the angle of the wedge. In this analysis, we account for the slip condition. Adopting a Cartesian coordinate system (x, y), where x and y denote the coordinates measured along the surface of the wedge and normal to it, respectively.

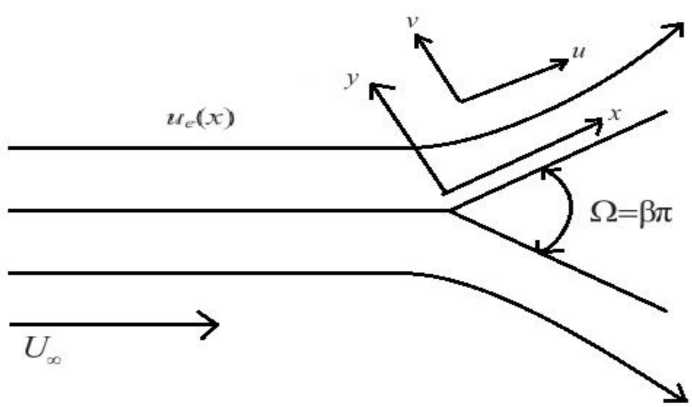

Fig. 1 The physical model and the coordinate system

The external flow velocity and the pressure gradient are given by

$u_e = U_\infty x^m$ (1)

$\frac{\partial P}{\partial x} = -\rho\, u_e \frac{du_e}{dx}$(2)

where u_e is the free stream or external velocity, P is the pressure, ρ is the density and x is the position along the wedge. The coefficient U_∞ is constant.

Under the above conditions, the flow near the wedge will be governed by the boundary layer equations:

Continuity equation:

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (3)

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Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial y}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + v\frac{\partial^2 u}{\partial y^2} \quad (4)$$

which becomes by (2)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} \quad (5)$$

subject to the boundary conditions

$$\begin{cases} u = N_1(x)v\frac{\partial u}{\partial y}, \quad v = 0 \text{ at } y = 0 \\ v = 0, \quad u \to u_e(x) \text{at} \quad y \to \infty \end{cases} \quad (6)$$

Here u and v are the velocity components along the $x-$ and $y-$ axes respectively, ρ is the fluid density, ν is the kinematic viscosity, N_1 is the slip parameter.

Let ψ be the stream function, such that

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x} \quad (7)$$

Then, the continuity equation (3) holds and equation (5) is transformed to:

$$\psi_y\psi_{xy} - \psi_x\psi_{yy} = u_e\frac{du_e}{dx} + v\,\psi_{yyy} \quad (8)$$

and the boundary conditions become

$$\begin{cases} \psi_y = N_1 v\psi_{yy}, \quad \psi_x = 0 \text{ at } y = 0 \\ \psi_y = u_e(x) \text{at} \quad y \to \infty \end{cases} \quad (9)$$

To transform the equation (8) to ordinary differential equation, we use the following similarity transformations

$$\hat{\eta} = x^{\frac{1-m}{2}}y, \qquad \psi = x^{\frac{1+m}{2}}f(\hat{\eta}) \quad (10)$$

Where the exponent m is function of β and

$$m = \frac{\beta}{\beta-2}. \quad (11)$$

Substituting (10) into (8), we get

$$v\,f''' + \frac{m+1}{2}f\,f'' - m\,(f')^2 + m\,U_\infty^2 = 0 \quad (12)$$

where primes denote the derivative with respect to $\hat{\eta}$.

The boundary conditions become

$$f(0) = 0, \; f'(0) = v\,N_1 x^{\frac{m+1}{2}} f''(0), \; f'(\infty) \to U_\infty. \quad (13)$$

We now consider the following dimensionless independent variable

$$\eta = \sqrt{\frac{U_\infty(m+1)}{2v}}\hat{\eta}. \quad (14)$$

Also, from the boundary condition (12), we have $N_1(x) = \left(\frac{\sqrt{2}N}{\sqrt{vU_\infty(m+1)x^{m+1}}}\right)$ where N is the constant velocity slip parameter.

Using equation (13),the equation (11) changes to

$$f''' + f\,f'' + \left(\frac{2m}{m+1}\right)[1 - (f')^2] = 0 \quad (15)$$

where the primes denote derivatives w.r.t η and the corresponding boundary conditions (12) become

$$f(0) = 0, \quad f'(0) = N\,f''(0), \quad f'(\infty) = 1. \quad (16)$$

Principles of analytical methods

The differential transform method

An arbitrary function $u(x)$ analytic in the domain T, can be written in the Taylor series form about the point $x = x_0$ as

$$u(x) = \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0}(x - x_0)^k \quad (17)$$

The differential transform of $u(x)$ is defined as

$$U(k) = \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \quad (18)$$

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where $u(x)$ is the original function and $U(k)$ is the transformed function.

