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Recent Advances in Fractional Order Control System: A Review

Roopam Shrivastav[1], Prashant Garg[2], Narottam Dutt Upadhyay[3]

Research Scholar, Department of Electrical Engineering[1]

Assistant Professor, Department of Electrical Engineering[2,3]

Vikrant Institute of Technology & Management, Gwalior, India

Abstract: *Several papers reviewing fractional order calculus in control applications have been published recently. These papers focus on general tuning procedures, especially for the fractional order proportional integral derivative controller. However, not all these tuning procedures are applicable to all kinds of processes, such as the delicate time delay systems. This motivates the need for synthesizing fractional order control applications, problems, and advances completely dedicated to time delay processes. The purpose of this paper is to provide a state of the art that can be easily used as a basis to familiarize oneself with fractional order tuning strategies targeted for time delayed processes. Solely, the most recent advances, dating from the last decade, are included in this review.*

Keywords: Fractional calculus, time delay process, fractional order control..

I. INTRODUCTION

Time delays are quite frequently encountered in industrial applications, such as heat exchanges, distillation units, min ing processes, steel manufacturing and so on. But they are not limited to industrial applications. Time delay processes span from biological to mechanical systems, including also economical or electrical fields. The physical phenomenon that generates time delays is the need to transport information, energy or different masses. Time lags accumulate also between interconnected systems or arise when sensors need measure and acquire signals and when microcontrollers(or other devices) compute the control signal and actuate upon the process.

From the frequency domain point of view, the presence of delay introduces an additional lag in the process phase. This results in lower phase and gain margins and ultimately complicates the closed loop control of these processes. The ideal situation is to design a controller that completely eliminates the effect of time delays. Many control strategies have been developed throughout the years to cope with time delay characteristics [1]–[4], but none of them proved to be an ideal solution. As the domain of fractional order controller gained more popularity, the control focus also reached the field of time delays processes. The desire is to combine the better performance of fractional calculus to the time delay control problem by extending fractional order design methods to the time delay field. Research output from the delay free processes suggest that using fractional order controllers can help improve robustness and closed loop response of time delay processes as well [4]–[7]. As it will be further detailedin this paper, some existing results have already accomplished better performance when using fractional order calculus in dealing with time delays compared to the traditional approach of using integer order controllers [8]–[12]. The advantages mentioned so far in existing literature refer to improved closed-loop results, considering modeling errors, due to the possibility of a more robust tuning of fractional order controllers [8]–[12]. The choice of the fractional order PID controller over classical, integer order control, is justified throughout literature in studies that focus on comparing the two control strategies tuned in similar manners for time delayed processes [8]–[10], [13]–[19]. For example, an anal-ysis of the effect of the fractional order derivative action over an unstable, first order, plant is realized in [31], where the closed loop stability is investigated for fractional derivative orders in the range (0, 2). The paper clearly proves that for the same derivative gain of the controller, a fractional order differentiator with the derivative gain smaller than 1 provides a more stable system than an integer order controller. A comparison of six different tuning rules for fractional order PI and PID controllers is presented in [32] based on different approaches.

