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A Simple Linear Controller Design to Ensure Global Exponential Stability for a Class of Uncertain Fifth-Order Nonlinear Systems

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Abstract*: This paper intends to explore the controller design problem for a class of uncertain fifth-order nonlinear control systems. Combining the theory of differential and integral inequalities, a linear controller will be proposed to promote a class of nonlinear control systems with multiple uncertainties to achieve the goal of global exponential stability. In addition, the guaranteed exponential convergence rate of such uncertain nonlinear systems will also be precisely calculated. Finally, some numerical simulation results will also be presented to verify and illustrate the correctness of this main theorem and the design process of the controller*

Keywords: Nonlinear system, Global exponential stabilization, Uncertain systems, Linear control, Multiple uncertainties

I. INTRODUCTION

Since all real physical systems are nonlinear systems, and the system model is simplified or the system parameters are difficult to estimate, the analysis and design of uncertain nonlinear dynamic systems is very important for researchers. In recent years, various problems related to uncertain nonlinear systems have been widely and extensively raised, and impressive results have been presented; see, for example, [1]-[13] and the references therein. It is worth mentioning that, based on integral and differential inequalities, a linear filter has been designed in [8] to achieve the global exponential stabilization for a class of uncertain nonlinear systems. Besides, the concept of global exponential ε-stabilization has been introduced in [9] and a nonlinear control has also been established to attain the global stabilization for a type of uncertain nonlinear systems.

In this paper, motivated by [8] and [9], a simple linear controller will be designed to achieve global exponential stability for a class of nonlinear control systems with multiple uncertainties. Not only will the exponential convergence rate be calculated, but we will also provide multiple numerical simulation results to demonstrate the practicality and correctness of this paper. Throughout this paper, $|a|$ denotes the modulus of a complex number *a* and $\|x\|$ means the Euclidean norm of the vector $x \in \Re^n$.

II. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we explore the following nonlinear systems with multiple uncertainties:

$$\dot{x}_1 = \Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_5 + f_1(x_1, x_2, x_3, x_4, x_5), \quad \text{(1a)}$$

$$\dot{x}_2 = \Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + \Delta d_7 x_5 + f_2(x_1, x_2, x_3, x_4, x_5) + \Delta\phi_1(u_1), \quad \text{(1b)}$$

$$\dot{x}_3 = \Delta d_8 x_2 + \Delta b x_3 + \Delta d_9 x_4 + \Delta d_{10} x_5 + f_3(x_1, x_2, x_3, x_4, x_5), \quad \text{(1c)}$$

$$\dot{x}_4 = \Delta d_{11} x_2 + \Delta c x_4 + \Delta d_{12} x_5 + f_4(x_1, x_2, x_3, x_4, x_5), \quad \text{(1d)}$$

$$\dot{x}_5 = \Delta d_{13} x_1 + \Delta d_{14} x_2 + \Delta d_{15} x_3 + \Delta d_{16} x_4 + \Delta d_{17} x_5 + f_5(x_1, x_2, x_3, x_4, x_5) + \Delta\phi_2(u_2), \quad \text{(1e)}$$

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where $x(t):=[x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t)]^T \in \Re^{5\times 1}$ is the state vector, $u(t):=[u_1(t) \quad u_2(t)]^T \in \Re^{2\times 1}$ is the input vector, $\Delta a, \Delta b, \Delta c,$ and Δd_i are uncertain parameters, f_i is smooth nonlinear function with $\Delta f_i(0,0,0,0,0)=0, \forall\, i \in \{1,2,3,4,5\}$, and the smooth operator $\Delta\phi_i(u): \Re \to \Re, \forall\, i \in \{1,2\}$ is the uncertain input nonlinearity.

The following assumptions are made for the above uncertain and nonlinear terms:

(A1) There are constants $\underline{a}, \underline{b}, \underline{c},$ and $\overline{d_i}$ such that

$$\Delta a \le -\underline{a} < 0, \quad \Delta b \le -\underline{b} < 0, \quad \Delta c \le -\underline{c} < 0, \quad |\Delta d_i| \le \overline{d_i}, \quad \forall\, i \in \{1,2,3,\cdots,17\}.$$

(A2) There are positive numbers r_1 and r_2 such that

$$r_i \cdot u^2 \le u \cdot \Delta\phi_i(u), \quad \forall\, i \in \{1, 2\}.$$

(A3) There are positive numbers $k_1, k_2, k_3, k_4,$ and k_5 such that

$$\sum_{i=1}^{5} k_i^2 \cdot x_i \cdot f_i(x_1, x_2, x_3, x_4, x_5) = 0.$$

The definition of global exponential stability and exponential convergence rate are introduced below.