Pade approximant

Solution obtained by DTM is in terms of power series. Since the radius of convergence of the power series may not be large enough to contain the two boundaries, Pade approximants are applied to manipulate the obtained series for numerical approximations to overcome this difficulty.

Pade approximant [36] and [37] is the best approximation for a polynomial approximation of a function into rational functions of polynomials of given order.

Let us suppose that we are given a power series $\sum_{i=0}^{\infty} a_i x^i$, representing a function $f(x)$,

$$f(x) = \sum_{i=0}^{\infty} a_i x^i \qquad (19)$$

The Pade approximant is a rational function given by

$$\left[\frac{L}{M}\right] = \frac{P_L(x)}{Q_M(x)} \qquad (20)$$

where $P_L(x)$ is a polynomial of degree at most L and $Q_M(x)$ is a polynomial of degree at most M.

II. RESULT AND DISCUSSION

Utilizing the DTM-Pade technique, we successfully addressed the transformed equation (15) along with its associated boundary conditions (16). A comprehensive comparison of the wall shear stress, denoted as $f''(0)$, was conducted, referencing the results presented by Bashir et al. [34], specifically for $N = 0$ and various values of the power law parameter, m (Table 1).

The outcomes presented in Table 2 reveal a commendable agreement between the values of $f'(0)$ and $f''(0)$ obtained through the DTM-Pade analysis for diverse m and N, in comparison to the findings reported by Bashir et al. [34]. Their results, derived through an implicit finite difference method in MATLAB, align remarkably well with our analyses. Additionally, Table 2 underscores the observation that the velocity profile experiences an augmentation with increasing values of both m and N. Notably, the wall shear stress exhibits an increment with rising m, whereas it demonstrates a decrement with increasing N.

The influence of the power law parameter, m, on velocity profiles is visually illustrated in Fig. 2 for $N = 1$ and Fig. 3 for $N = -1$. As depicted, an increase in m corresponds to an elevation in velocity. Furthermore, Figs. 4 and 5 elucidate the impact of the slip parameter, N, on velocity profiles for $m = 0.5$. The observed trend indicates that velocity experiences an upsurge with an increase in N. These findings contribute valuable insights into the intricate relationships between key parameters and the resultant flow characteristics, enhancing our understanding of the dynamics involved.

TABLE I: Values of $f''(0)$ for various values of m when $N = 0$

m	Bashir et al. (34)	Present Result (DTM Pade)
0	**0.4696**	**0.4651**
1/11	**0.6549**	**0.6541**
0.2	**0.8021**	**0.8032**
1/3	**0.9276**	**0.9255**
0.5	**1.0389**	**1.0427**
1	**1.2325**	**1.2396**

TABLE III: Variation of $f'(0)$ and $f''(0)$ for various values of m and N

		Bashir et al. (34)		Present Result (DTM Pade)	
m	N	$f'(0)$	$f''(0)$	$f'(0)$	$f''(0)$
0.2	**0.1**	**0.076740**	**0.767407**	**0.0765528**	**0.765528**
0.2	**0.4**	**0.264234**	**0.660585**	**0.264329**	**0.660822**
0.2	**0.7**	**0.397370**	**0.567671**	**0.397499**	**0.567857**
0.2	**1**	**0.492994**	**0.492994**	**0.492370**	**0.492370**
0.5	**0.1**	**0.097208**	**0.972083**	**0.0972354**	**0.972354**
0.5	**0.4**	**0.316409**	**0.791024**	**0.316458**	**0.791147**

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0.5	**0.7**	**0.457747**	**0.653924**	**0.45823**	**0.654622**
0.5	**1**	**0.553109**	**0.553109**	**0.554038**	**0.554038**
0.8	**0.1**	**0.108360**	**1.083608**	**0.109092**	**1.09092**
0.8	**0.4**	**0.342919**	**0.857299**	**0.343755**	**0.859329**
0.8	**0.7**	**0.487064**	**0.695806**	**0.487843**	**0.696919**
0.8	**1**	**0.581472**	**0.581472**	**0.582198**	**0.582198**
1.5	**0.1**	**0.121875**	**1.218758**	**0.122515**	**1.22515**
1.5	**0.4**	**0.373337**	**0.933344**	**0.37407**	**0.935174**
1.5	**0.7**	**0.519600**	**0.742286**	**0.520002**	**0.74286**
1.5	**1**	**0.612313**	**0.612313**	**0.612571**	**0.612571**