Among the years, many well-known scientists contributedto the mathematical development of non-integer order dif-ferentiation. Euler and Lagrange were the first to intro-duce theoretical contributions in 18^{th} century, followed by Riemann, Liouville and Holmgren, with systematic studiesat the beginning of the 19^{th} century. Liouville developed function expansion in series of exponentials and definitionof n^{th} order derivative by operating term-by-term, while Riemann introduced the definite integral applicable to powerseries with non-integer exponents. Later, Grunwald and Krug unified the results of Liouville and Riemann, with the firstapplication of fractional calculus dating from 1823. Heav-iside developed symbolic methods for solving linear dif-ferential equations of constant coefficients, while Weil and Hardy defined the differ-integral operator properties andRiesz extended the result to multivariable functions, etc. [20].In the last decades, the number of applications for fractional-order calculus has been growing exponentially, mainly in the fields of control engineering, signal processing and systemtheory. The main advances were made by Bode's ideal loop transfer function, followed by Manabe's results on frequencyand transient response of the non-integer integral and its application in control. The first occurrence of fractional ordercontroller may be attributed to Oustaloup, who introduced and demonstrated the superiority of the Commande Robuste dOrdre Non Entier (CRONE) controller. The generalization of the integer order proportional-integrative-derivative (PID) controller to fractional order has been proposed by Podlubny[21]. The fractional order basic control actions, proportional,integral and derivative, add more flexibility to the set ofperformance specifications the closed loop system is able tofulfill. This is mainly due to the extra tuning parameters of the fractional order PID (FOPID): the fractional order of integra- tion and the fractional order of differentiation. Even thoughthe FOPID represents the most common fractional ordercontrol algorithm, other types of fractional order controllershave been designed, as it will be indicated later in this paper.Review papers focusing on the use of fractional calculus in control engineering have been published recently suchas [22]–[25] and provide an insight into fractional order control of different types. Analytic, numerical and rule-based tuning methods for fractional order PID controllers only hasalso been published [26]. Some of these methods can also be used to control time delay systems.

The main contribution of the present paper is the focus ontime delayed processes and available fractional order meth-ods for their control. As such, all fractional order controller algorithms, not just the FOPID, are included in the review. The paper provides the reader with a brief summary of the control strategies as well as relevant literature. The purpose of this paper is to provide a state of the art that can be easilyused as a basis to familiarize oneself with fractional order tuning strategies targeted for time delayed processes. The most recent advances, dating solely from the last decade, are evidenced focusing mainly on the controller tuning approach, without providing any numerical examples, which can be found in the quoted literature. In this review paper, we con- sider processes with significant time delays and as such, tuning methods based on neglecting the time delay are not presented here.

The paper is structured as follows. The second section, after the Introduction, details the FOPID tuning methods, starting with the most commonly used design technique based on a frequency domain approach (sub-section A). Then, tuning methods for FOPIDs based on minimizing some timedomain cost functions, such as Integral of Square Error (ISE), Integral of Time Absolute Error (ITAE), Integral of AbsoluteError (IAE), etc. are presented in sub-section B. A third sub-section includes the tuning of FOPIDs using an extension ofthe popular M_s Constrained Integral Optimization (MIGO) method, called here the F-MIGO (Fractional-MIGO). Sub- section D gathers the Pontryagin and Hermite-Biehler theo-rems used in the tuning of FOPID controllers. Other tuning methods are included in sub-section E. All these tuning methods require a process model. Sub-section F details the auto-tuning methods for fractional order controllers for time delay processes. A process model is not required here. The common factor of all these tuning methods is that the classical closed loop system is used. However, other control schemescan be used in the control of time delay processes, such as the Internal Model Control (IMC) strategy or the Smith Predictor (SP). The next two sections of the paper include particularities in the design of fractional order controllers inthe framework of an IMC (Section III) or a SP (Section IV) control structure. Section V details the existing control algorithms that combine fractional calculus and advanced control methods. Section VI is a collection of existing papersthat provide the experimental validation of various types of fractional order controllers on time delay processes. Finally,the last section presents the concluding remarks and future challenges.

II. TUNING METHODS FOR FRACTIONAL ORDER CONTROLLERS (FOC) FOR TIME DELAY SYSTEMS

Classical tuning algorithms are used in the design of fractional order controllers, such as the Ziegler-Nichols rules [27], Hermite-Biehler and Pontryagin theorems [18] linear programming formulation [32], F-MIGO optimization [16], [33]. Other tuning methods based on optimizing a certain performance index were developed [8], [13]. The PID block diagram from Fig. 1 is based on the classic feedback control structure with the mention that the controller is of fractional order

