

# New Methods to Solve and Find Initial Basic Feasible Solution of Transportation Problems

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**Abstract:** In this paper to solve the initial basic solution for transportation problem and Minimizing transportation time and getting optimal solutions are always considered as important factors while solving transportation problem. This paper shows a new approach for finding initial basic solution for transportation problem which reduces cost of transportation more than any transportation method such as Northwest Corner Method(NCM), Least Cost Method(LCM), northwest, Vogel's approximation, Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) and so on. This method has been illustrated by taking an Numerical example.

**Keywords:** Transportation problem, initial basic feasible solution, optimality test, optimal solution..

## I. INTRODUCTION

The Transportation problem grab the attention of most of the researchers due to its tremendous application in discrete areas of science and modern life of human beings to survive conveniently specially in industrial area. After the globalisation the international trade sprouted globally. The transportation problem is nothing but a special class of linear programming problem. it deals with optimization particularly in minimize the cost of transporting homogeneous commodity from multiple sources (Factories/Warehouses) to the multiple selling points or destinations (Customer service points) with satisfying demand and supply constraints.

To obtain minimum cost of transportation problem it is essential to start with an initial basic feasible solution. To find initial basic feasible solution various methods are available like Vogel's Approximation Method -VAM (Vogel and Reinfeld), Northwest Corner Method, and Least Cost Method. The Vogel's approximation method mostly preferred. The transportation problem can have some practical applications such as staff assignment, cash flow job scheduling, inventory control etc. the transportation problems based on the assumption that the model is balanced, means the total demand equals to the total supply. If the transportation problem is unbalanced, we can always add dummy row or column to make balanced transportation problem.

The transportation problem was first introduced by Hitchcock in 1941 in his research article. "The distribution of a product from several sources to numerous localities". It was further developed by Koopman in 1949. To find the solution of transportation problem Dantzig introduced a method followed by Charnes and others in 1953. The optimal solution to the transportation problem requires the determination of a number of units of commodities to be transported from each origin to various destination satisfying source availability and destination demands. So that the total cost of transportation problem is minimum.

## II. MATHEMATICAL FORMULATION OF TRANSPORTATION PROBLEM

Minimize

$$Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (1)$$

Subject to  $\sum_{j=1}^n X_{ij} = S_i, \quad i = 1, 2, 3, \dots, m$

$$\sum_{i=1}^m X_{ij} = D_j, \quad j = 1, 2, \dots, n \quad (2)$$

Where  $X_{ij} \geq 0, \forall i, j$

**Notations:**

$m$  = Total number of Supply  
 $n$  = Total number of demand  
 $S_i$  = Supply  $i$   
 $D_j$  = Demand  $j$   
 $C_{ij}$  = Transportation Cost from supply  $i$  to demand  $j$   
 $X_{ij}$  = Allocation made from supply  $i$  to demand  $j$

A transportation problem is balanced if the total supply is equal to the total demand.

$$\sum_{j=1}^n S_i = \sum_{i=1}^m D_j$$

**III. BASIC TERMINOLOGY**

**Feasible Solution:** Any set of non-negative allocations ( $x_{ij} \geq 0$ ) which satisfies the row and column restrictions is called a 'feasible solution'.

**Existence of Feasible Solution:** A necessary and sufficient condition to exist the feasible solution to the transportation problem is

$$\sum_{j=1}^n S_i = \sum_{i=1}^m D_j \quad (3)$$

The balanced transportation problem always has a feasible solution.

**Basic Feasible Solution:** A feasible solution to a transportation problem is called a "basic feasible solution" if the number of non-negative allocations is equal to  $m + n - 1$ , where  $m$  is the number of rows and  $n$  is the number of columns in a transportation problem.

**Optimal Solution:** A feasible solution (not necessarily basic) that minimize the total cost of transportation.

**Non-degenerate Basic Feasible Solution:** A feasible solution to a transportation problem containing  $m$  origins and  $n$  destinations is said to be "non degenerate" if it contains  $m + n - 1$  occupied cells and each allocation is in an independent position.

**Note:** The allocations are in independent positions means it is tricky to form a closed path.

**Closed Path:** A path which is in the fashion by allowing transverse and vertical lines occupied only the corner cells.

**Degenerate Basic Feasible Solution:** A basic feasible solution in which the number of nonnegative allocations are less than  $m + n - 1$ .

**Non-negativity:** A non-negative allocations exist in the problem the values of all variables to be greater than or equal to zero. I.e. the minimum value is zero and maximum value may be any positive integer.