Definition 1 [8, 9]: If there are positive numbers α, k and an appropriate controller u such that $\|x(t)\| \le k \cdot e^{-\alpha t}, \quad \forall\, t \ge 0,$

then the uncertain nonlinear systems (1) are said to be globally exponentially stabilized. In this situation, the positive number α is called the exponential convergence rate.

The primary goal of this paper is to design a linear controller for the uncertain nonlinear control systems of (1), so that the entire closed-loop system can achieve global exponential stability. Furthermore, we will calculate the exponential convergence rate of the above uncertain nonlinear systems based on theoretical derivation.

In the following we present the main result for the global exponential stabilization of uncertain nonlinear systems of (1).

Theorem 1: The uncertain nonlinear systems (1) with (A1)-(A3) are globally exponentially stabilized at the zero equilibrium point, provided that

$$\sqrt{\underline{b} \times \underline{c}} \ge \frac{k_3 \overline{d_9}}{k_4}. \qquad (2)$$

Besides, in this situation, a suitable linear controller is given as

$$u = [-\eta_1 x_2 \quad -\eta_2 x_5]^T, \; (3)$$

with

$$\eta_1 \ge \frac{\delta_1 + \dfrac{\left(\dfrac{k_1\overline{d_1}}{k_2} + \dfrac{k_2\overline{d_3}}{k_1}\right)^2}{\underline{a}} + \overline{d_4} + \dfrac{2\left(\dfrac{k_2\overline{d_5}}{k_3} + \dfrac{k_3\overline{d_8}}{k_2}\right)^2}{\underline{b}} + \dfrac{2\left(\dfrac{k_2\overline{d_6}}{k_4} + \dfrac{k_4\overline{d_{11}}}{k_2}\right)^2}{\underline{c}} + \dfrac{\left(\dfrac{k_2\overline{d_7}}{k_5} + \dfrac{k_5\overline{d_{14}}}{k_2}\right)}{2}}{r_1}, \qquad (4)$$

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$$\eta_2 \geq \frac{\delta_2 + \frac{\left(\frac{k_1\overline{d_2}}{k_5}+\frac{k_5\overline{d_{13}}}{k_1}\right)^2}{\underline{a}} + \frac{2\left(\frac{k_3\overline{d_{10}}}{k_5}+\frac{k_5\overline{d_{15}}}{k_3}\right)^2}{\underline{b}} + \frac{2\left(\frac{k_4\overline{d_{12}}}{k_5}+\frac{k_5\overline{d_{16}}}{k_4}\right)^2}{\underline{c}} + \overline{d_{16}} + \frac{\left(\frac{k_2\overline{d_7}}{k_5}+\frac{k_5\overline{d_{14}}}{k_2}\right)}{2}}{r_2}, \quad (5)$$

$\delta_1 > 0$, and $\delta_2 > 0$. Meanwhile, the guaranteed exponential convergence rate is calculated as

$$\alpha := \min\left\{\frac{\underline{a}}{2}, \delta_1, \frac{\underline{b}}{4}, \frac{\underline{c}}{4}, \delta_2\right\}. \quad (6)$$

Proof. Let

$$V(x(t)) := \sum_{i=1}^{5} k_i^2 \cdot x_i^2(t) . (7)$$

The time derivative of $V(x(t))$ along the trajectories of uncertain systems (1), with (A1)-(A3) and (2)-(7), is given by

