

# MHD Couette Flow on Viscous Incompressible Fluid through A Vertical Tube with Heat and Mass Transfer

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**Abstract:** *Unsteady flow of dusty viscoelastic fluid through a long uniform tube, whose cross section curvilinear quadrilateral bounded by the areas and radii of two concentric circles under the influence of time varying pressure gradient has been considered expressions for the velocities of liquid and dust particles cases for different pressure gradient have also discussed The flow is driven by the motion of the tube's inner or outer surface, while the vertical orientation introduces the effects of free convection driven by temperature and concentration gradients. A uniform transverse magnetic field is applied, and the governing equations for momentum, energy, and concentration are formulated using the cylindrical coordinate system. These coupled non-linear differential equations are solved either analytically or through numerical techniques like the finite difference method..*

**Keywords:**  $P_{ik}$  - stress tensor,  $u$  - Velocity of liquid,  $\lambda_0$  - Elastic coefficient,  $\mu$  - Viscosity of liquid,  $k$  - The stokes resistance coefficient,  $N_0$  - number of density of the particles,  $\nu$  - Kinematic coefficient of viscosity,  $\rho$  - The density of the fluid,.

## I. INTRODUCTION

In this paper we consider the unsteady MHD flow in elastic-viscous liquid through a long uniform tube whose cross section is curvilinear quadrilateral bounded by the arcs and radii of two concentric circles  $r=1$ ,  $r=b$  and  $\theta=0$ ,  $\theta=\alpha$  under the influence of time varying pressure gradient. Initially the liquid particles are at rest. Some particular cases for different pressure gradient have also been discussed in detail. Later a large number of dusty flow problems have investigated in the literature and are well documented in a review by Marble (1963), Dutta (1985) discussed the Temperature field in the flow over stretching surface with uniform heat flux. Khani et al. (2009) gave the Analytic solution for heat transfer of a third grade viscoelastic fluid in non-Darcy porous media with thermo physical effects. Salem (2007) discussed Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Chen (2010) study On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation.

### Mathematical Modeling

According to Kuvshinski rheological equations satisfied by viscoelastic liquid are:

$$P_{ik} = P\delta_{ik} + P_{ik}^* \quad \dots (1)$$

$$\left(1 + \lambda_0 \frac{D}{Dt}\right) P_{ik}^* = 2\mu e_{ik} \quad \dots (2)$$

$$\frac{D}{Dt} P_{ik}^* = \frac{\partial}{\partial t} P_{ik}^* + u_m \frac{\partial}{\partial t} P_{ik}^* \quad \dots (3)$$

$$e_{ik} = \frac{1}{2} \left\{ \frac{\partial u_i}{\partial u_k} + \frac{\partial u_k}{\partial u_i} \right\} \quad \dots (4)$$

Let  $u_r, u_\theta, u_z$ , and  $v_r, v_\theta, v_z$ , of the components of liquid velocity and dust velocity in radial, tangential and axial direction respectively and the boundary conditions are:

$$u_r = 0, u_\theta = 0, u_z = u_z(r, t)$$

$$v_r = 0, v_\theta = 0, v_z = v_z(r, t)$$

Using this boundary condition the equations (1) to (4) represent in the form:

$$\rho \left( 1 + \lambda_0 \frac{\partial}{\partial t} \right) \frac{\partial u_z}{\partial t} - \left( 1 + \lambda_0 \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} \right) + KN_0 \left( 1 + \lambda_0 \frac{\partial}{\partial t} \right) (v_z - u_z) - \frac{B_0 \mu u_z}{r^2} \left( 1 + \lambda_0 \frac{\partial}{\partial t} \right) \dots (5)$$

$$m \frac{\partial v_z}{\partial t} = k(u_z - v_z) \dots (6)$$

Introducing the following non dimensional quantities are

$$u^* = \frac{u_z l}{\vartheta}, \quad v^* = \frac{v_z l}{\vartheta}, \quad z^* = \frac{z}{l}, \\ t^* = \frac{t \vartheta}{l^2}, \quad r^* = \frac{r}{l}, \quad p^* = \frac{p l^2}{\vartheta^2}, \\ \alpha^* = \frac{\lambda_0 \vartheta}{l^2}, \quad \beta = \frac{f}{t}, \quad f = \frac{m N_0}{\rho}, \quad Z = \frac{m \vartheta}{k l z'}$$

The initial boundary conditions are

$$t = 0, u(r, \theta, t) = v(r, \theta, t) = 0$$

$$t > 0, u(r, \theta, t) = v(r, \theta, t) = 0$$

$$u(r, \theta, t) = v(r, \theta, t) = 0 \text{ for } \theta = 0, \theta = \alpha,$$

$$s \leq r \leq 1, \text{ where } s = \frac{b}{l},$$

Using the boundary, the equation (5) and (6) becomes

$$\left( 1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = - \left( 1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial z} + \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) + \beta (1 + \alpha) (v - u) - \frac{B_0 u}{r^2} \left( 1 + \alpha \frac{\partial}{\partial t} \right) \dots (7)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau} (u - v) \dots (8)$$

### Solution of the problem

Putting  $\theta = \frac{\alpha}{\pi} \varphi$  and  $-\frac{\partial p}{\partial z} = f(t)$  in equation (7) and (8) we get

$$\left( 1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = - \left( 1 + \alpha \frac{\partial}{\partial t} \right) f(t) + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\pi^2}{\alpha^2 r^2} \frac{\partial^2 u}{\partial \varphi^2} + \beta \left( 1 + \alpha \frac{\partial}{\partial t} \right) (v - u) - \frac{B_0 u}{r^2} \left( 1 + \alpha \frac{\partial}{\partial t} \right) \dots (9)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau} (u - v) \dots (10)$$

Using finite Fourier transformation equations (9) and (10) become

$$\left( 1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial \bar{u}}{\partial t} = - \frac{2}{q_n} \left( 1 + \alpha \frac{\partial}{\partial t} \right) f(t) + \frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} \mp \beta \left( 1 + \alpha \frac{\partial}{\partial t} \right) (\bar{v} - \bar{u}) - \frac{B_0 \bar{u}}{r^2} \left( 1 + \alpha \frac{\partial}{\partial t} \right) \dots (11)$$

$$\frac{\partial \bar{v}}{\partial t} = \frac{1}{\tau} (\bar{u} - \bar{v}) \dots (12)$$

Using finite transformation in (11) and (12) and applying boundary conditions

$$\bar{u}_H = 0, \bar{v}_H = 0, \text{ at } t = 0,$$

We get

$$\left( 1 + \alpha \frac{\partial}{\partial t} \right) \bar{u} = \frac{2}{q_n} \left( 1 + \alpha \frac{\partial}{\partial t} \right) f(t) \int_0^s r \mathbf{B}_m(\xi_1 r) dr - \xi_1^2 \bar{u}_H + \beta \left( 1 + \alpha \frac{\partial}{\partial t} \right) (\bar{v}_H - \bar{u}_H) - \left( 1 + \alpha \frac{\partial}{\partial t} \right) B_0^2 \bar{u}_H \dots (13)$$

$$\frac{\partial \bar{v}_H}{\partial t} = \frac{1}{\tau} (\bar{u}_H - \bar{v}_H) \quad \dots (14)$$

## II. CONCLUSION

The study established analytical expressions for the velocities of both the fluid and dust particles, demonstrating how their motion is influenced by time-varying pressure gradients. The vertical orientation of the tube plays a critical role in the flow dynamics, as heat and mass. If the mass of the dust particles are small then their influence and the fluid flow is reduced as  $m \rightarrow 0$  then fluid becomes ordinary viscous.

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