FREQUENCY DOMAIN TUNING FOR FOPID

As a generalization of the classical PID (Proportional Integral Derivative) control approach to arbitrary orders of integration and differentiation, the fractional order PID, frequently encountered under the notation of $PI^{\lambda}D^{\mu}$, outbounds the limitations induced by integral order differentiation. The transferfunction of the FOPID controller can be written as

$$H_{FOPID} = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu}$$

where kp, ki and kd are the proportional, integral and derivative gains, while λ and μ are the fractional orders of integration and differentiation. Theoretically, λ and μ can take any real, positive value. However, for the controller to have physical meaning, the interval of the fractional orders of integration and differentiation is usually limited to (0, 2) [21]. One of the most popular methods for tuning the fractional order PID controller is determining the parameters of the con- troller by solving a system of nonlinear equations expressing specifications related to phase margin, gain crossover fre- quency, sensitivity functions and robustness to gain changes in a limited interval. The particularities of the system to be controlled dictates the requirements the controller needs to fulfill, determining the choice of tuning specifications Several works such as [14], [21], approach the tuning of the fractional order PID controller through frequency domain specifications, a method with high applicability to any time delay process, regardless of the time delay amount. The constraints related to the imposed phase margin, denoted by φm, and gain crossover frequency ωcg influence the damping ratio of the closed loop system, the settling time, as well as the robustness.

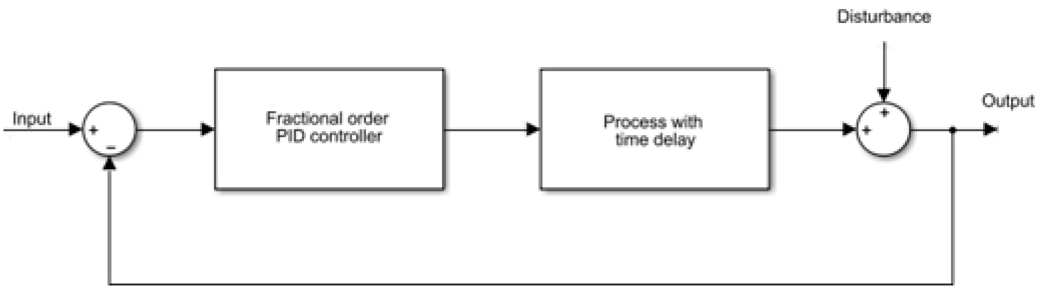

Fig-1 Fractional order $PI^{\lambda}D^{\mu}$control structure

III. TUNING FOPIDS BASED ON TIME DOMAIN COSTFUNCTIONS AND OPTIMIZATION ROUTINES

Variations of using the previously described frequency domain specifications can be found by combining the frequency constraints with other tuning techniques. Such an example is presented in [3] where the authors tune a PI controller based on Bode's ideal transfer function for the open loop system. Iterative optimization routines are usedto minimize a quadratic cost function based on the sum of squares of the control signal input and the sum of the integral squared error between the closed loop system with the PI con- troller and the time response desired process response. The optimization features two additional constraints regarding the sensitivity and the complementary sensitivity equations from (5) and (6). The proposed method is highly versatile, being validated on both integer and fractional order models as well as processes with considerable time delays. A set of tuning rules to determine both integer and fractional order controllers for first order plus time delay plants are high- lighted in [13].

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F-MIGO METHODS

Fractional M_s Constrained Integral Optimization, defined as F-MIGO through the specialized literature is the fractional extension of the MIGO algorithm designed by Astrom et. al. as an improvement to the simple and tuning-friendly Ziegler-Nichols rules. The procedure consists of optimizing the load disturbance rejection by constraining the maximum sensitivity function. Comparisons between theF-MIGO and other fractional order design strategies is realized in [19] by analyzing and comparing the closed loop system's performance in terms of time domain and frequency domain specifications. The method is based on the assump- tion that the transfer function of the process is known, linear, with a finite number of poles and exhibits a single singularityat infinity.