$$\begin{aligned}
\dot{V}(x(t)) &= 2k_1^2 \cdot x_1\dot{x}_1 + 2k_2^2 \cdot x_2\dot{x}_2 + 2k_3^2 \cdot x_3\dot{x}_3 + 2k_4^2 \cdot x_4\dot{x}_4 + 2k_5^2 \cdot x_5\dot{x}_5 \\
&= 2k_1^2 x_1(\Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_5 + f_1) \\
&\quad + 2k_2^2 x_2[\Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + \Delta d_7 x_5 + f_2 + \Delta\phi_1(u_1)] \\
&\quad + 2k_3^2 x_3(\Delta d_8 x_2 + \Delta b x_3 + \Delta d_9 x_4 + \Delta d_{10} x_5 + f_3) \\
&\quad + 2k_4^2 x_4(\Delta d_{11} x_2 + \Delta c x_4 + \Delta d_{12} x_5 + f_4) \\
&\quad + 2k_5^2 x_5[\Delta d_{13} x_1 + \Delta d_{14} x_2 + \Delta d_{15} x_3 + \Delta d_{16} x_4 + \Delta d_{17} x_5 + f_5 + \Delta\phi_2(u_2)] \\
&\leq -2k_1^2 \underline{a} x_1^2 + 2k_1^2\overline{d_1}|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_3}|x_1||x_2| + 2k_2^2\overline{d_4}x_2^2 + 2k_2^2\overline{d_5}|x_2||x_3| \\
&\quad + 2k_2^2\overline{d_6}|x_2||x_4| + 2k_2^2\overline{d_7}|x_2||x_5| + 2k_3^2\overline{d_8}|x_2||x_3| \\
&\quad - 2k_3^2\underline{b}x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| + 2k_3^2\overline{d_{10}}|x_3||x_5| + 2k_4^2\overline{d_{11}}|x_2||x_4| \\
&\quad - 2k_4^2\underline{c}x_4^2 + 2k_4^2\overline{d_{12}}|x_4||x_5| + 2k_5^2\overline{d_{13}}|x_1||x_5| + 2k_5^2\overline{d_{14}}|x_2||x_5| \\
&\quad + 2k_5^2\overline{d_{15}}|x_3||x_5| + 2k_5^2\overline{d_{16}}|x_4||x_5| + 2k_5^2\overline{d_{17}}x_5^2 \\
&\quad + 2(k_1^2 x_1 f_1 + k_2^2 x_2 f_2 + k_3^2 x_3 f_3 + k_4^2 x_4 f_4 + k_5^2 x_5 f_5) \\
&\quad + 2k_2^2 x_2\Delta\phi_1(u_1) + 2k_5^2 x_5\Delta\phi_2(u_2) \\
&= -2k_1^2\left(\frac{\underline{a}}{4}+\frac{\underline{a}}{4}+\frac{\underline{a}}{2}\right)x_1^2 + 2k_1^2\overline{d_1}|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_3}|x_1||x_2| + 2k_2^2\overline{d_4}x_2^2 + 2k_2^2\overline{d_5}|x_2||x_3| \\
&\quad + 2k_2^2\overline{d_6}|x_2||x_4| + 2k_2^2\overline{d_7}|x_2||x_5| + 2k_3^2\overline{d_8}|x_2||x_3| \\
&\quad - 2k_3^2\left(\frac{\underline{b}}{8}+\frac{\underline{b}}{8}+\frac{\underline{b}}{2}+\frac{\underline{b}}{4}\right)x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| + 2k_3^2\overline{d_{10}}|x_3||x_5| + 2k_4^2\overline{d_{11}}|x_2||x_4| \\
&\quad - 2k_4^2\left(\frac{\underline{c}}{8}+\frac{\underline{c}}{8}+\frac{\underline{c}}{2}+\frac{\underline{c}}{4}\right)x_4^2 + 2k_4^2\overline{d_{12}}|x_4||x_5| + 2k_5^2\overline{d_{13}}|x_1||x_5| + 2k_5^2\overline{d_{14}}|x_2||x_5| \\
&\quad + 2k_5^2\overline{d_{15}}|x_3||x_5| + 2k_5^2\overline{d_{16}}|x_4||x_5| + 2k_5^2\overline{d_{17}}x_5^2 + 2k_2^2 x_2\Delta\phi_1(u_1) + 2k_5^2 x_5\Delta\phi_2(u_2)
\end{aligned}$$