III. CONCLUDING REMARKS

In conclusion, this research delved into the analysis of flow over a wedge, considering the influence of slip effects. The application of the DTM-Pade technique proved to be a straightforward and effective approach, eliminating the need for linearization, discretization, and perturbation commonly associated with other numerical and approximate methods. The obtained results were meticulously compared with the findings of Bashir et al. [34], revealing an outstanding agreement. Furthermore, the investigation highlighted a noteworthy correlation between the slip parameter and an increase in velocity. This study not only contributes valuable insights to the understanding of wedge flow dynamics but also underscores the efficacy of the DTM-Pade technique in such analyses

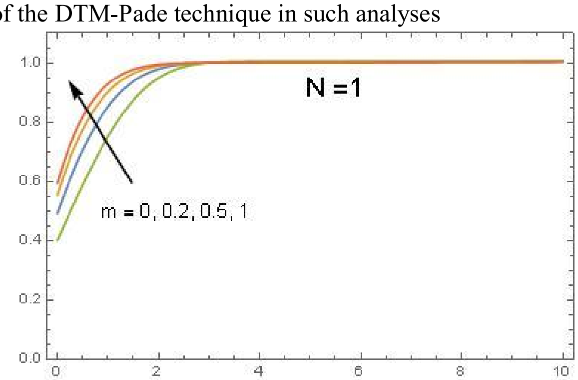

Fig. 2.Effects of m on the velocity when $N = 1$

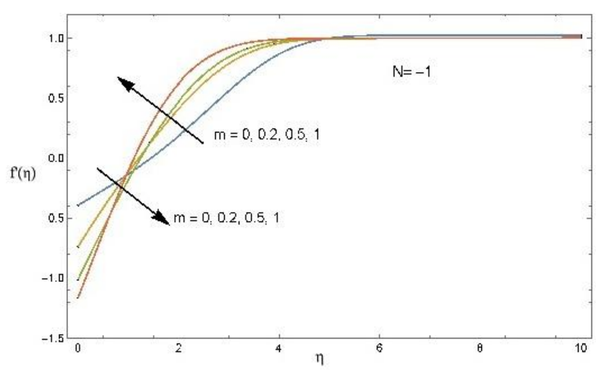

Fig. 3. Effects of m on the velocity when $N = -1$

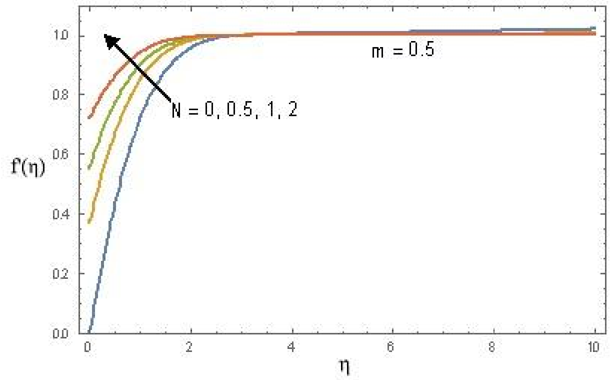

Fig. 4. Effects of N (for $N \geq 0$) on the velocity, when $m = 0.5$

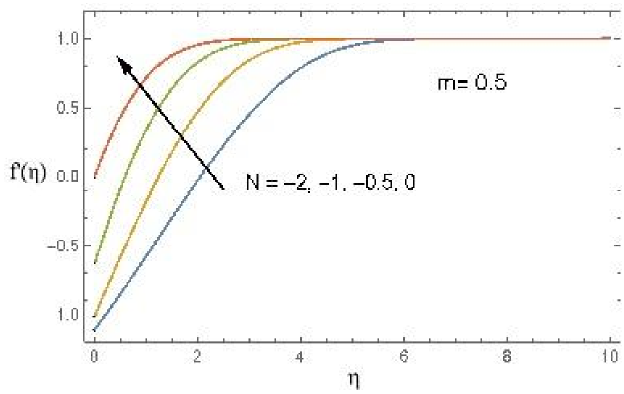

Fig. 5. Effects of N (for $N \leq 0$) on the velocity, when $m = 0.5$

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