AUTOTUNING OF FOPID CONTROLLERS

Whilst the other subsections are abundant with recent worksrevolving around fractional order tuning for time delay processes using a mathematical model of the system to be con- trolled, autotuning methods for FOPIDs are relatively scarce.One recent study is presented in [17] where a fractional order PI controller is determined. Even if the method is of an experimental nature, the authors use only numerical simulations to prove the veracity of the proposed approach. Also, the method is focused around plants exhibiting large amounts of time delay. A relay test is used to find the fre- quency of the process to be controlled and an integer order PI control law is computed using the previously determined process frequency and the classical Ziegler-Nichols tuning rules. Starting from the integer order controller, an extension into the fractional dimension is done by varying the fractional order of integration and analyzing the time domain closed loop system response. The advantage of the FO-KC autotuning principle is that thenonlinear system of equations that needs to be solved to estimate the FOPI/FOPD parameters is no longer required. Instead, the design is based on using the phase margin requirement to define a forbidden region in the Nyquist plane that the loop frequency response should avoid. More- over, to ensure the iso-damping property, the optimal FOPI/FOPD controller is determined such that the differ- ence between the slope of the loop frequency response and the slope of the forbidden region border is mini- mum, at the gain crossover frequency. The method can be applied to time delay processes as well. In [6], the pro- posed autotuning procedure is limited to systems with delay and order greater than one. Firstly, the identification of the process at the desired crossover frequency is performed. Next, the parameters of the fractional order PID controller are determined using the same set of perfor- mance specifications: crossover frequency, phase margin andthe iso-damping property.

PMITH-pREDICTORS AND FOC

A couple of researchers have proposed to solve the problemof controlling time delay systems based on combining various types of FO controllers and classical or modified Smith Predictor structures (Fig.2).

In [10], two Smith Predictor based controllers are used inthe comparison. The paper uses a mismatched model to analyze the performance of the control strategy. The controller chosen is a fractional order CRONE controller, which is well known for increased robustness and performance tradeoff. The conclusion of the research is that the use of an improved Smith predictor is not necessary to obtain good performance,even though it can be used as a means of simplifying the design of the (robust) controller. In [11], a new fractional- order PI controller embedded in a Smith predictor is proposed. The design is based on fractional calculus and Bode'sideal transfer function. In this case, the model considered in the design is of a first-order-plus-dead-time, with the analytical tuning rules derived by using the frequency domain. Later, the tuning rules are easily applied to various dynamics, including both the integer-order and fractional-order dynamic processes. The approach benefits from a couple of advantages, such as a simple design scheme, a straightforward method, which can be easily implementedin the process industry. Numerous numerical examples are included to show the superior closed-loop performance, for both servo and regulatory problems, in comparison with other reported controllers in terms of the minimum integral absolute error with a constraint on the maximum sensitivity value.