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$$
\begin{aligned}
&= -2k_1^2\left(\frac{a}{4}+\frac{a}{4}+\frac{a}{2}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_8}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{11}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{14}}\right)|x_2||x_5| - 2k_3^2\left(\frac{b}{8}+\frac{b}{8}+\frac{b}{2}+\frac{b}{4}\right)x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{10}}+k_5^2\overline{d_{15}}\right)|x_3||x_5| - 2k_4^2\left(\frac{c}{8}+\frac{c}{8}+\frac{c}{2}+\frac{c}{4}\right)x_4^2 + 2\left(k_4^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_4||x_5| \\
&\quad + 2k_5^2\overline{d_{13}}|x_1||x_5| + 2k_5^2\overline{d_{17}}x_5^2 - 2k_2^2\frac{u_1\Delta\phi_1(u_1)}{\eta_1} - 2k_5^2\frac{u_2\Delta\phi_2(u_2)}{\eta_2}
\end{aligned}
$$

$$
\begin{aligned}
&\le -2k_1^2\left(\frac{a}{4}+\frac{a}{4}+\frac{a}{2}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_8}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{11}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{14}}\right)|x_2||x_5| - 2k_3^2\left(\frac{b}{8}+\frac{b}{8}+\frac{b}{2}+\frac{b}{4}\right)x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{10}}+k_5^2\overline{d_{15}}\right)|x_3||x_5| - 2k_4^2\left(\frac{c}{8}+\frac{c}{8}+\frac{c}{2}+\frac{c}{4}\right)x_4^2 + 2\left(k_4^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_4||x_5| \\
&\quad + 2k_5^2\overline{d_{13}}|x_1||x_5| + 2k_5^2\overline{d_{17}}x_5^2 - 2k_2^2\frac{r_1u_1^2}{\eta_1} - 2k_5^2\frac{r_2u_2^2}{\eta_2}
\end{aligned}
$$

$$
\begin{aligned}
&= -2k_1^2\left(\frac{a}{4}+\frac{a}{4}+\frac{a}{2}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_8}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{11}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{14}}\right)|x_2||x_5| - 2k_3^2\left(\frac{b}{8}+\frac{b}{8}+\frac{b}{2}+\frac{b}{4}\right)x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{10}}+k_5^2\overline{d_{15}}\right)|x_3||x_5| - 2k_4^2\left(\frac{c}{8}+\frac{c}{8}+\frac{c}{2}+\frac{c}{4}\right)x_4^2 + 2\left(k_4^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_4||x_5| \\
&\quad + 2k_5^2\overline{d_{13}}|x_1||x_5| + 2k_5^2\overline{d_{17}}x_5^2 - 2k_2^2r_1\eta_1x_2^2 - 2k_5^2r_2\eta_2x_5^2
\end{aligned}
$$

$$
\begin{aligned}
&\le -2k_1^2\left(\frac{a}{4}+\frac{a}{4}+\frac{a}{2}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2}+k_5^2\overline{d_{13}}\right)|x_1||x_5| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_8}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{11}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{14}}\right)|x_2||x_5| - 2k_3^2\left(\frac{b}{8}+\frac{b}{8}+\frac{b}{2}+\frac{b}{4}\right)x_3^2 + 2k_3^2\overline{d_9}|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{10}}+k_5^2\overline{d_{15}}\right)|x_3||x_5| - 2k_4^2\left(\frac{c}{8}+\frac{c}{8}+\frac{c}{2}+\frac{c}{4}\right)x_4^2 + 2\left(k_4^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_4||x_5| \\
&\quad + 2k_5^2\overline{d_{17}}x_5^2
\end{aligned}
$$

$$-2k_2^2\left[\delta_1+\frac{\left(\frac{k_1\overline{d_1}}{k_2}+\frac{k_2\overline{d_3}}{k_1}\right)^2}{\underline{a}}+\overline{d_4}+\frac{2\left(\frac{k_2\overline{d_5}}{k_3}+\frac{k_3\overline{d_8}}{k_2}\right)^2}{\underline{b}}+\frac{2\left(\frac{k_2\overline{d_6}}{k_4}+\frac{k_4\overline{d_{11}}}{k_2}\right)^2}{\underline{c}}\right.$$

$$\left.+\frac{\left(\frac{k_2\overline{d_7}}{k_5}+\frac{k_5\overline{d_{14}}}{k_2}\right)}{2}\right]x_2^2-2k_5^2\left[\delta_2+\frac{\left(\frac{k_1\overline{d_2}}{k_5}+\frac{k_5\overline{d_{13}}}{k_1}\right)^2}{\underline{a}}+\frac{2\left(\frac{k_3\overline{d_{10}}}{k_5}+\frac{k_5\overline{d_{15}}}{k_3}\right)^2}{\underline{b}}\right.$$