There are two steps to obtained optimal solution of transportation problem. The first step is finding Initial basic feasible solution (IBFS) and in the second step we find the optimal solution. It is necessary to find IBFS to solve TP. Finding Initial basic feasible solution is important because it is basic solution to obtained optimal solution. Sometimes IBFS can be near or similar to optimal solution. This research focus on finding Initial basic solution to obtain the minimum cost of the transportation problem. There are three popular methods which are used to find initial basic feasible solution of transportation problem.

1) Northwest Corner Method.

2) Least Cost Method.

3) Vogel's Approximation Method.

Here, we will study a new approach **Total Opportunity Cost Matrix-Minimal Total (TOCM-MT)** to find initial basic feasible solution of transportation problem. This new method is proposed by **Bilqis Amaliah, Chastine Fatichah and Erma Suryani**. The Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) is the integration of Total Opportunity Cost Matrix (TOCM) introduced by Kirca and Satir and Total Difference Method 1. The Total Difference Method 1 (TDM1) to obtain initial basic feasible solution of transportation problem was proposed by E. Hosseini in 2017.

Vogel's Approximation Method (VAM) is best for to find initial basic feasible solution for transportation problem. It calculates penalties for all rows and columns. While TDM1 calculate penalty just for rows. Another difference is that how to calculate penalty. The penalty of TDM1 is more complete than that of VAM. Because the VAM penalty is the difference between two least cost and TDM1 penalty is the difference between the least and other costs. The Advantage of Total Opportunity Cost Matrix-Minimal Total over Total Difference Method 1 are as follows.

- 1) The TOCM-MT use the TOCM as the initial matrix to increase the chance of selecting the least cost while TDM1 uses the original transportation matrix.
- 2) The TOCM-MT has the rules to choose highest penalty (HP) while TDM1 don't have the rules to choose the HP when there are several HP with the same worth.
- 3) TDM1 has no any mechanism to allocate maximum units to the least cost cell when the least cost is equal to zero but TOCM-MT has the mechanism. Therefore, it has the possibility to get minimal cost.

**Total Opportunity Cost Matrix (TOCM)** is obtained by transforming the original transportation matrix into an initial matrix by adding the row and column opportunity cost matrix. The row opportunity cost matrix subtracts every element in the row by the least cost in that row. Similarly for column opportunity cost matrix subtracts every element in the column by the least cost in it. Therefore, the Total Opportunity Cost Matrix is the sum of row and column opportunity cost matrix.

#### IV. TOTAL OPPORTUNITY COST MATRIX-MINIMAL TOTAL (TOCM-MT)

The proposed method is the combination of Total Opportunity Cost Matrix (TOCM) and modified Total Difference Method 1(TDM1).

**Step-1:** Construct an original transportation matrix( $m \times n$ ) with cost  $C_{ij}$ , Demand  $D_j, j = 1, 2, \dots, n$  and supply  $S_i, i = 1, 2, \dots, m$  if there is discrepancy in demand and supply, then we always add dummy row or column.

**Step-2:** find out the row opportunity cost matrix for original transportation matrix. Subtracts least cost of each row from each cost in that row.

**Step-3:** construct column opportunity cost matrix from original transportation matrix by subtracting least cost of every column from every element of that column

**Step-4:** Adding the row and column opportunity cost matrix to obtained Total Opportunity Cost Matrix (TOCM)

**Step-5:** calculate the penalty  $P$  for every row, which is the total difference between the least cost  $LC_i$  and other costs in the corresponding row.

$$LC_i = \min(C_{ij}), \quad j = 1, 2, \dots, n \quad (4)$$

$$P_i = \sum_{j=1}^n (LC_i - C_{ij}) \quad (5)$$

**Step-6:** Choose the Highest Penalty (HP) as in equation (6)

$$HP = \max(P_i), \quad i = 1, 2, \dots, m \quad (6)$$

when there is tie in equal HP use the following rules to select HP.

choose HP with smallest least cost cell ( $C_{ij}$ ).

if there is tie in (i) select HP with greater total cost ( $TC_i$ ) as in below equation.

$$TC_i = \sum_{j=1}^n C_{ij} \quad (7)$$

if tie in (ii) choose the HP with maximum allocation of  $X_{ij}$ .

**Step-7:** Choose the least cost ( $LC$ ) from the  $HP$ . if tie occur i.e equal  $LC$ . choose  $LC$  with maximum allocation possible.