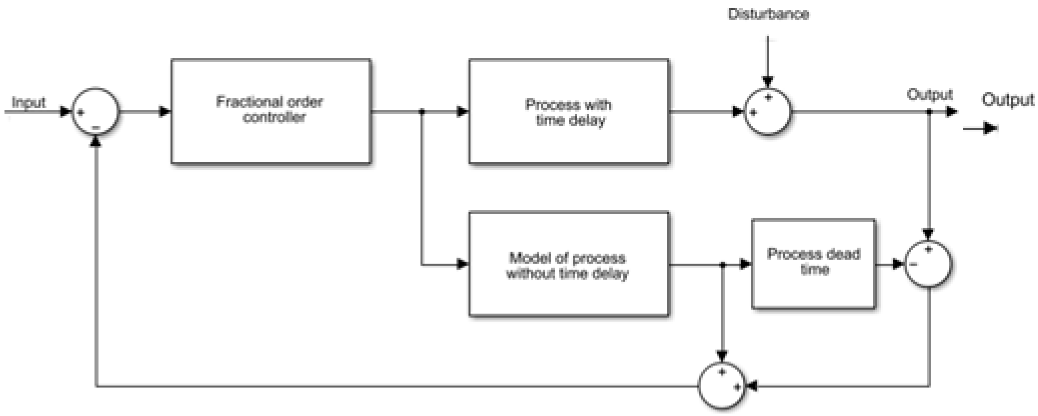

Fig-2 Smith predictor control structure

Most of the times, the design of the fractional order controllers is done using frequency domain approach, dueto a simpler representation of fractional order systems inthis domain. However, time domain approaches have been also reported, in [5] and [12]. In the case of [5], a new approach to the design of fractional order PI controllers in Smith Predictor structures for varying time delay systemsis proposed. The design is focused on ensuring robustnessof the closed loop system against time delay uncertainties and it is based on time domain specifications, rather thanthe more widely used frequency approach. Apart from this, the proposed tuning method relies on additional robustness to plant uncertainties, achieved by maximizing open-loop gainmargin, through an iterative procedure. The simulation example provided demonstrates the efficiency of the proposed method, in comparison to classical integer order PI controller In [12], a simple and efficient analytical method to design a fractional order controller for time delay integer order systems is proposed, considering a Smith predictor structure.The design procedure is based on time-domain specifications, such as the percentage of overshoot and settling time. The tuning formulas are then derived based on ideal closed-loop transfer function. The major advantage of this design methodconsists in only two tuning parameters, which can be obtained using an explicit set of tuning formulas. Two simulation examples are used, including a robustness test considering plant uncertainties, to demonstrate the performance of the proposed controller compared to those provided by several well-known design techniques. A time domain approach is also considered in [13] and [14]. In [13], a new methodology to design fractional order integral controllers combined with Smith predictors is proposed with the target of ensuring the robustness to high frequency model changes, especially changes in the time delay. The case study considered hereis the water distribution in a main irrigation canal pool. The fractional order integral controllers perform better and are more robust than the integer order PI or PID controllers. Additionally, they also show less sensitivity to high fre- quency measurement noise and disturbances. Comparisons with more complex control techniques such as predictive con- trol and robust *H* controllers, demonstrate that the proposed fractional order integral controllers have better or similar performances than these. In [64], the idea in [13] is reiterated. Time domain specifications are used in the design of a frac-tional order PI controller combined with a Smith Predictor, which is later compared to an integer order PI controller in a Smith Predictor structure. The extra tuning parameter of the fractional order controller is used to maximize the robustness to variations in plant parameters. Simulated results show the robustness improvements achieved with this controller compared with a conventional PI controller.

In [17], an original model based analytic method is devel- oped. The method attempts to design a fractional order controller combined with a Smith predictor and a modified Smith predictor that yield control systems, which are robust to changes in the process parameters. The proposed method is not limited to fractional order systems and works well for integer order systems as well. The tuning procedure is based on Bode's ideal transfer function and the IMC (internal model control) principle. In this case also, only simulation results are presented, which demonstrate the successful performance of the proposed method for controlling integer as well as fractional order linear stable systems with long time delay.

In [22], the differential evolution algorithm is used to deter- mine the five parameters of the FOPID controller. The tuning is based on user-specified peak overshoot and settling time and has been formulated as a single objective optimization problem. The control structure used is the Smith Predictor. Simulation results are provided that show that the FOPID controller performs better than its integer order version.

Another FOPID controller is incorporated in the Smith Predictor control structure generalized for plants exhibiting time delay in [6]. The fractional order controller is determined by solving the set of the five non-linear equations related to gain crossover frequency, phase margin, robustness, sensitivity and complementary sensitivity. The proposed control structure is validated on numerical simulations for plants with different amounts of time delay.

For first order uncertain systems, an interval-based stabilization method using stability conditions of the non-commensurate elementary fractional order transfer function of the second kind is developed in [16]. The design of all stabilizing controllers is based upon some analytic expressions, including those derived for a robust performance, such as the iso-damping property. The fractional order controller obtained is combined with the Smith Predictor to control a first-order system with time delay. Only numerical examples are included here, also.

The Smith Predictor is also the chosen control structure in [15], where a FOPI controller is designed for a second order time delay system. The tuning is based on phase margin and gain crossover frequency specifications, com- bined with a condition to maximize the loop gain margin. A hardware-in-the-loop approach is considered to validate the results, with the digital FOPI controller combined with the Smith Predictor implemented in LabVIEW.

The long time delay compensating Smith Predictor based control scheme is also proposed in [69]. The FOMCON toolbox with MATLAB/Simulink is used to determine a fractional order PID controller for a coupled tank system. The design is based on an iterative optimization technique. Comparisons with a PID controller based on the Smith Pre-dictor, designed in a same way, are also included.