$$\left.+\frac{2\left(\frac{k_4\overline{d_{12}}}{k_5}+\frac{k_5\overline{d_{16}}}{k_4}\right)^2}{\underline{c}}+\overline{d_{16}}+\frac{\left(\frac{k_2\overline{d_7}}{k_5}+\frac{k_5\overline{d_{14}}}{k_2}\right)}{2}\right]x_5^2$$

$$=-2k_1^2\left(\frac{a}{2}\right)x_1^2-2k_2^2\delta_1x_2^2-2k_3^2\left(\frac{b}{4}\right)x_3^2-2k_4^2\left(\frac{c}{4}\right)x_4^2-2k_5^2\delta_2x_5^2$$

$$-2\left[\frac{\sqrt{\underline{a}}}{2}k_1|x_1|-\frac{\left(\frac{k_1\overline{d_1}}{k_2}+\frac{k_2\overline{d_3}}{k_1}\right)k_2|x_2|}{\sqrt{\underline{a}}}\right]^2$$

$$-2\left[\frac{\sqrt{\underline{b}}}{2\sqrt{2}}k_3|x_3|-\frac{\sqrt{2}\left(\frac{k_2\overline{d_5}}{k_3}+\frac{k_3\overline{d_8}}{k_2}\right)k_2|x_2|}{\sqrt{\underline{b}}}\right]^2$$

$$-2\left[\frac{\sqrt{\underline{c}}}{2\sqrt{2}}k_4|x_4|-\frac{\sqrt{2}\left(\frac{k_2\overline{d_6}}{k_4}+\frac{k_4\overline{d_{11}}}{k_2}\right)k_2|x_2|}{\sqrt{\underline{c}}}\right]^2$$

$$-2\left[\frac{\sqrt{\underline{a}}}{2}k_1|x_1|-\frac{\left(\frac{k_1\overline{d_2}}{k_5}+\frac{k_5\overline{d_{13}}}{k_1}\right)k_5|x_5|}{\sqrt{\underline{a}}}\right]^2$$

$$-2\left[\frac{\sqrt{\underline{b}}}{2\sqrt{2}}k_3|x_3|-\frac{\sqrt{2}\left(\frac{k_3\overline{d_{10}}}{k_5}+\frac{k_5\overline{d_{15}}}{k_3}\right)k_5|x_5|}{\sqrt{\underline{b}}}\right]^2$$

$$-2\left[\frac{\sqrt{c}}{2\sqrt{2}}k_4|x_4|-\frac{\sqrt{2}\left(\frac{k_4\overline{d_{12}}}{k_5}+\frac{k_5\overline{d_{16}}}{k_4}\right)k_5|x_5|}{\sqrt{c}}\right]^2$$

$$-2\left[\frac{\sqrt{b}}{\sqrt{2}}k_3|x_3|-\frac{\sqrt{c}}{\sqrt{2}}k_4|x_4|\right]^2-2\left(\sqrt{\underline{b}\times\underline{c}}-\frac{k_3\overline{d_9}}{k_4}\right)k_3k_4|x_3||x_4|$$

$$-\left(\frac{k_2\overline{d_7}}{k_5}+\frac{k_5\overline{d_{14}}}{k_2}\right)\left(k_2|x_2|-k_5|x_5|\right)^2$$

$$\le -2k_1^2\left(\frac{\underline{a}}{2}\right)x_1^2-2k_2^2\delta_1x_2^2-2k_3^2\left(\frac{\underline{b}}{4}\right)x_3^2-2k_4^2\left(\frac{\underline{c}}{4}\right)x_4^2-2k_5^2\delta_2x_5^2$$

$$\le -2\alpha\left(k_1^2x_1^2+k_2^2x_2^2+k_3^2x_3^2+k_4^2x_4^2+k_5^2x_5^2\right)$$

$$=-2\alpha V,\quad \forall\, t\ge 0.$$

Thus, we have

$$e^{2\alpha t}\cdot\dot{V}+e^{2\alpha t}\cdot 2\alpha\cdot V=\frac{d}{dt}\left[e^{2\alpha t}\cdot V\right]\le 0,\quad \forall\, t\ge 0.$$

It yields that

$$\int_0^t\frac{d}{d\tau}\left[e^{2\alpha\tau}\cdot V(x(t))\right]d\tau=e^{2\alpha t}\cdot V(x(t))-V(x(0))\le\int_0^t 0\,d\tau=0,\quad \forall\, t\ge 0. \tag{8}$$

It is easy to obtain that

$$\left(\min_{1\le i\le 5}k_i\right)^2\|x(t)\|^2\le V(x(t))\le e^{-2\alpha t}V(x(0)),\ \ \forall\, t\ge 0,$$

in view of (7) and (8). Therefore, we conclude that

$$\|x(t)\|\le\frac{\sqrt{V(x(0))}}{\left(\min_{1\le i\le 5}k_i\right)}\cdot e^{-\alpha t},\ \ \forall\, t\ge 0.$$

Thus the proof is completed. □

Remark 1: It is worth mentioning that since the controller mentioned in Theorem 1 is a linear controller, it has the dual advantages of easy hardware implementation and low price.