**Step-8:** if least cost ( $LC$ ) is not zero, then go to next step 9. if least cost ( $LC$ ) is zero then select Highest Penalty ( $HP$ ) from the  $HP(HP_1)$  or  $HP(HP_2)$ . choose the  $HP$  by comparing every cost cell in  $HP_1$  and  $HP_2$ .  $C_{1j}, C_{2j}$  are the cost cells in  $HP_1$  and  $HP_2$  respectively. The value of  $GV_{1j}$  (greater value) is 1 if the cost in  $HP_1$  is bigger than the cost in  $HP_2$  otherwise zero. similarly, the value of  $GV_{2j}$  is 1 if the cost in  $HP_2$  is greater than the cost in  $HP_1$  and 0 if the cost in  $HP_2$  is smaller than  $HP_1$ .

$$TotalGV_{1j} = \sum_{j=1}^n GV_{1j} \quad (8)$$

Where,

$$GV_{1j} = \begin{cases} 1 & \text{if } CV_{1j} > CV_{2j}, j = 1, 2, \dots, n \\ 0 & \text{if } CV_{1j} \leq CV_{2j}, j = 1, 2, \dots, n \end{cases}$$

Similarly

$$TotalGV_{2j} = \sum_{j=1}^n GV_{2j} \quad (9)$$

$$and GV_{2j} = \begin{cases} 1 & \text{if } CV_{2j} > CV_{1j}, j = 1, 2, \dots, n \\ 0 & \text{if } CV_{2j} \leq CV_{1j}, j = 1, 2, \dots, n \end{cases}$$

$$HP = \begin{cases} HP_1 & \text{if } TotalGV_{1j} \geq TotalGV_{2j} \\ HP_2 & \text{if } TotalGV_{1j} < TotalGV_{2j} \end{cases} \quad (10)$$

**Step-9:** Allocate minimum units  $X_{ij} = \min(S_i, D_j)$  to the least cost cell of Highest Penalty (HP).

**Step-10:** Reduce the TOCM after removing satisfied row or column and adjust the supply and demand.

**Step-11:** Again, calculate the penalty without noticing the deleted row or column.

**Step-12:** Repeat the steps 6-11 till all the allocation has been made.

**Step-13:** After all, determined the total cost.

$$Total\ cost = \sum_{i=1}^m \sum_{j=1}^n C_{ij} \quad (11)$$

**Numerical:**

**Step-1:** Construction of original transportation problem.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	19	30	50	10	7
$S_2$	70	30	40	60	9
$S_3$	40	8	70	20	18
Demand	5	8	7	14	34

The given transportation problem is balanced so feasible solution exist.

**Step-2:** The row opportunity cost matrix. subtracts the least cost of every row from every cost of that row.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	19	30	50	10	7
$S_2$	70	30	40	60	9
$S_3$	40	8	70	20	18
Demand	5	8	7	14	

**Step-3:** The column opportunity cost matrix obtained by subtracting least cost of each column from each cost of the same column

**Step-4:** Obtained the Total Opportunity Cost Matrix (TOCM) by adding the row and column opportunity cost matrix.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	9	42	50	0	7
$S_2$	91	22	10	80	9
$S_3$	53	0	92	22	18
Demand	5	8	7	14	

**Step-5:** calculate the penalty for every row. Penalty is the total difference between the least cost and other cost in the same row as given below.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply	$P_1$
$S_1$	9	42	50	0	7	101
$S_2$	91	22	10	80	9	163
$S_3$	53	0	92	22	18	167
<b>Demand</b>	5	8	7	14		

**Step-6:** The highest penalty is 167.

**Step-7:** The least cost corresponding to highest penalty is '0'.

**Step-8:** The Least Cost is zero then using the given procedure in step 8 of the proposed method. The selection of Highest Penalty (HP) between  $HP_1 = 167$  and  $HP_2 = 163$ .

so, we compare the cost cells of  $HP_1 = 167$  and  $HP_2 = 163$  as below.

$$\begin{aligned}
 GV_{11} &= 0, (\because 53 < 91) \text{ and } GV_{21} = 1, (\because 91 > 53) \\
 GV_{12} &= 0, (\because 0 < 22) \text{ and } GV_{22} = 1, (\because 22 > 0) \\
 GV_{13} &= 1, (\because 92 > 10) \text{ and } GV_{23} = 0, (\because 10 < 92) \\
 GV_{14} &= 0, (\because 22 < 80) \text{ and } GV_{24} = 1, (\because 80 > 22)
 \end{aligned}$$

Where  $j = 1, 2, 3, 4$ .