In [70], several fractional order control algorithms are investigated and compared, when integrated within a Smith Predictor control structure. Simulations are presented to eval- uate the performance of the proposed fractional order control algorithms, on a heat diffusion system, selected as the case study.

Two tuning algorithms for fractional-order internal model control (IMC) controllers for time delay processes are pre-sented in [12]. One of these tuning algorithms is based on Smith Predictor structure, where the equivalency between IMC and Smith predictor control structures is used to tune a fractional-order IMC controller as the primary controller of the Smith predictor structure. The design of the fractional order IMC controller is done with the purpose of enhanc- ing the closed-loop performance and robustness of classical integer order IMC controllers. Several numerical examples, as well as an experimental validation are provided for a multivariable system.

A multivariable case study is presented in [24], as well, where the design method of fractional order Smith Predic- tor controller was proposed. The idea is also based on the equivalency between the Smith Predictor and IMC structures. A decoupler is used to achieve non-square system decoupling, while the particle swarm optimization technique is used to reduce the complexity of the decoupled system from a high order to a first order plus time delay model. Finally, based on the IMC principle, a fractional order controller was designed, and the controller parameters were tuned by using the dom- inant pole placement method. The simulation results show that the proposed method is simple, and can provide a better dynamic performance and robustness.

IV. CONCLUSION

The review study familiarizes the reader with recent advances in fractional order control for time delayed plants. Due to recent advances in fractional order control, the paper focuses on publications later than 2010. The study is categorized into sections based on the closed loop control structure used (feedback control, IMC or the SP control scheme). The study also delimits the several existing tuning methods for FOPID controllers and the fractional order extensions to advanced control strategies.

Fractional order Proportional Integrator Derivative, Inter- nal Model Control, Smith Predictor, Sliding Mode Control are the main structures detailed, providing the user a brief summary of the design method and its requirements while also specifying relevant literature where the design methods were applied on time delayed processes. Analyz- ing the amount of relevant literature present in every cate- gory, it is clear that the fractional order PID in the classi- cal negative feedback structure is the most popular control approach. Another important remark is the purely analytic aspect of all the cited works.

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REFERENCES

[1] C. C. Prasad and P. R. Krishnaswamy, ''Control of pure time delay processes,'' Chem. Eng. Sci., vol. 30, no. 2, pp. 207–215, Feb. 1975.

[2] M. Bozorg and E. J. Davison, ''Control of time delay processes with uncer- tain delays: Time delay stability margins,'' J. Process Control, vol. 16, no. 4, pp. 403–408, Apr. 2006C.

[3] K. Watanabe, E. Nobuyama, and A. Kojima, ''Recent advances in control of time delay systems—A tutorial review,'' in Proc. 35th IEEE Conf. Decis. Control, vol. 2, Dec. 1996, pp. 2083–2089.

[4] W. Tang, S.-J. Shi, and M.-X. Wang, ''Autotuning PID control for large time-delay processes and its application to paper basis weight control,'' Ind. Eng. Chem. Res., vol. 41, no. 17, pp. 4318–4327, Jul. 2002.

[5] C. I. Pop, C. Ionescu, R. De Keyser, and E. H. Dulf, ''Robustness evaluation of fractional order control for varying time delay processes,'' Signal, Image Video Process., vol. 6, no. 3, pp. 453–461, Sep. 2012.

[6] C. I. Muresan, S. Folea, G. Mois, and E. H. Dulf, ''Development and imple- mentation of an FPGA based fractional order controller for a DC motor,'' Mechatronics, vol. 23, no. 7, pp. 798–804, 2013.

[7] S. Folea, C. I. Muresan, R. De Keyser, and C. M. Ionescu, ''Theoreti- cal analysis and experimental validation of a simplified fractional order controller for a magnetic levitation system,'' IEEE Trans. Control Syst. Technol., vol. 24, no. 2, pp. 756–763, Mar. 2016.

[8] S. Sadati, A. Ranjbar, and R. Ghaderi, ''Fractional-order control of a nonlinear time-delay system: Case study in oxygen regulation in the heart- lung machine,'' vol. 2012. Jul. 2012.