III. NUMERICAL EXAMPLE

Consider the uncertain nonlinear systems of (1) with

$$f_1=x_2x_3x_4,\quad f_2=-x_1x_3x_4,\quad f_3=x_1x_2x_4,\quad f_4=x_1x_2x_3,\quad f_5=0, \tag{9a}$$

$$\Delta\phi_1(u_1)=\Delta d_{18}u_1+\Delta d_{19}u_1^3,\quad \Delta\phi_2(u_2)=\Delta d_{20}u_2+\Delta d_{21}u_2^5 \tag{9b}$$

$$1\le\Delta d_{18}\le 3,\quad 0\le\Delta d_{19}\le 1,\quad 1\le\Delta d_{20}\le 2,\quad 0\le\Delta d_{21}\le 2, \tag{9c}$$

$$\underline{a}=30,\quad \underline{b}=5,\quad \underline{c}=4.45,\quad \overline{d_i}=1,\quad \forall\, i\in\{2,5,6,7,10,11,12,15,16,17\}, \tag{9d}$$

$$\overline{d_1}=32,\quad \overline{d_3}=\overline{d_4}=10,\quad \overline{d_8}=15.7,\quad \overline{d_9}=2.5,\quad \overline{d_{13}}=\overline{d_{14}}=38.5. \tag{9e}$$

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By properly selecting parameters $r_1 = r_2 = 1$ with (9b) and (9c), (A2) is evidently satisfied. Obviously, by choosing parameters $k_1 = k_3 = k_4 = k_5 = 1$ and $k_2 = \sqrt{3}$, (A3) is evidently satisfied. From (4)-(5) and combining the parameters $\delta_1 = \delta_2 = 1$, one has

$$\frac{\delta_1 + \dfrac{\left(\dfrac{k_1 \overline{d_1}}{k_2} + \dfrac{k_2 \overline{d_3}}{k_1}\right)^2}{a} + \overline{d_4} + \dfrac{2\left(\dfrac{k_2 \overline{d_5}}{k_3} + \dfrac{k_3 \overline{d_8}}{k_2}\right)^2}{b} + \dfrac{2\left(\dfrac{k_2 \overline{d_6}}{k_4} + \dfrac{k_4 \overline{d_{11}}}{k_2}\right)^2}{c} + \dfrac{\left(\dfrac{k_2 \overline{d_7}}{k_5} + \dfrac{k_5 \overline{d_{14}}}{k_2}\right)}{2}}{r_1} = 104.72,$$

$$\frac{\delta_2 + \dfrac{\left(\dfrac{k_1 \overline{d_2}}{k_5} + \dfrac{k_5 \overline{d_{13}}}{k_1}\right)^2}{a} + \dfrac{2\left(\dfrac{k_3 \overline{d_{10}}}{k_5} + \dfrac{k_5 \overline{d_{15}}}{k_3}\right)^2}{b} + \dfrac{2\left(\dfrac{k_4 \overline{d_{12}}}{k_5} + \dfrac{k_5 \overline{d_{16}}}{k_4}\right)^2}{c} + \overline{d_{16}} + \dfrac{\left(\dfrac{k_2 \overline{d_7}}{k_5} + \dfrac{k_5 \overline{d_{14}}}{k_2}\right)}{2}}{r_2} = 68.39.$$

Therefore, by Theorem 1 with $\eta_1 = 105$ and $\eta_2 = 69$, we conclude that the uncertain nonlinear systems (1) with (9) via the linear control

$$u = [-105 x_2 \quad -69 x_5]^T \qquad (10)$$

is globally exponentially stable. In this situation, from (6), the guaranteed exponential convergence rate is calculated as

$$\alpha = \min\left\{\frac{a}{2}, \delta_1, \frac{b}{4}, \frac{c}{4}, \delta_2\right\} = 1.$$

Typical state trajectories of uncontrolled and controlled systems are shown in Figure 1 and 2, respectively. It can be seen from Figure 2 that the uncertain nonlinear systems (1) with (9) can indeed achieve the goal of global exponential stability under the linear controller of (10). Besides, the control signal and the electronic circuit to realize the simple linear control are depicted in Figure 3 and 4, respectively.