$$\begin{aligned}
 \therefore \text{Total}GV_{1j} &= \sum_{j=1}^n GV_{1j} \text{ and } \text{Total}GV_{2j} = \sum_{j=1}^n GV_{2j} \\
 \text{Total}GV_{1j} &= 0 + 0 + 1 + 0 \text{ and } \text{Total}GV_{2j} = 1 + 1 + 0 + 1 \\
 \therefore \text{Total}GV_{1j} &= 1 \text{ and } \text{Total}GV_{2j} = 3 \\
 \therefore \text{Total}GV_{1j} &< \text{Total}GV_{2j}, (\because 1 < 3) \\
 \Rightarrow \text{HP} &= \text{HP}_1 = 163
 \end{aligned}$$

From the above argument we choose the  $HP = 163$ .

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply	$P_1$
$S_1$	9	42	50	0	7	101
$S_2$	91	22	10(7)	80	9/2	163↑
$S_3$	53	0	92	22	18	167
<b>Demand</b>	5	8	7/0	14		

The TOCM after removing the  $D_3$  column again calculate the penalty is  $HP = 127$  and corresponding least cost is 22.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_4$	Supply	$P_1$	$P_2$
$S_1$	9	42	0	7	101	51
$S_2$	91	22(2)	80	2/0	163↑	127↑
$S_3$	53	0	22	18	167	75
<b>Demand</b>	5	8/6	14			

After satisfying  $S_2$  row the TOCM and penalty as.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_4$	Supply	$P_1$	$P_2$	$P_3$
$S_1$	9	42	0	7	101	51	51
$S_3$	53	0	22	18	167	75	75
<b>Demand</b>	5	8/6	14				

The highest penalty in above matrix is 75 but the corresponding least cost is Zero. so using procedure in step 8. we select the Highest Penalty (HP) between  $HP_1 = 75$  and  $HP_2 = 51$ .

$$GV_{11} = 1, (\because 53 > 9) \text{ and } GV_{21} = 0, (\because 9 < 53)$$

$$GV_{12} = 0, (\because 0 < 42) \text{ and } GV_{22} = 1, (\because 42 > 0)$$

$$GV_{14} = 1, (\because 22 > 0) \text{ and } GV_{24} = 0, (\because 0 < 22)$$

$$TotalGV_{1j} = GV_{11} + GV_{12} + GV_{14} \text{ and}$$

$$TotalGV_{2j} = GV_{21} + GV_{22} + GV_{24}, \text{ where } j = 1, 2, 4.$$

$$TotalGV_{1j} = 1 + 0 + 1 = 2 \text{ and } TotalGV_{2j} = 0 + 1 + 0 = 1$$

$$\Rightarrow TotalGV_{1j} > TotalGV_{2j}, (\because 2 > 1)$$

$$\Rightarrow HP = HP_1 = 75$$

Destinations→ Sources ↓	$D_1$	$D_2$	$D_4$	Supply	$P_1$	$P_2$	$P_3$
$S_1$	9	42	0	7	101	51	51
$S_3$	53	0(6)	22	18/12	167	75	75↑
Demand	5	8/6/0	14				

We remove the satisfied column  $D_2$  and calculate the penalties for rows. The highest penalty is 31 and corresponding least cost is 22.

Destinations→ Sources ↓	$D_1$	$D_4$	Supply	$P_1$	$P_2$	$P_3$	$P_4$
$S_1$	9	0	7	101	51	51	9
$S_3$	53	22(12)	18/12/0	167	75	75↑	31↑
Demand	5	14/2					

Now we remove the satisfied row  $S_3$  and the penalty is 9 with least cost zero.

Destinations→ Sources ↓	$D_1$	$D_4$	Supply	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$S_1$	9	0(2)	7/5	101	51	51	9	9↑
Demand	5	14/2/0						

Now delete the  $D_4$  which is satisfied column. finally TOCM reduce as below and penalty 9 with least cost 9.

Destinations→ Sources ↓	$D_1$	Supply	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
$S_1$	9(5)	7/5/0	101	51	51	9	9↑	9↑
Demand	5/0							

All the Allocation made and all the row/column are satisfied.

The final allocation table is

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	19(5)	30	50	10(2)	7
$S_2$	70	30(2)	40(7)	60	9
$S_3$	40	8(6)	70	20(12)	18
Demand	5	8	7	14	34

Therefore, the total cost is given by

$$Total\ Cost = 19(5) + 10(2) + 30(2) + 40(7) + 8(6) + 20(12)$$

$$Total\ Cost = 95 + 20 + 60 + 280 + 48 + 240$$

$$Total\ Cost = 743$$

**Vogel's Approximation Method:**

VAM is a heuristic in nature and it is mostly preferred to find the initial basic feasible solution for the transportation problem. it gives better solution than Northwest Corner Method (NWCM) and Least Cost Method (LCM).

so we find the IBFS of the above numerical by VAM.