[9] R. Gopinath, ''Article: Design of fractional order controllers for first order plus time delay systems,'' in Proc. IJCA Proc. Int. Conf. Innov. Intell. Instrum., Optim. Elect. Sci. (ICIIIOES), vol. 13, Dec. 2013, pp. 29–33.

[10] S. E. Hamamci and M. Koksal, ''Calculation of all stabilizing fractional- order PD controllers for integrating time delay systems,'' Comput. Math. Appl., vol. 59, no. 5, pp. 1621–1629, Mar. 2010.

[11] T. Vinopraba, N. Sivakumaran, N. Selvaganesan, and S. Narayanan, ''Sta- bilization using fractional-order PIDα controllers for first order time delay system,'' in Proc. Int. Conf. Adv. Comput., Control, Telecommun. Tech- nol. (ACT), Dec. 2009, pp. 725–728.

[12] C. I. Muresan, A. Dutta, E. H. Dulf, Z. Pinar, A. Maxim, and C. M. Ionescu, ''Tuning algorithms for fractional order internal model controllers for time delay processes,'' Int. J. Control, vol. 89, no. 3, pp. 579–593, Oct. 2016.

[13] F. Padula and A. Visioli, ''Tuning rules for optimal PID and fractional- order PID controllers,'' J. Process Control, vol. 21, no. 1, pp. 69–81, Jan. 2011.

[14] Y. Luo and Y. Chen, ''Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems,'' Automatica, vol. 48, no. 9, pp. 2159–2167, Sep. 2012.

[15] T. Bhaskaran, Y. Chen, and D. Xue, ''Practical tuning of fractional order proportional and integral controller (i): Tuning rule development,'' in Proc. ASME Int. Design Eng. Tech. Conf. Comput. Inf. Eng. Conf., IDETC/CIE, Sep. 2007, pp. 1245–1258.

[16] V. Bhambhani, Y. Chen, and D. Xue, ''Optimal fractional order propor- tional integral controller for varying time-delay systems,'' IFAC Proc. Volumes, vol. 41, no. 2, pp. 4910–4915, 2008.

[17] H. Malek, Y. Luo, and Y. Chen, ''Identification and tuning fractional order proportional integral controllers for time delayed systems with a fractional pole,'' Mechatronics, vol. 23, no. 7, pp. 746–754, Oct. 2013.

[18] A. Ruszewski and A. Sobolewski, ''Comparative studies of control systems with fractional controllers,'' Przeglad Elektrotechniczny, vol. 88, no. 4b, pp. 204–208, Jan. 2012.

[19] B. Hmed, M. Amairi, M. Aoun, and S. E. Hamdi, ''Comparative study of some fractional PI controllers for first order plus time delay systems,'' in Proc. 18th Int. Conf. Sci. Techn. Autom. Control Comput. Eng. (STA), Dec. 2017, pp. 278–283.

[20] D. Stéphane and C. Houzel, ''Les différentielles métaphysiques histoire et philosophie de la généralisation de l'ordre de la dérivation,'' Ph.D. dissertation, Dept. Math., Univ. Paris, Paris, France, 1994.

[21] A. Monje, Y. Chen, B. M. Vinagre, D. Xue, and V. Feliu-Batlle, Funda- mentals of Fractional-Order Systems. London, U.K.: Springer, 2010.

[22] S. Sondhi and Y. V. Hote, ‘‘Fractional order controller and its applications: A review,’’ in Proc. AsiaMIC, 2012.

[23] A. Soukkou, M. C. Belhour, and S. Leulmi, ‘‘Review, design, optimization and stability analysis of fractional-order PID controller,’’ Int. J. Intell. Syst. Technol. Appl., vol. 8, no. 7, pp. 73–96, Jul. 2016.

[24] A. Y. Leng, Z. D. Qi, L. Shan, and H. J. Bian, ‘‘Review of fractional order control,’’ in Modern Technologies in Materials, Mechanics and Intelligent Systems (Advanced Materials Research), vol. 1049. Cham, Switzerland: Trans Tech Publications, 2014, pp. 983–986.