IV. CONCLUSION

In this paper, the controller design problem for a class of uncertain fifth-order nonlinear control systems has been explored. Combining the theory of differential and integral inequalities, a simple linear controller has been proposed to promote a class of nonlinear control systems with multiple uncertainties to achieve the goal of global exponential stability. In addition, the guaranteed exponential convergence rate of such uncertain nonlinear systems has been precisely calculated. Finally, some numerical simulation results have also been presented to verify and illustrate the correctness of this main theorem and the design process of the controller.

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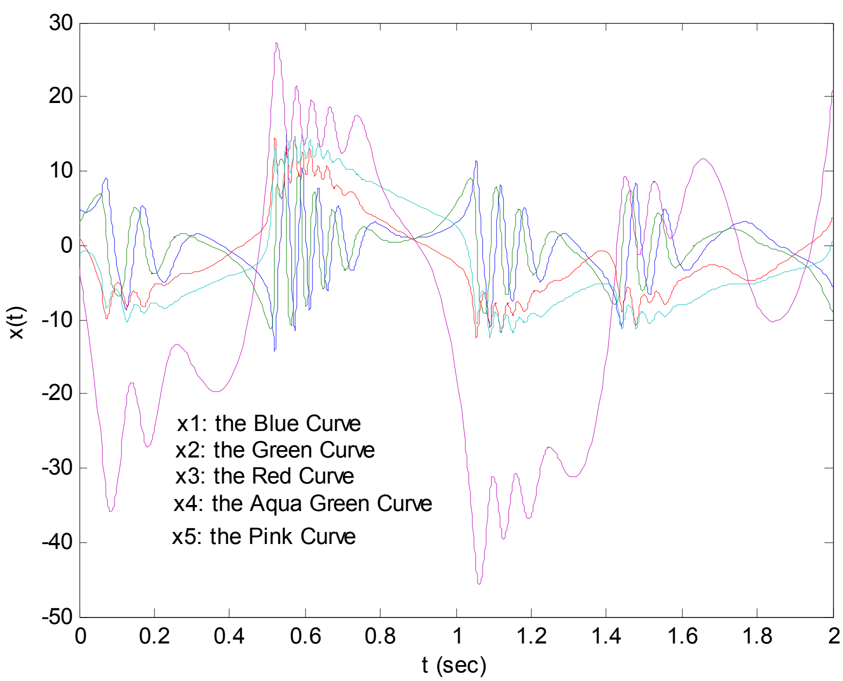

Figure 1: Typical state trajectories of the uncontrolled systems of (1) with (9).

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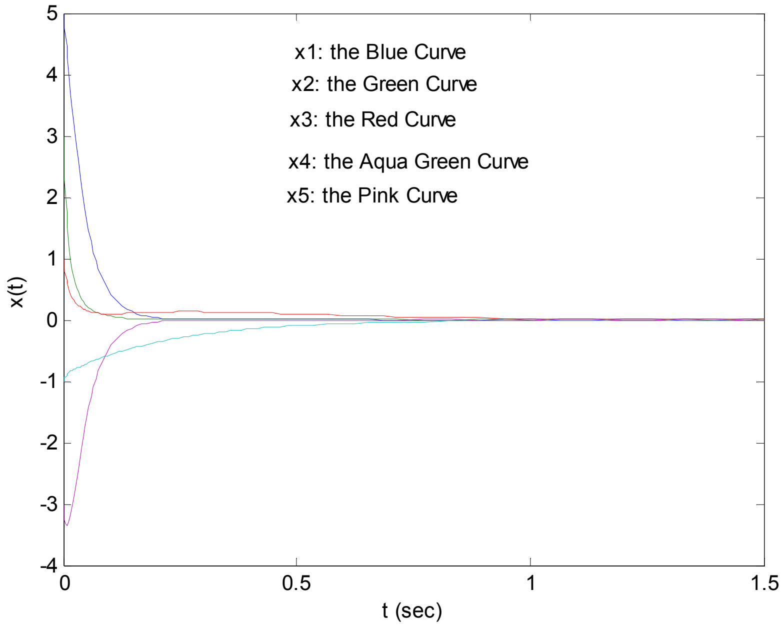

Figure 2: Typical state trajectories of the feedback-controlled systems of (1) with (9) and (10).