**Step-1:** Construction of transportation matrix, if transportation problem is not balanced then must add dummy row or column to get balanced transportation problem.

**Step-2:** Determine the penalty, is the difference between the FLC (First Least Cost) and SLC (Second Least Cost) for each row and column. Write penalty below the corresponding column and right to corresponding row.

**Step-3:** Now choose the row or column for which penalty is maximum. If there is a tie in maximum penalty i.e equal penalty then choose the penalty with maximum allocation possible. Allocate minimum  $\min(S_i, D_j)$  to the corresponding least cost cell.

**Step-4:** Cross out the satisfied row or column. If both row and column satisfied simultaneously, only one of the row or column crossed out. The surviving row or column zero demand (supply) allocation made.

The steps 2-4 repeat till all demand and supply satisfied.

The Transportation matrix of given numerical as below.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	19	30	50	10	7
$S_2$	70	30	40	60	9
$S_3$	40	8	70	20	18
Demand	5	8	7	14	

We find the IBFS of above transportation problem by VAM as follows.

The allocation matrix (table) is

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
$S_1$	19(5)	30	50	10(2)	7	9	9	40	40	-	-
$S_2$	70	30	40(7)	60(2)	9	10	20	20	20	20	40
$S_3$	40	8(8)	70	20(10)	18	12	20	50↑	-	-	-
Demand	5	8	7	14							
$P_1'$	21	22↑	10	10							
$P_2'$	21↑	-	10	10							
$P_3'$	-	-	10	10							
$P_4'$	-	-	10	50							
$P_5'$	-	-	40	60							
$P_6'$	-	-	40	-							

$$\therefore Total\ Cost = 19(5) + 10(2) + 40(7) + 60(2) + 8(8) + 20(10)$$

$$\therefore Total\ Cost = 95 + 20 + 280 + 120 + 64 + 200$$

$$\Rightarrow Total\ Cost = 779$$

**Total Difference Method 1:**

This method was proposed by E. Hosseini in 2017. This method is mostly similar to Vogel’s Approximation Method. In this method the penalty is the total difference between the least cost and the other cost of the same row. The penalty is calculated only for each row not for the column.

We find the initial basic feasible solution of above transportation problem by this method.

Destinations→ Sources ↓	$D_1$	$D_2$	$D_3$	$D_4$	Supply	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$S_1$	19(5)	30	50	10(2)	7/5/0	60	49	49	9	9
$S_2$	70	30	40(7)	60(2)	9/7/0	80	50	50↑	10	-
$S_3$	40	8(8)	70	20(10)	18/10/0	106↑	70↑	-	-	-
<i>Demand</i>	5/0	8/0	7/0	14/4/2/0	34					

where  $P_1, P_2, P_3, P_4, P_5$  are penalties for the rows.

$$\begin{aligned} \therefore \text{Total Cost} &= 19(5) + 10(2) + 40(7) + 60(2) + 8(8) + 20(10) \\ \therefore \text{Total Cost} &= 95 + 20 + 280 + 120 + 64 + 200 \\ &\Rightarrow \text{Total Cost} = 779 \end{aligned}$$

The improvement percentage in the IBFS of given numerical by Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) over Vogel’s Approximation Method (VAM) and Total Difference Method 1 (TDM1) is calculated as.

$$I_p = \frac{IBFS - T_m}{IBFS} \times 100$$

Where  $I_p$  = Improvement

$IBFS$  = Initial Basic Feasible Solution

$T_m$  = TOCM – MT

$$I_p = \frac{779 - 743}{779} \times 100$$

$$I_p = \frac{36}{779} \times 100 = 4.62$$

Which is the improvement percentage of initial basic feasible solution of above transportation problem by Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) over Vogel’s Approximation Method (VAM) and TDM1 (Total Difference Method 1).

**V. CONCLUSION**

We find the initial basic feasible solution of above transportation problem by this method and Minimizing transportation time and getting optimal solutions are always considered as important factors while solving transportation problem. This paper shows a new approach for finding initial basic solution for transportation problem which reduces cost of transportation more than any transportation method such as Northwest Corner Method(NCM), Least Cost Method(LCM), northwest, Vogel’s approximation, Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) and so on. The final allocation table and Numerical examples.

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