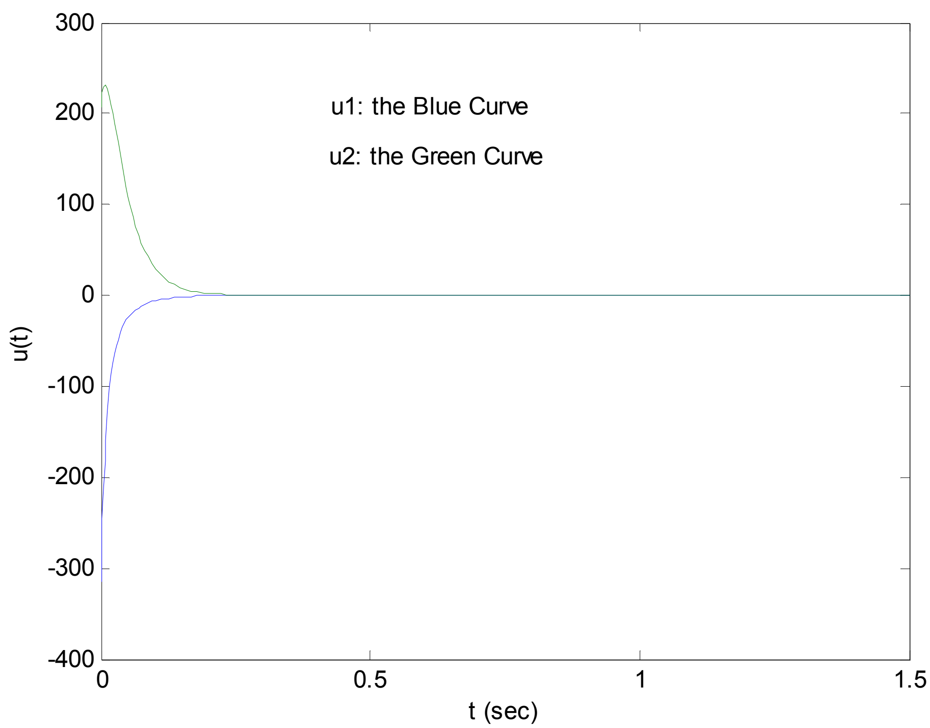

Figure 3: The time response of the control signal of (10).

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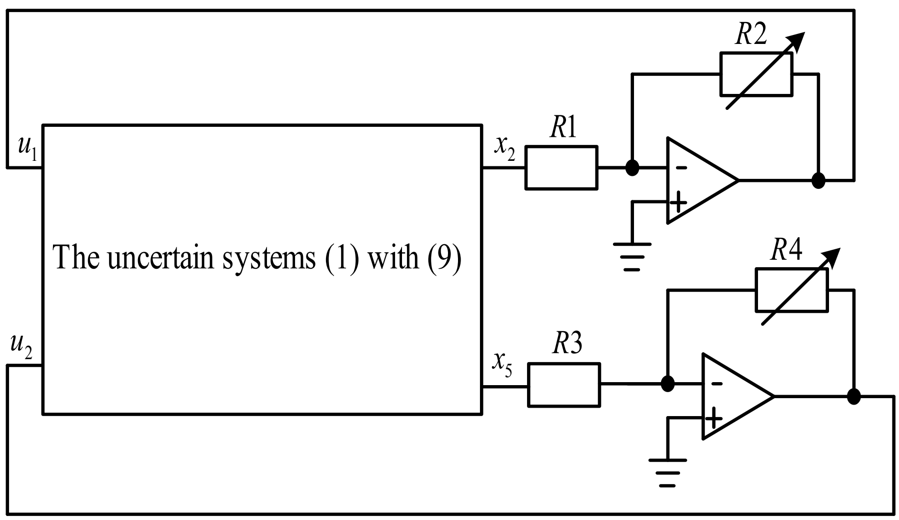

Figure 4: The diagram of implementation of numerical example, where $R1 = 1k\Omega$, $R2 = 105k\Omega$, $R3 = 1k\Omega$, , and $R4 = 69k\Omega$.